Technische Universität Berlin
Fakultät II – Institut für Mathematik
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Summer 2017
Hand in solutions: Tue, May 23
Discussion in class: Tue, May 23

Kontrolltheorie – Control theory

3. Exercise

– Stabilizability & Observability –

Each group (max. 2 students) hands in the solution of **the assigned** exercise (max. 3 points).

3.1) Hautus-test for stabilizability (3 points)

Groups: 5, 6, 11, 12, 17, 18

Prove Theorem 2.41:
For a LTI system of the form $\dot{x} = Ax + Bu$ the following statements are equivalent:

i) The system is stabilizable.

ii) If $A^T z = \lambda z$ for $z \neq 0$ and $\Re(\lambda) \geq 0$, then $z^T B \neq 0$.

iii) $\operatorname{rank} \begin{bmatrix} \lambda I - A & B \end{bmatrix} = n$ for all $\lambda \in \mathbb{C}$ with $\Re(\lambda) \geq 0$. (Hautus-test)

iv) In the corresponding Kalman decomposition we have $\Lambda(A_3) \subseteq \mathbb{C}^-$.

Hint: Show i) ⇒ ii) ⇒ iv) ⇒ i) and ii) ⇔ iii).

3.2) examples (3 points)

Groups: 1, 2, 7, 8, 13, 14, 19

a) Which of the control problems $\dot{x} = Ax + B_i u$, $i \in \{1, 2\}$, with matrices

$$A = \begin{bmatrix} -1 & 0 & & \\ 0 & -2 & & \\ & & 0 & 2 \\ & & 2 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

are stabilizable? In the case of stabilizability, is the system also controllable?

b) parabolic antenna
We consider once more the parabolic antenna from exercise 1.3, i.e., the control problem $\dot{x} = Ax + Bu$ with matrices

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{r}{j} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \frac{k}{j} \end{bmatrix}.$$

The parameters satisfy $k, j, r > 0$. The task is to find all stabilizing feedback matrices $F \in \mathbb{R}^{1,2}$ such that $\Lambda(A - BF) \subseteq \mathbb{C}^-$.

3.3) Kalman decomposition – controllability and observability (3 points)

Groups: 3, 4, 9, 10, 15, 16,

Prove Corollary 2.55:
Given matrices $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,m}$, and $C \in \mathbb{R}^{p,n}$ there exists an orthogonal matrix $V \in \mathbb{R}^{n,n}$ such that

$$V^T A V = \begin{bmatrix} A_{c\bar{o}} & A_{12} & A_{13} & A_{14} \\ 0 & A_{co} & A_{23} & A_{24} \\ 0 & 0 & A_{\bar{c}\bar{o}} & A_{34} \\ 0 & 0 & 0 & A_{\bar{c}o} \end{bmatrix}, \qquad V^T B = \begin{bmatrix} B_1 \\ B_2 \\ 0 \\ 0 \end{bmatrix}, \qquad CV = \begin{bmatrix} 0 & C_2 & 0 & C_4 \end{bmatrix},$$

where $(A_{\mathrm{co}}, B_2, C_2)$ is controllable and observable.