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Summer term 2014

Control Theory

3. Exercise
(Discussion on May 26, 2014)

Exercise 3.1: (Kalman decomposition)
Let $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,m}$, $C \in \mathbb{R}^{p,n}$. Show that there exists an orthogonal matrix $V \in \mathbb{R}^{n,n}$ such that

$$V^T A V = \begin{bmatrix} A_{c\bar{o}} & A_{12} & A_{13} & A_{14} \\ 0 & A_{co} & A_{23} & A_{24} \\ 0 & 0 & A_{\overline{co}} & A_{34} \\ 0 & 0 & 0 & A_{\bar{c}o} \end{bmatrix}, \quad V^T B = \begin{bmatrix} B_1 \\ B_2 \\ 0 \\ 0 \end{bmatrix},$$
$$CV = \begin{bmatrix} 0 & C_2 & 0 & C_4 \end{bmatrix}$$

whereby (A_{co}, B_2, C_2) is controllable and observable.

Exercise 3.2: (Stabilization)
Consider the control problem

$$\dot{\varphi}(t) = \omega(t)$$
$$j\dot{\omega}(t) = -r\omega(t) + ku(t)$$

with $k, j, r > 0$. Compute all stabilizing feedback matrices $F \in \mathbb{R}^{1,2}$ for the system.

Exercise 3.3: (Equivalence of LTI-systems)
Consider two equivalent LTI-systems given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \qquad (1) \qquad\qquad \begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \\ \tilde{y} &= \tilde{C}\tilde{x} + \tilde{D}\tilde{u} \end{aligned} \qquad (2)$$

i.e., it holds that

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \begin{bmatrix} P & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P^{-1} & 0 \\ F & Q \end{bmatrix},$$

with $P \in \mathbb{R}^{n \times n}$, $R \in \mathbb{R}^{p \times p}$, $Q \in \mathbb{R}^{m \times m}$ nonsingular, and $F \in \mathbb{R}^{m \times n}$.
Show: If (1) is controllable (stabilizable), then (2) is also controllable (stabilizable).