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Complexity theory and fixed parameter algorithms
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Exercise VII

Problem 1

Consider the group theory vocabulary. The following are the group axioms:

i) $\forall x \forall y \forall z ((x \circ y) \circ z \equiv x \circ (y \circ z))$

ii) $\forall x (x \circ 1 \equiv x)$

iii) $\forall x \exists y (x \circ y \equiv 1)$

Show that no proper subset of these axioms implies another of the axioms, that is, the axioms are "independent".

Problem 2

Prove theorem 17 and 18 from the lecture:

i) Theorem 17: Suppose that $\Delta \cup \{\phi\} \vdash \psi$, then $\Delta \vdash \underline{\phi \Rightarrow \psi}$.
Hint: Use induction on the first-order logic proof used for $\Delta \cup \{\phi\} \vdash \psi$.

ii) Theorem 18: If $\Delta \cup \{\neg\phi\}$ is inconsistent, then $\Delta \vdash \phi$.
Hint: This is a corollary from theorem 17.

Problem 3

Gödel's completeness theorem says that if a set Δ of axioms is consistent, then it has a model. Use this to prove the following "compactness theorem for first-order logic":
If all finite subsets of a set of sentences Δ are satisfiable, then Δ is satisfiable.