

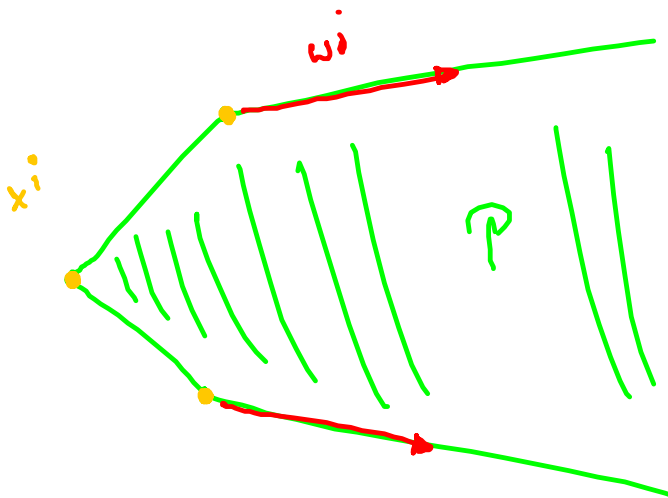
# Resolution Theorem

$\mathcal{P} = \{x \in \mathbb{R}^n \mid Ax \geq b\}$  with at least one extreme point

Then  $\mathcal{P} = \left\{ \sum \lambda_i x^i + \sum \theta_j w^j \mid \theta_j, \lambda_i \geq 0, \sum \lambda_i = 1 \right\}$

where  $x^i$  are the extreme points

and  $w^j$  are the extreme rays of  $\mathcal{P}$



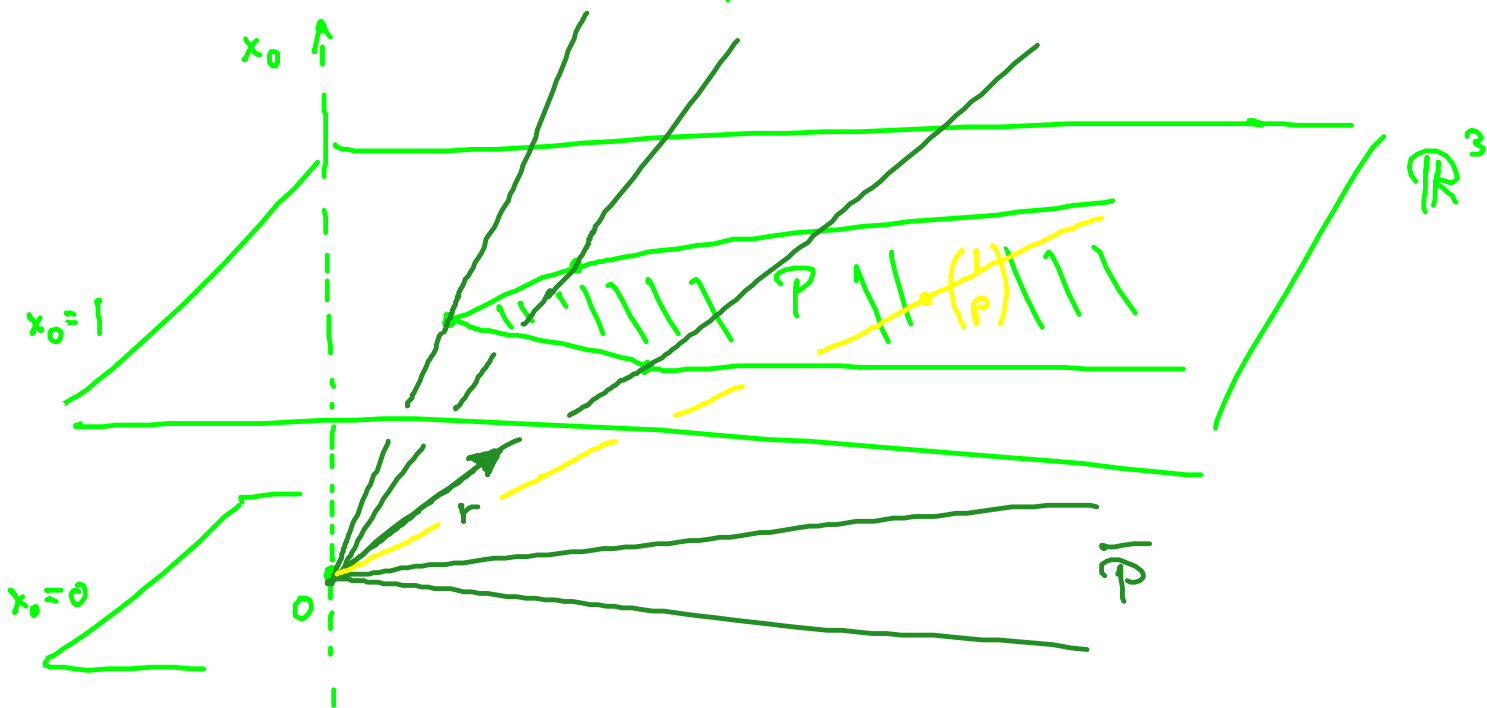
## ► Lifting

(offine) inequality  $a^T x \geq b_i$  (i-th row)  
"  $(a_1, \dots, a_n)$

→ (linear) inequality  $-b_i x_0 + a_1 x_1 + \dots + a_n x_n \geq 0$

$\bar{\mathcal{P}} := \left\{ x = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^{n+1} \mid \begin{pmatrix} -b_i & A \end{pmatrix} x \geq 0, x_0 \geq 0 \right\}$

$$P = \bar{P} \cap \{x \in \mathbb{R}^{u+1} \mid x_0 = 1\}$$



$$\text{ray in } \mathbb{R}^{u+1} : r = \left\{ \lambda \cdot \begin{pmatrix} r_0 \\ r_1 \\ \vdots \\ r_u \end{pmatrix} \mid \lambda \geq 0 \right\}$$

$$\text{if } r_0 \neq 0 \longrightarrow \text{normalise } r = \left\{ \lambda \cdot \begin{pmatrix} 1 \\ r_1 \\ \vdots \\ r_u \end{pmatrix} \right\}$$

extreme rays of  $\bar{P}$  give extreme  $\left\{ \begin{array}{l} \text{points} \quad \text{if } r_0 = 1 \\ \text{rays} \quad \text{if } r_0 = 0 \end{array} \right\}$  of  $P$

$$p \in P : \begin{pmatrix} 1 \\ p \end{pmatrix} = \sum \mu_i r^i = \sum \mu_i \begin{pmatrix} 1 \\ r_1 \\ \vdots \\ r_u \end{pmatrix} + \sum \mu_i \begin{pmatrix} 0 \\ r_1 \\ \vdots \\ r_u \end{pmatrix}$$

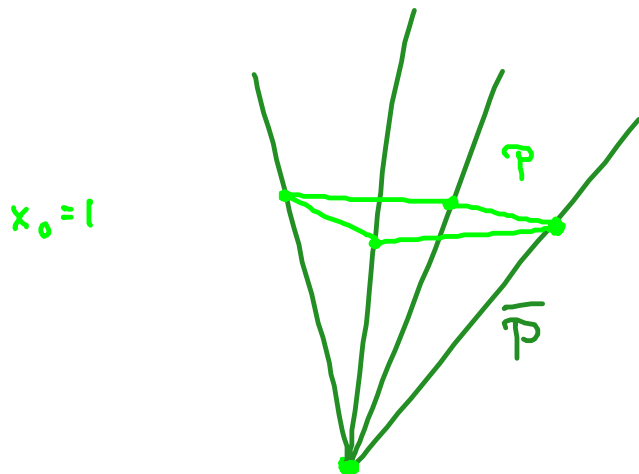
↑  
extreme rays of  $\bar{P}$

$$\Leftrightarrow p = \sum \lambda_i x^i + \sum \theta_i w^i$$

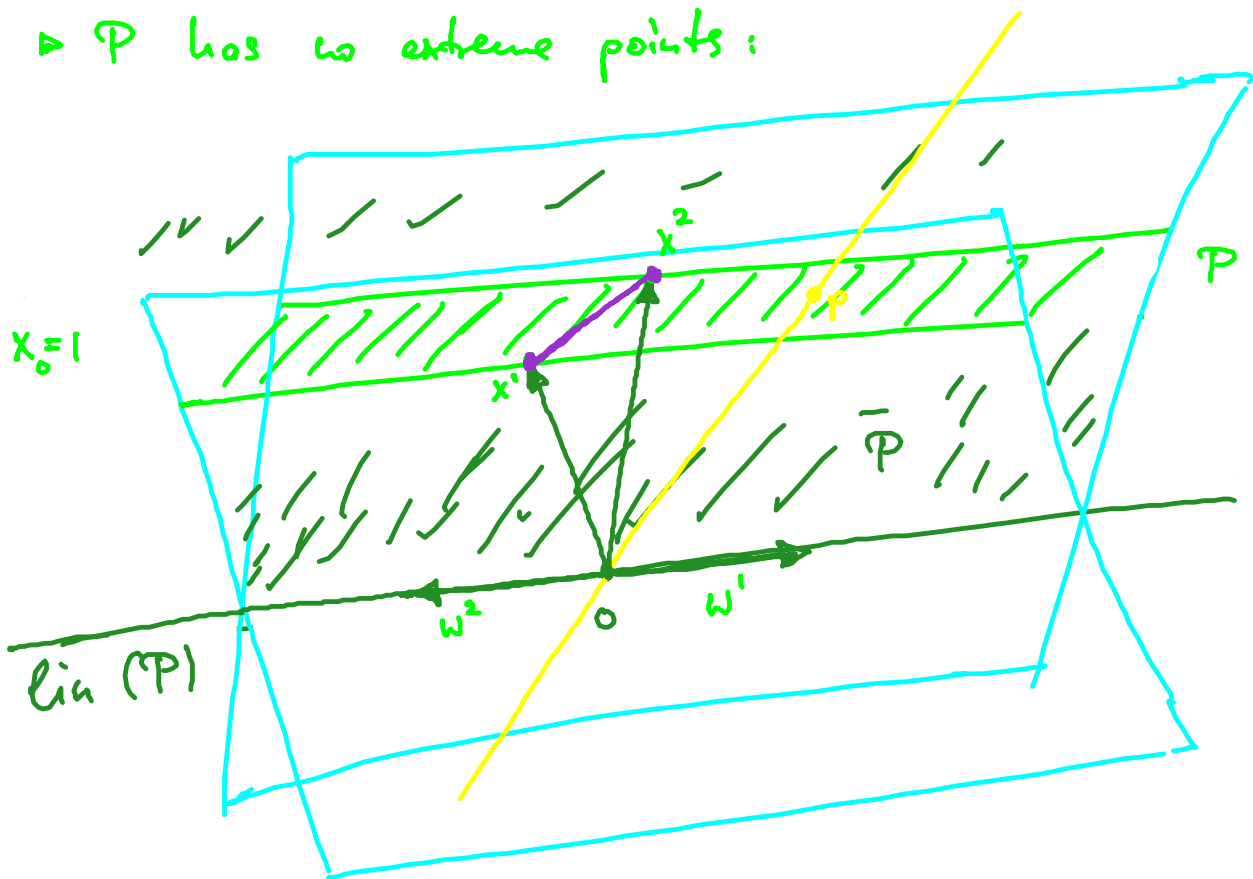
(res. thm.)

$$1 = \sum \lambda_i$$

► P has no extreme rays:



► P has no extreme points:



$$\begin{pmatrix} 1 \\ p \end{pmatrix} = \sum \mu_i \begin{pmatrix} 1 \\ r_i \end{pmatrix} = \sum \lambda_i \begin{pmatrix} 1 \\ x_i \end{pmatrix} + \sum \theta_i \begin{pmatrix} 0 \\ w_i \end{pmatrix}$$

$$\rightarrow p = \sum \lambda_i x^i + \sum \theta_i w^i$$

$$1 = \sum \lambda_i$$

$\rightarrow P$  can be written as the sum of a pointed polyhedron and a subspace  $\text{lin}(P)$

$$P = \underbrace{\text{lin}(P)} + (P \cap \text{lin}(P)^\perp)$$

$$= \langle l_1, \dots, l_k \rangle$$

where  $l_1, \dots, l_k$  are the lines contained in  $P$

... projective space:  $\mathbb{R}P^h$

$\mathbb{F}_2 P^h$

$\mathbb{R}P^h := \{ \{ \lambda r \mid \lambda \in \mathbb{R} \} \mid r \in \mathbb{R}^{h+1} \setminus \{0\} \}$  the set of all

1-dimensional subspaces of  $\mathbb{R}^{h+1}$  ("points")

"lines" in  $\mathbb{R}P^h$  are the 2-dim. subspaces of  $\mathbb{R}^{h+1}$