Technische Universität Berlin
Fakultät II – Institut für Mathematik
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Summer 2017
Hand in solutions: Wed, June 28
Discussion in class: Wed, June 28

Kontrolltheorie – Control theory

5. Exercise

– Partial stabilization, pole placement –

Each group (max. 2 students) hands in the solution of **the assigned** exercise (max. 3 points).

5.1) Sign iteration and sign projection (3 points)

Groups: 1-6

a) Prove Theorem 3.24:
Let M be invertible. Then, the iteration

$$M_{(i+1)} = \frac{1}{2}\Big(M_{(i)} + M_{(i)}^{-1}\Big), \qquad M_{(0)} = M$$

converges to $\operatorname{sgn}(M)$.

Hint: You should use Lemmata 3.25 and 3.26.

b) Prove that the matrix

$$P := \frac{1}{2}(I - \operatorname{sgn} A)$$

is a projection to the A-invariant subspace w.r.t. the eigenvalues in $\mathbb{C}^-$.

5.2) examples (3 points)

Groups: 7-12

Prove Lemma 3.39
Let the pair (A, B) be controllable and in *controllable canonical form* ($m = 1$). Further, let $F \in \mathbb{R}^{1,n}$ be a feedback matrix with $\Lambda(A - BF) = \mathcal{P} = \bar{\mathcal{P}}$. Then,

i) Is $\lambda \in \mathcal{P} \cap \Lambda(A)$, then every eigenvector of A w.r.t. λ is also an eigenvectorof $A - BF$ w.r.t. λ.

ii) Is $\lambda \in \mathcal{P} \setminus \Lambda(A)$, then $x := (A - \lambda I)^{-1}B$ is an eigenvector of $A - BF$ w.r.t. λ and satisfies $Fx = 1$.

Hint: You are allowed to use the Ackermann formula from exercise 5.3).

5.3) Ackermann formula (3 points)

Groups: 13-19

We consider an alternative way to compute a feedback matrix F. Let (A, B) be controllable, $m = 1$, and $\mathcal{P} = \bar{\mathcal{P}} = \{\mu_1, \dots, \mu_n\}$ the set of desired poles of $A - BF$. Let T be the transformation matrix, which brings (A, B) into *controllable canonical form*, i.e.,

$$T^{-1}AT = C_1(A), \qquad T^{-1}B = e_n.$$

a) Show $e_1^T T^{-1} K(A, B) = e_n^T$.

b) Prove Theorem 3.46
Let Ψ be the polynom, which corresponds to $\mathcal{P}$, i.e,

$$\Psi(x) := \Pi_{k=1}^n (x - \mu_k) = (x - \mu_1) \cdots (x - \mu_n).$$

Then, the feedback matrix $F := e_n^T K(A, B)^{-1} \Psi(A)$ satisfies $\Lambda(A - BF) = \mathcal{P}$.