Maintain tree structure (T, L, U)

T: tree edges, L: lower, U: upper

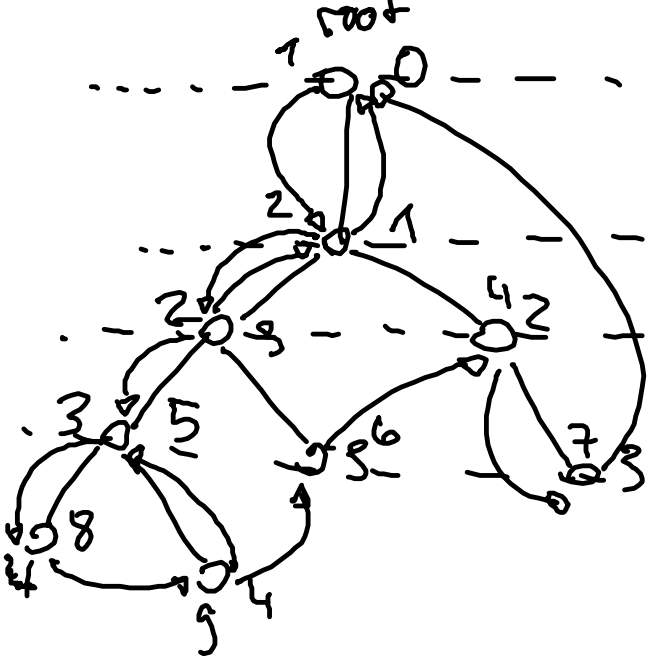
(1) identify a cycle
↳ pivot cycle

(2) compute and update
x flow and p dual values

(3) calculate θ and determine
the leaving arc

Saving the tree by indices at the nodes.

- pred
- depth
- thread



depth first search

i	1	2	3	4	...
pred(i)	-	1	2	2	
depth(i)	0	1	2	2	
thread(i)	2	3	5	7	

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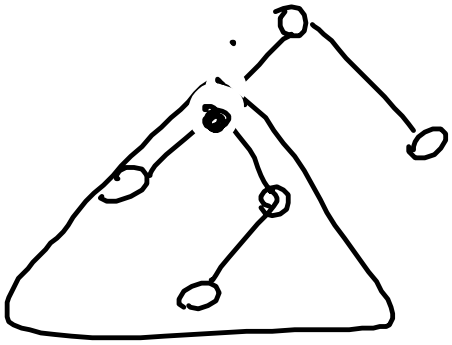


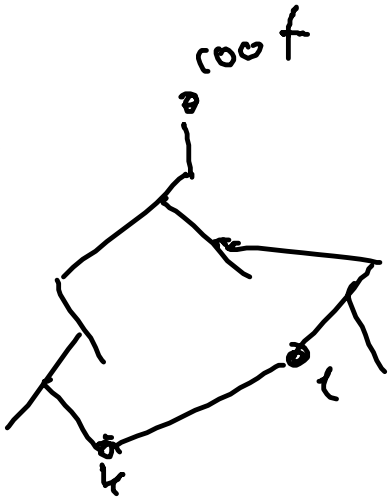
We can visit all nodes of a subtree without visiting the others by following the thread index.

Given the tree structure
compute the dual node potentials

```
begin
    p(1) := 0;
    j := thread(1);
    while j ≠ 1 do
```

```
begin
    i := pred(j);
    IF (i,j) ∈ T THEN
        p(j) := p(i) - c_ij;
    IF (j,i) ∈ T THEN
        p(j) := p(i) + c_ij;
    j := thread(j);
END;
END;
```

Given the entering edge (k,l)
We want to identify the pivot cycle



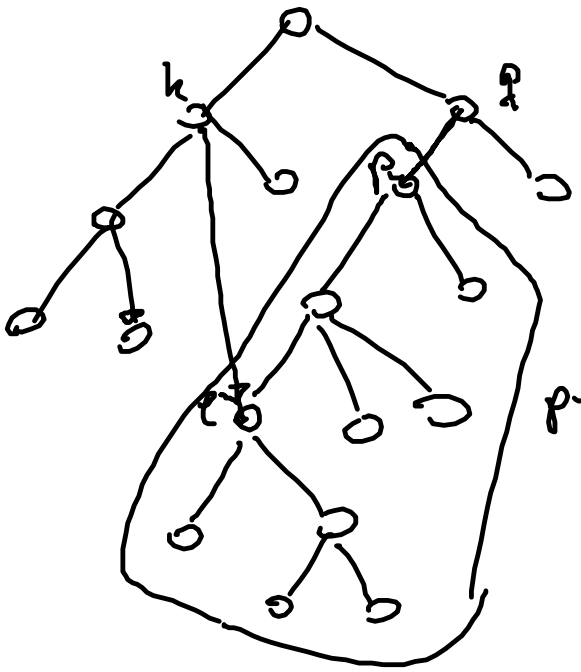

```
begin
    i := k;
    j := l;
    while i ≠ j do
    begin
        if depth(i) > depth(j) then
            i := pred(i);
        else if depth(j) > depth(i) then
            j := pred(j);
        else  i := pred(i);
              j := pred(j);
    end;
end;
```

We uses this method twice with two different side effects:

FIRST : Compute ϵ and leaving edge

SECOND : Update x flow values

FIRST update the node potentials



(p,q) leaving arc

(k,ℓ) entering arc

$p(i) - p(j) = c_{ij} \quad \forall (i,j) \in T$

p-values will change

$$p(\ell)_{new} = p(k) - c_{k\ell}$$

$$p(\ell)_{new} - p(\ell)_{old} = p(k) - c_{k\ell} - p(\ell)_{old}$$

$$= -(c_{k\ell} - p(k) + p(\ell)_{old})$$

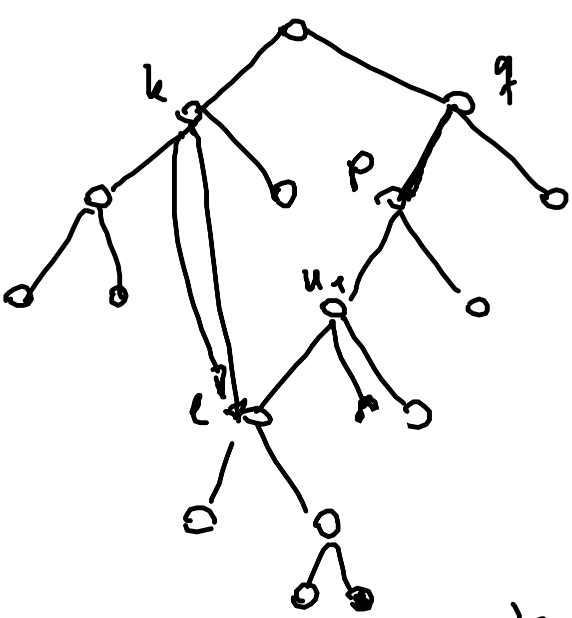
$$= -\bar{c}_{k\ell}$$

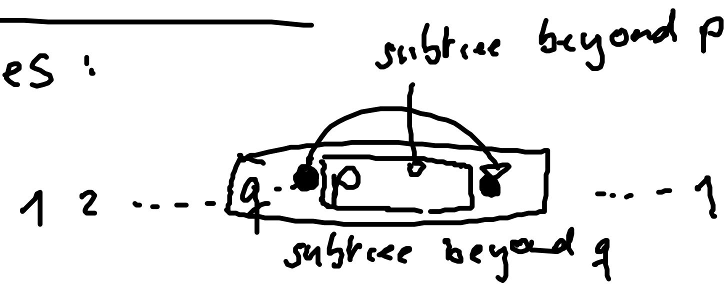
IF ℓ lies in the blue area all p-value decrease by $\bar{c}_{k\ell}$.

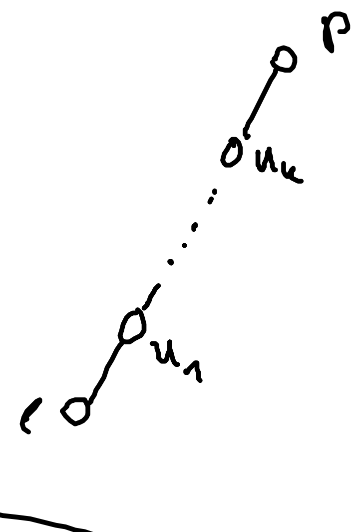
IF k lies in the blue area all p-value increase by $\bar{c}_{k\ell}$.

```
begin
  y := p;
  IF l lies in blue area THEN change := - c̄_kl ;
  IF k lies      "        THEN change := c̄_kl ;
  p(y) := p(y) + change ;
  y := thread (y);
  while depth(y) > depth(p) do
   begin
       p(y) := p(y) + change ;
       y := thread(y) ;
   end ;
end ;
```

Update the thread indices:

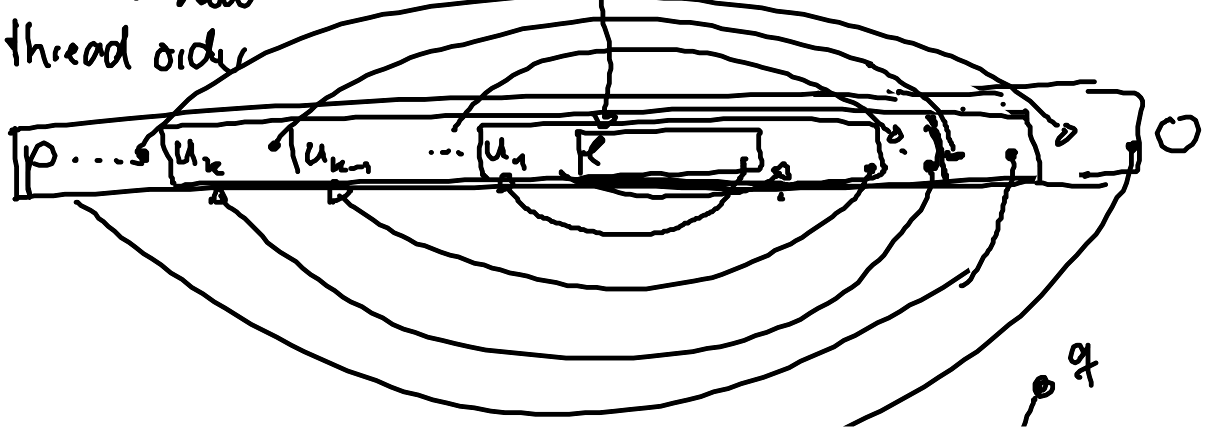


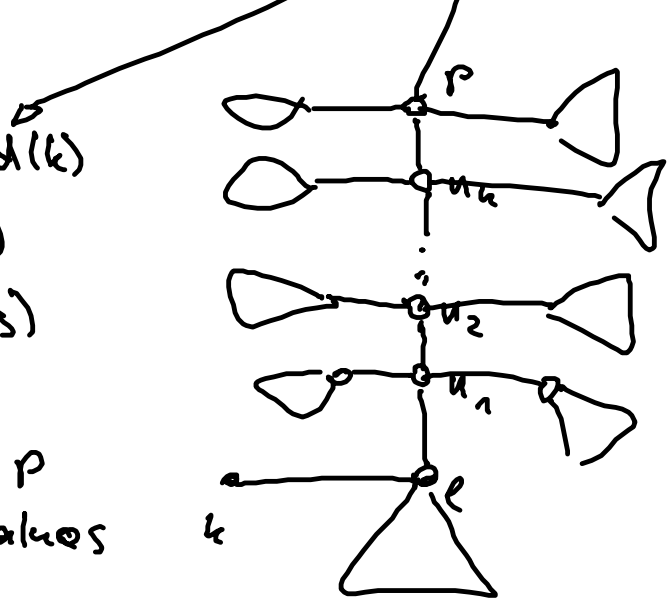




$u_1, \ldots, u_k$ are the inner nodes of the l-p-Path in tree T.

→ new thread order





1) store k, ℓ, p, thread(k)
2) identify (by pred-indices) and store $u_1, \dots, u_k$
3) Walk over subtree beyond p using the old thread-values and update them

thread-indices are updated correctly.

pred-indices are easy to update:

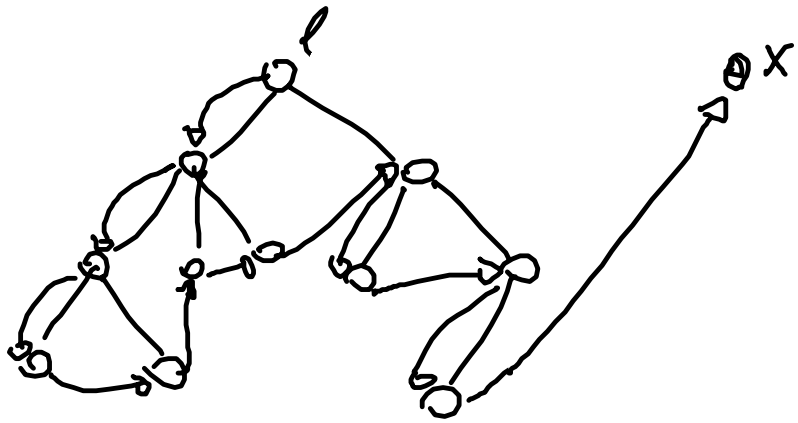
One has just to invert the ℓ–p-path

Do this as a side effect when identifying $u_1, \dots, u_k$.

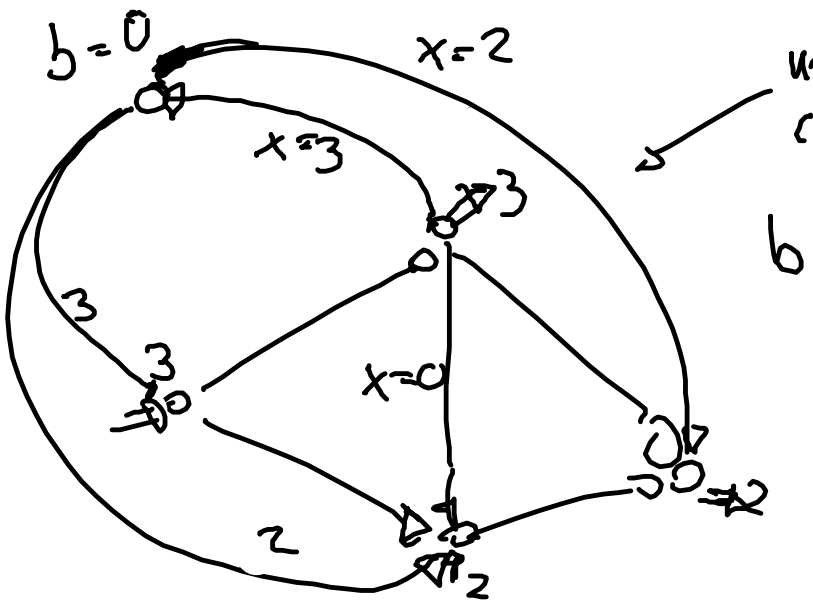
Pred-indices are updated correctly

depth-indices:

```
begin
   y := l;
   depth(y) := depth(h) + 1;
   y := thread(y);
   while y ≠ x do
   begin
        depth(y) := depth(pred(y)) + 1;
        y := thread(y);
   end;
end;
```



Depth-indices are updated correctly.



upper capacities = ∞ $(= M)$
cost = ∞ $\left(= \tilde{M} = \sum_{c_{ij}>0} c_{ij} + 1\right)$

b

Apply network simplex.

If one artificial arc is used in optimal solution the original problem is infeasible.