

Example: Berlin airlift

- four quarters of year, cargo to be transported in each:
100, 150, 150, 200 units of cargo, where
1 unit $\hat{=}$ 1 plane carriage unit $\hat{=}$ 14 t
- each airplane needs 3 people to operate
- initially 110 planes \rightarrow 330 crew members
- 1 crew member, not operating planes, can train
19 new crew members in 1 quarter
- crew operating in 1 quarter get leave in the next
- 20% get lost one the way back
- Cost: new plane: 200 MU
op./idle crew member: 7 MU
new — " — : 10 MU
resting — " — : 5 MU

- VARIABLES: p_i : # of new planes built in quarter i
($i=1, \dots, 4$) \bar{p}_i : # of idle planes — " —
 m_i : # of new crew members — " —
 \bar{m}_i : # of idle — " —

\bar{m}_i : # of resting _____
 c_i : # of cargo units to be transported in quarter i

• OBJECTIVE:

minimize $200 \sum_{i=1}^4 p_i + 7 \sum_{i=1}^4 \bar{m}_i + 10 \sum_{i=1}^4 m_i + 5 \sum_{i=1}^4 \tilde{m}_i$

• CONSTRAINTS:

$c_1 = 100$
 $c_2 = 150$
 $c_3 = 150$
 $c_4 = 200$

(cargo that has to be transported)

$c_1 + \bar{p}_1 = 110$

$c_2 + \bar{p}_2 = p_1 + \bar{p}_1 + 0.8 c_1$

(planes available/needed)

$\begin{matrix} 3 & \vdots & 3 & \vdots & 2 & 2 & \vdots & 2 \\ 4 & \vdots & 4 & \vdots & 3 & 3 & \vdots & 3 \end{matrix}$

$3c_1 + \bar{m}_1 + \frac{1}{20} m_1 = 330$

$\tilde{m}_1 = 0$

(crew available/needed)

$3c_2 + \bar{m}_2 + \frac{1}{20} m_2 = m_1 + \bar{m}_1$

$\tilde{m}_2 = 0.8 \cdot 3 \cdot c_1 = 2.4 c_1$

$3c_3 + \bar{m}_3 + \frac{1}{20} m_3 = m_2 + \bar{m}_2 + \tilde{m}_2$

$\begin{matrix} \vdots & \vdots & \vdots \end{matrix}$

• LP: (matrix)

	c_1	c_2	c_3	c_4	\bar{P}_1	\bar{P}_2	\bar{P}_3	\bar{P}_4	P_1	P_2	P_3	P_4	\bar{m}_1	\bar{m}_2	\bar{m}_3	\bar{m}_4	m_1	m_2	m_3	m_4	\tilde{u}_1	\tilde{u}_2	\tilde{u}_3	\tilde{u}_4
	1																							
		1																						
			1																					
				1																				
	-0.8	1			-1	1			-1															
		-0.8	1			-1	1			-1														
			-0.8	1			-1	1			-1													
3	3												0.05											
		3											-1	0.05										
			3										-1		0.05									
				3											-1	0.05								
2.4																								
		2.4																						
			2.4																					

(right hand side)

$$b = \begin{pmatrix} 100 \\ 150 \\ 150 \\ 200 \\ \hline 110 \\ 0 \\ 0 \\ 0 \\ \hline 330 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A \cdot x = b, \quad x \geq 0$$

↑
feasible set