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Complexity theory and fixed parameter algorithms
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Exercise XI

Problem 1

Consider the following scheduling problem: Given n jobs with processing times p_i, due dates d_i and penalties w_i, $1 \leq i \leq n$, the cost of a schedule is the sum of the w_i over all jobs i that finished after their due dates. The objective is to find a schedule that minimizes this sum (using only one machine and jobs cannot be preempted).

Give a dynamic programming algorithm that runs in $\mathcal{O}^*(2^n)$.

Problem 2

The *Hamiltonian Path Problem* asks whether an undirected graph G has a path containing all vertices. We want to develop an algorithm that computes the number of Hamiltonian Paths in G and, in particular, this algorithm can decide the Hamiltonian Path Problem.
Let $G = (V, E)$ with $V = \{1, \ldots, n\}$. Assume that a Hamiltonian path (if it exists) starts in vertex 1 and ends in vertex n.
A *walk* in G consists of vertices $v_1, \ldots, v_k$ such that there is an edge from $\{v_i, v_{i+1}\} \in E$ for all $1 \leq i \leq k - 1$. Note that vertices may be visited many times in a walk.
For $S \subseteq V$, let $W(S)$ denote the set of walks in G of length n, starting in 1 and ending in n, that do not visit any vertex in S.

i) Show that for any $S \subseteq V$, $|W(S)|$ can be computed in polynomial time using exponentiation of the adjacency matrix of G.

ii) Show that the number of Hamiltonian paths starting at 1 and ending in n is

$$|W(\emptyset)| - \left| \bigcup_{k=2}^{n-1} W(\{k\}) \right|.$$

iii) Show that this expression is the same as

$$\sum_{S \subseteq V} (-1)^{|S|} |W(S)|.$$

iv) Give an algorithm that computes the number of Hamiltonian paths in G using time $\mathcal{O}^*(2^n)$ and polynomial space.