Technische Universität Berlin
Fakultät II – Institut für Mathematik
R. Altmann

Summer 2017
Hand in solutions: Tue, Jun 13
Discussion in class: Tue, Jun 13

Kontrolltheorie – Control theory

4. Exercise

– Lyapunov equations –

Each group (max. 2 students) hands in the solution of **the assigned** exercise (max. 3 points).

4.1) feedback equivalence for LTI systems (3 points)

Groups: 3, 6, 9, 12, 15, 18

a) Prove Theorem 2.61:
Let a LTI system be controllable (stabilizable). Then, also every *equivalent* system is controllable (stabilizable).

b) Prove Theorem 2.62:
Let a LTI system be observable (reconstructable, detectable, asymptotically stable). Then, also every *strongly equivalent* system is observable (reconstructable, detectable, asymptotically stable).

4.2) Kronecker product and vectorization (3 points)

Groups: 2, 5, 8, 11, 14, 17

a) Prove (parts of) Lemma 3.2:
For corresponding dimensions the Kronecker product satisfies:

vi) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$,

vii) $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$,

viii) $\Lambda(A \otimes B) = \{\lambda_i \mu_j \mid \lambda_i \in \Lambda(A), \mu_j \in \Lambda(B)\}$,

ix) $\det(A \otimes B) = (\det A)^m (\det B)^n$ für $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{m,m}$,

x) $\operatorname{Rang}(A \otimes B) = \operatorname{Rang} A \cdot \operatorname{Rang} B$.

b) Prove (parts of) Lemma 3.4:
For matrices A, B, X of corresponding dimensionens, we have:

ii) $\operatorname{vec}(AXB) = (B^T \otimes A) \operatorname{vec}(X)$,

v) $\|(I \otimes A) \operatorname{vec}(I)\|_2 = \|(A \otimes I) \operatorname{vec}(I)\|_2 = \|A\|_F$,

vi) $\|A \otimes B\|_2 = \|A\|_2 \|B\|_2$.

4.3) condition number of a Lyapunov equation (3 points)

Groups: 1, 4, 7, 10, 13, 16, 19

Prove Lemma 3.16:
Let X be the solution of $AX + XA^T = W$ and $\tilde{X}$ the solution of the perturbed Lyapunov equation $\tilde{A}\tilde{X} + \tilde{X}\tilde{A}^T = \tilde{W}$ with $\|\tilde{A} - A\|_2 \le \varepsilon \|A\|_2$ and $\|\tilde{W} - W\|_F \le \varepsilon \|W\|_F$. Is $\varepsilon\kappa < 1$, where κ denotes the condition number of the Lyapunov equation, then

$$\frac{\|\tilde{X} - X\|_F}{\|X\|_F} \le \frac{2\varepsilon\kappa}{1 - \varepsilon\kappa}.$$