

Exercise I

Problem 1

Define Turing machines for the following problems and state their worst case running times.

- a) Given a natural number in unary encoding, increase it by 1.
- b) Given a natural number in binary encoding, decrease it by 1.

Problem 2

We want to consider the problem of deciding whether an undirected simple graph is Eulerian. Recall that this is the case iff the graph is connected and all vertices have even degree.

- a) Define a suitable input encoding for a graph.
- b) Think of a Turing machine M_0 that checks if all vertices have even degree and give a rough outline of its transition function. What problems do you encounter?
- c) What is the worst case running time of 'your' M_0 in \mathcal{O} -notation?
- d) Suppose you are given M_0 , a TM that checks the degree of the vertices, and M_1 a TM that checks (on the same input) whether a graph is connected. How can these be combined to a TM M that decides the language of all Eulerian graphs?

Problem 3

- a) Under what conditions does a Turing machine run forever? Come up with three different examples!
- b) Can you think of an easy characterization of such TMs?