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Integer Linear Programming
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# Assignment 5

### Exercise 1

For $k \in \mathbb{N}$ let $A = \begin{pmatrix} 1 & 0 \\ 1 & k \end{pmatrix}$ and consider the polyhedron $P = \{x \in \mathbb{R}^2 \mid Ax \leq 0\}$.

a) Compute a minimal Hilbert basis for the cone spanned by the rows of $A$.

b) Show that any TDI system $A'x \leq 0$ that describes $P$ has size exponential in the encoding size of $A$.

### Exercise 2

Let $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ such that the system $Ax \leq b$ is TDI. Show that for any $r \in \mathbb{N}$ the system $\frac{1}{r}Ax \leq b$ is also TDI.

### Exercise 3

For $k \in \mathbb{N}$ let $P_k = \text{conv.hull}\{(0,0), (0,1), (k, \frac{1}{2})\}$. Show that $P_k^{(t)} \neq P_I$ for any $t < k$. *Hint: Show that* $(k-1, \frac{1}{2}) \in P_k'$.

### Exercise 4

Let $G = (V, E)$ be a graph. A stable set is a subset $S \subseteq V$ such that every edge $e \in E$ has at most one endpoint in $S$. Observe that for the polyhedron

$$P^G = \{x \in \mathbb{R}^V \mid x_v + x_w \leq 1 \ \forall \{v, w\} \in E,\ x \geq 0\}$$

$P_I^G$ is the convex hull of all incidence vectors of stable sets in $G$. Let $C_5$ be the graph that consists of a cycle of five vertices and $P = P^{C_5}$. Determine

$$\max_{x \in P} \sum_{v \in V} x_v \text{ and } \max_{x \in P'} \sum_{v \in V} x_v.$$

### Exercise 5

Show that the following problem is $NP$-hard: Given a finite set $E$ and three Matroids $(E, \mathcal{F}_1), (E, \mathcal{F}_2), (E, \mathcal{F}_3)$ by an independency oracle, find a set $F \in \mathcal{F}_1 \cap \mathcal{F}_2 \cap \mathcal{F}_3$ of maximum cardinality. *Hint: Use a reduction from Hamiltonian Path.*