Example: Berlin airlift

- four quarters of year, cargo to be transported in each:
  100, 150, 150, 200 units of cargo, where
  1 unit ≙ 1 plane carriage unit ≙ 14 t
- each airplane needs 3 people to operate
- initially 110 planes ⟶ 330 crew members
- 1 crew member, not operating planes, can train
  19 new crew members in 1 quarter
- crew operating in 1 quarter get leave in the next
- 20% get lost one the way back
- cost: new plane : 200 MU
  op./idle crew member: 7 MU
  new — " — : 10 MU
  resting — " — : 5 MU

- VARIABLES: p_i : # of new planes built in quarter i
  $(i = 1, \dots, 4)$ $\bar{p}_i$: # of idle planes — " —
  m_i : # of new crew members — " —
  $\bar{m}_i$: # of idle — " —

$\tilde{m}_i$: # of resting ——— " ———

(c_i : # of cargo units to be transported in quarter i)

- OBJECTIVE:

minimize $\quad 200 \sum_{i=1}^{4} p_i + 7 \sum_{i=1}^{4} \bar{m}_i + 10 \sum_{i=1}^{4} m_i + 5 \sum_{i=1}^{4} \tilde{m}_i$

- CONSTRAINTS:

$c_1 = 100$
$c_2 = 150$
$c_3 = 150$
$c_4 = 200$

(cargo that <u>has</u> to be transported)

$$c_1 + \bar{p}_1 = 110$$

$$c_2 + \bar{p}_2 = p_1 + \bar{p}_1 + 0.8\, c_1$$

(planes available/needed)

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$3 \qquad 3 \qquad 2 \qquad 2 \qquad 2$$

$$4 \qquad 4 \qquad 3 \qquad 3 \qquad 3$$

$$3c_1 + \bar{m}_1 + \frac{1}{20} m_1 = 330$$

(crew available/needed)

$$\tilde{m}_1 = 0$$

$$3c_2 + \bar{m}_2 + \frac{1}{20} m_2 = m_1 + \bar{m}_1$$

$$\tilde{m}_2 = 0.8 \cdot 3 \cdot c_1 = 2.4\, c_1$$

$$3c_3 + \bar{m}_3 + \frac{1}{20} m_3 = m_2 + \bar{m}_2 + \tilde{m}_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

- LP: (matrix)

$$
A = \left(\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc}
c_1 & c_2 & c_3 & c_4 & \bar{p}_1 & \bar{p}_2 & \bar{p}_3 & \bar{p}_4 & p_1 & p_2 & p_3 & p_4 & \bar{m}_1 & \bar{m}_2 & \bar{m}_3 & \bar{m}_4 & m_1 & m_2 & m_3 & m_4 & \tilde{m}_1 & \tilde{m}_2 & \tilde{m}_3 & \tilde{m}_4 \\
1 & & & & & & & & & & & & & & & & & & & & & & & \\
 & 1 & & & & & & & & & & & & & & & & & & & & & & \\
 & & 1 & & & & & & & & & & & & & & & & & & & & & \\
 & & & 1 & & & & & & & & & & & & & & & & & & & & \\
\hline
1 & & & & & & & & & & & & & & & & & & & & & & & \\
-0.8 & 1 & & & -1 & 1 & & & -1 & & & & & & & & & & & & & & & \\
 & -0.8 & 1 & & & -1 & 1 & & & -1 & & & & & & & & & & & & & & \\
 & & -0.8 & 1 & & & -1 & 1 & & & -1 & & & & & & & & & & & & & \\
\hline
3 & & & & & & & & & & & & 1 & & & & 0.05 & & & & & & & \\
 & 3 & & & & & & & & & & & -1 & 1 & & & -1 & 0.05 & & & & & & \\
 & & 3 & & & & & & & & & & & -1 & 1 & & & -1 & 0.05 & & & -1 & & \\
 & & & 3 & & & & & & & & & & & -1 & 1 & & & -1 & 0.05 & & & -1 & \\
\hline
2.4 & & & & & & & & & & & & & & & & & & & & & -1 & & \\
 & 2.4 & & & & & & & & & & & & & & & & & & & & & -1 & \\
 & & 2.4 & & & & & & & & & & & & & & & & & & & & & -1
\end{array}\right)
$$

(right hand side)

$$
b = \left(\begin{array}{c}
100 \\ 150 \\ 150 \\ 200 \\ \hline
110 \\ 0 \\ 0 \\ 0 \\ \hline
330 \\ 0 \\ 0 \\ 0 \\ \hline
0 \\ 0 \\ 0 \\ 0
\end{array}\right)
$$

$$A \cdot x = b \quad , \; x \geq 0$$

$\uparrow$

feasible set