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Complexity theory and fixed parameter algorithms
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Exercise X

Problem 1

Consider the following decision problems and show that these problems are in NP ∩ coNP. What could be the difference between them?

a) Given an undirected bipartite graph G, does G have a perfect matching? Hint: One can use the "Marriage Theorem".

b) Given $n, k \in \mathbb{N}$ (in binary encoding), is there an $m \in \{2, \ldots, k\}$ such that $m|n$? (I.e., does n have a factor of size at most k?) Hint: Use PRIMES ∈ P.

Problem 2

The Completeness Theorem states "$\Delta \models \Phi \iff \Delta \vdash \Phi$". In particular, all valid sentences can be proven. However, the length of such a proof is not considered.

Recall the satisfiability problem SAT: Given a Boolean expression Φ in conjunctive normal form, is there a truth assignment satisfying Φ?

We want to show: There is no polynomial p such that every valid sentence with n letters has a proof with at most $p(n)$ letters, unless SAT ∈ NP ∩ coNP. (Note that this would imply NP = coNP, because SAT is NP-complete.)

Your task: Find a set of axioms and a first-order logic sentence for an instance of SAT such that the sentence is a valid consequence of the given axioms if Φ is satisfiable. If Φ is unsatisfiable the sentence should also be unsatisfiable for the given axioms. (It might be convenient to choose a system of axioms that allows only models with exactly two elements.) Then consider a non-deterministic Turing machine that "guesses" proofs for this axiomatic system. What effect does it have on the complexity of SAT if there are polynomial-size proofs for all valid sentences?