Proof:

① Suppose that rows $1,\dots,m$ are lex. positive. Suppose that x_j enters the basis and $x_{B(\ell)}$ exits the basis. By construction $u_\ell > 0$ $\quad (u := B^{-1} \cdot A_j)$ and

$$\frac{\ell\text{-th row}}{u_\ell} \overset{L}{<} \frac{i\text{-th row}}{u_i} \quad \text{for } i \neq \ell \text{ with } u_i > 0$$

Notice that

$$(\text{new } \ell\text{-th row}) = \frac{1}{u_\ell} \cdot (\text{old } \ell\text{-th row}) \overset{L}{>} 0$$

and for $i \neq \ell$

$$(\text{new } i\text{th row}) = (\text{old } i\text{-th row}) - \frac{u_i}{u_\ell} (\text{old } \ell\text{-th row})$$

Case 1: $u_i \leq 0 \Rightarrow -\frac{u_i}{u_\ell} \geq 0 \Rightarrow$ new i-th row is the sum of a lexicogr. positive row and a lex. pos. or zero row ✓

Case 2: $u_i > 0 \Rightarrow \frac{1}{u_i} \cdot (\text{old } i\text{-th row}) \overset{L}{>} \frac{1}{u_\ell} (\text{old } \ell\text{-th row})$

$$\Rightarrow \text{old } i\text{-th row} - \frac{u_i}{u_\ell} \cdot (\text{old } \ell\text{-th row}) \overset{L}{>} 0$$

② $\bar{c}_j < 0$ and

$$(\text{new } 0\text{-th row}) = (\text{old } 0\text{-th row}) - \underbrace{\underbrace{\frac{\bar{c}_j}{u_\ell}}_{>0} \cdot \underbrace{(\text{old } \ell\text{-th row})}_{\overset{L}{>} 0}}_{\overset{L}{>} 0}$$

③ The zeroth row is uniquely determined by the current basis. As a consequence of ② the same basis cannot occur more than once. □