TU Berlin
Department of Mathematics
Prof. Dr. Martin Skutella
Daniel Dressler

Complexity theory and fixed parameter algorithms
Winter term 2008

Exercise IV

Problem 1
Try to find the shortest possible Boolean expressions for each of the following Boolean expressions.

i) $x \lor \neg x$

ii) $(y \Rightarrow x) \lor x$

iii) $((x \Rightarrow y) \Rightarrow (y \Rightarrow z)) \Rightarrow (x \Rightarrow z)$

Problem 2
Rewrite

$$(x_1 \lor y_1) \land (x_2 \lor y_2) \land \ldots \land (x_n \lor y_n)$$

in disjunctive normal form.

Problem 3
Let $\mathcal{T} \subseteq \{\text{true}, \text{false}\}^n$ be a set of truth assignments. Then there is a Boolean expression Φ such that $\mathcal{T}$ is exactly the set of truth assignments satisfying Φ. We want to investigate under what conditions on $\mathcal{T}$ we can express Φ as a conjunction of Horn clauses.

Some notation: $T := T_1 \land T_2$ is the truth assignment $T(x) := T_1(x) \land T_2(x)$. And we define a partial order $\leq$ with $T_1 \leq T_2$ if $T_1(x) = \text{true}$ implies $T_2(x) = \text{true}$.

Now show the equivalence of the following three statements:

i) $\mathcal{T}$ is the set of truth assignments satisfying a conjunction of Horn clauses.

ii) If $T_1, T_2 \in \mathcal{T}$, then $T_1 \land T_2 \in \mathcal{T}$.

iii) If $T \notin \mathcal{T}$ (more precisely $T \in \{\text{true}, \text{false}\}^n \setminus \mathcal{T}$), then $S := \{T' \in \mathcal{T} : T \leq T'\}$ is either empty or there exists a unique $T^* \in S$ such that $T^* \leq T' \; \forall T' \in S$.