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Functional Analysis I

6th problem sheet

Please return your answers in the tutorials on June, 5th / 6th
(together with the answers to the 7th problem sheet)

Problem 1: **5 pt.**

Let $(X, \|\cdot\|)$ be a complex Banach space and let $A \in L(X)$. Show there exists an $M > 0$ such that

$$(A-\lambda)^{-1} \quad \text{exists and} \quad \|(A-\lambda)^{-1}\| \leq \frac{1}{|\lambda| - M}$$

for all $\lambda \in \mathbb{C}$ with $|\lambda| > M$.
Here (and everywhere) $A - \lambda$ always means $A - \lambda I$.

Problem 2: **5 pt.**

Let c_0 be the space of all sequences that converge to 0 and equip c_0 with the $\|\cdot\|_\infty$-norm. Show that the mapping

$$T : l^1(\mathbb{N}) \to (c_0)', \quad (Tx)(y) := \sum_{n=1}^{\infty} x_n y_n, \quad x \in l^1(\mathbb{N}),\ y \in c_0$$

is well defined and an isometric isomorphism between $l^1(\mathbb{N})$ and $(c_0)'$. Hence $l^1(\mathbb{N})$ and $(c_0)'$ are isometric isomorph.

Problem 3: **5 pt.**

Let

$$V : L^2((0,\infty)) \to L^2((0,\infty)), \quad (Vf)(t) := \begin{cases} f(t-1), & \text{if } t \geq 1, \\ 0, & \text{if } 0 < t < 1, \end{cases} \qquad f \in L^2((0,\infty)).$$

Is V an isomorphism? Is V isometric?

Problem 4: **5 pt.**

Solve the integral equation

$$u(t) - \int_0^1 2stu(s)ds = \sin(\pi t), \quad t \in [0,1],\ u \in C([0,1]),$$

with the help of the Neumann series (why does it converge?)