**Solution to the second problem :**
**(ii)⇒(i):** We have to show: $X$ is not strictly convex $\Rightarrow$ there exist linear independent $x, y \in X$ with $\|x+y\| = \|x\| + \|y\|$.

Let $x, y \in X$ be as in $(ii)$: $\|x+y\| = \|x\| + \|y\|$ and $x, y$ are linearly independent. W.l.o.g. we assume $x, y \neq 0$ and $\|x\| \leq \|y\|$. Set $\tilde{x} := \frac{x}{\|x\|}$ and $\|\tilde{y}\| := \frac{y}{\|y\|}$. Both $\tilde{x}$ and $\tilde{y}$ then are normed and $\tilde{x} \neq \tilde{y}$ as $x$ and $y$ are linearly independent. With the help of the inverse triangular inequality we calculate

$$\begin{aligned}
\|\tilde{x} + \tilde{y}\| &= \|\frac{x}{\|x\|} + \frac{y}{\|y\|}\| \\
&= \|\frac{x+y}{\|x\|} - y\big(\frac{1}{\|x\|} - \frac{1}{\|y\|}\big)\| \\
&\geq \frac{\|x+y\|}{\|x\|} - \|y\|\,\big(\frac{1}{\|x\|} - \frac{1}{\|y\|}\big) \\
&= \frac{\|x\| + \|y\|}{\|x\|} - \frac{\|y\|}{\|x\|} + \frac{\|y\|}{\|y\|} \\
&= 2.
\end{aligned}$$

Of course, we always have $\|\tilde{x} + \tilde{y}\| \leq \|\tilde{x}\| + \|\tilde{y}\| = 2$ and hence

$$\|\tilde{x} + \tilde{y}\| = 2.$$