Technische Universität Berlin

Institut für Mathematik
Wintersemester 07/08

Linear and Integer Programming (ADM II)

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Exercise sheet 12

Deadline: Wed, 23 Jan 2008, 12:15 in MA 042

Exercise 62. **8 points**

Among the population of a village there are n unmarried men and m unmarried women. The *marriage problem* is to arrange as many marriages among them as possible. Since the village is a bit conservative, marriages are allowed only between partners of different sexes and also every person may be married only once. Additionally, every couple should (at least initially) like each other, where sympathy relations are given as a set of pairs $(a, b) \in A \times B$ with A and B the sets of men and women, respectively.

Show that a maximal number of marriages can be found by solving a maximum flow problem (cf. Exercise 38).

Exercise 63. **7 points**

Consider the affine scaling algorithm under the assumptions as given in the lecture. Let $\mathbf{x}^*$ be the optimal solution and assume that $x_1, \ldots, x_m$ are the basic variables at $\mathbf{x}^*$. Suppose that $|x_i^k - x_i^*|/x_i^k \leq \gamma$ for $i = 1, \ldots, m$, where γ is a positive constant.

(a) Show that

$$\mathbf{c}'\mathbf{x}^k - \mathbf{c}'\mathbf{x}^* \;\leq\; \frac{1}{\beta}\left(\mathbf{c}'\mathbf{x}^k - \mathbf{c}'\mathbf{x}^{k+1}\right)\left(n - m + m\gamma^2\right)^{1/2}$$

(b) Show that

$$\mathbf{c}'\mathbf{x}^{k+1} - \mathbf{c}'\mathbf{x}^* \;\leq\; \left(1 - \frac{\beta}{(n - m + m\gamma^2)^{1/2}}\right)\left(\mathbf{c}'\mathbf{x}^k - \mathbf{c}'\mathbf{x}^*\right)$$

Exercise 64. **7 points**

Consider the linear programming problem

$$\begin{array}{rl} \text{minimize} & \mathbf{c}'\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

Assume that the rows of $\mathbf{A}$ are linearly independent and that the non-degeneracy assumptions ((v) and (vi) in the lecture) hold. Let $\mathbf{x}$ be some feasible solution and suppose that we have a vector $\mathbf{p}$ such that

$$x_i(c_i - \mathbf{A}_i'\mathbf{p}) = 0 \qquad \forall i.$$

Show that $\mathbf{x}$ is a basic feasible solution.

Exercise 65. **(Tutorial session)**

Suppose an interior point method, applied to a linear program

$$\begin{array}{rl} \text{minimize} & \mathbf{c}^\top\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

terminates with an ε-optimal pair of primal/dual solutions $(\mathbf{x}^*, \mathbf{p}^*)$. Describe how an optimal solution to the linear program can be found in polynomial time. How large should ε be chosen to guarantee the correctness of this method?