Technische Universität Berlin **Sommersemester 2012**
Fakultät II – Mathematik und Naturwissenschaften
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Systems and control theory

Series 5

Task 1: ⊚

Let $P \in \mathbb{C}[\lambda]^{p,q}$ and let $y \in \mathcal{C}^q_\infty$ be in the form $y(t) = e^{\lambda_0 t}\alpha_0$. Show that $P\left(\frac{d}{dt}\right) y(t) = P(\lambda_0)y(t)$.

Task 2: ⊚

Let $P \in \mathbb{C}[\lambda]^{p,q}$, $t_0 < t_1$, and let $S \in \mathbb{C}[\lambda]^{p,p}$ and $T \in \mathbb{C}[\lambda]^{q,q}$ be unimodular matrices.

1. Show that $\mathcal{B}(P)$ is controllable form t_0 to t_1 if and only if $\mathcal{B}(P)$ is controllable from 0 to $t_1 - t_0$.
2. Show that $\mathcal{B}(P)$ is controllable from t_0 to t_1 if and only if $\mathcal{B}(SPT)$ is controllable from t_0 to t_1.

Task 3: ⊖

Determine for each block in the Kronecker canonical form the set of zeros. How can one then specify the complete set of zeros of a pencil $\lambda F + G \in \mathbb{C}[\lambda]_1^{p,q}$ via the Kronecker canonical form?

Task 4: ⊚

Which of the blocks in the Kronecker canonical form are left prime, which are right prime?

Task 5: ⊕

Let $P \in \mathbb{C}[\lambda]^{p,q}$. Show that $\mathcal{B}(P) = \{0\}$ if and only if P is right prime.

Task 6: ⊚

Let $P \in \mathbb{C}[\lambda]^{p,q}$ be such that $\mathcal{B}(P)$ is controllable. Show that in this case there exists a left prime matrix $R \in \mathbb{C}[\lambda]^{r,q}$ such that $\mathcal{B}(R) = \mathcal{B}(P)$.

Task 7: ⊖

For $P \in \mathbb{C}[\lambda]^{p,q}$ we define the so-called *para-conjugate transposed* $P^\sim \in \mathbb{C}[\lambda]^{q,p}$ by $P^\sim(\lambda) := P^*(-\overline{\lambda})$. Show that P is right prime if and only if $P^\sim$ is left prime.

Task 8: ⊚

Let $A \in \mathbb{C}^{n,n}$ and $B \in \mathbb{C}^{n,m}$. Define $F := \begin{bmatrix} I & 0 \end{bmatrix}$ and $G := \begin{bmatrix} -A & -B \end{bmatrix}$. Show that the Kronecker canonical form of $\lambda F + G$ only has blocks of the type $\mathcal{L}$ and $\mathcal{J}$.

Task 9: ⊖

Consider the canonical form in Theorem 6 from the handout "Checking controllability numerically". What is the number of inputs. Also try to make a connection to the Kalman decomposition (no precise answer is expected here).

Task 10: ⊕

For each block in the Kronecker canonical from choose a fixed size (at least 3) and then apply the algorithm of Theorem 6 from the handout "Checking controllability numerically" to each block.

Task 11: ⊚

Show that if $U_1 \in \mathbb{C}[\lambda]^{p_1,q_1}$ and $U_2 \in \mathbb{C}[\lambda]^{p_2,q_2}$ are right prime and $\tilde{U} \in \mathbb{C}[\lambda]^{p_1,q_2}$ is arbitrary, then also

$$U := \begin{bmatrix} U_1 & \tilde{U} \\ 0 & U_2 \end{bmatrix}$$

is right prime. Give multiple proofs in the following ways:

1. Use the the Definition 1.12.
2. Complete U to a unimodular matrix by using the matrices by complete U_1 and U_2 to a unimodular matrix (cf. right prime version of Theorem 1.13 d)).
3. Specify a left inverse via left inverses of U_1 and U_2 (cf. right prime version of Theorem 1.13 e)).

Task 12: ⊕

In Series 3 we considered an electrical circuit with behavior $\mathcal{B}(\lambda F + G)$, where

$$\lambda F + G(\lambda) := \begin{bmatrix} 1 & 1 & 1 & 0 \\ -R & 0 & 0 & 1 \\ 0 & \lambda L & 0 & -1 \end{bmatrix}.$$

Use Lemma 1.9 to determine if the circuit is controllable or not. Then apply the algorithm from the handout "Checking controllability numerically" to compute the canonical form which reveals the controllability.

Task 13: ⊚

In Series 4 we considered two mass-spring-damper systems. The controllability was then analyzed by using the Kalman matrix. In this exercise the controllability shall be analyzed from a behavioral point of view (note that the second system now includes damping, in contrast to the example from Series 4). The task is the following:
Formulate both of the following systems as behavior systems and determine for which choices of the constants they are controllable.

1. With $m, d, k > 0$ being real constants and $q, f \in \mathcal{C}^1_\infty$ the first system is
$$m\ddot{q}(t) + d\dot{q}(t) + kq(t) = f(t).$$
2. With $m_1, m_2, d_1, d_2, k_1, k_2 > 0$ being real constants and $q_1, q_2, f \in \mathcal{C}^1_\infty$ the second system is
$$\begin{aligned} m_1\ddot{q}_1(t) + d_1\dot{q}_1(t) + k_1q_1(t) &= f(t), \\ m_2\ddot{q}_2(t) + d_2\dot{q}_2(t) + k_2q_2(t) &= f(t). \end{aligned}$$
Consider only the so-called *underdamped* case, i.e., the case where $\left(\frac{d_1}{2m_1}\right)^2 < \frac{k_1}{m_1}$, and use that the zeros $\lambda^{(i)}_{1,2}$ of $\lambda^2 m_i + \lambda d_i + k_i$ are given by
$$\lambda^{(i)}_{1,2} = -\frac{d_i}{2m_i} \pm \sqrt{\left(\frac{d_i}{2m_i}\right)^2 - \frac{k_i}{m_i}},$$
for $i = 1, 2$.