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Exercise II

Problem 1

Show the following inclusions:

a) $\text{TIME}(g) \subseteq \text{SPACE}(\mathcal{O}(g))$ for all functions $g : \mathbb{N} \to \mathbb{N}$.

b) $\text{LOGSPACE} \subseteq \text{P}$.

c) $\text{NPSPACE} \subseteq \text{PSPACE}$.

Problem 2

Languages that can be decided by a 1-string input/output TM are called "regular languages".

a) Recall the restrictions imposed on a 1-string input/output TM.

b) Show that $L = \{x \in \{0,1\}^* : x \text{ contains at least two 0s but not two consecutive 0s}\}$ is regular.

c) Show that for any infinite regular language there exist $x, y, z \in \Sigma^*$ such that $y \neq \epsilon$ and $xy^iz \in L \; \forall i \geq 0$.

d) Show that $L = \{\text{bin}(1)\text{bin}(2)\ldots\text{bin}(n) : n \geq 1\} = \{1, 110, 11011, 11011100, \ldots\}$ is not regular.

Note: It can be shown that $\text{SPACE}(\mathcal{O}(1))$ is exactly the set of regular languages.

Problem 3

Show that a k-string nondeterministic TM can be emulated by a 2-string nondeterministic TM with the same alphabet and the same running time (up to a constant factor).
Hint: Use the 2nd string to guess the behavior of Δ, then check all k strings independently.