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Complexity theory and fixed parameter algorithms
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Exercise VIII

Problem 1
Formulate the graph theoretic problem REACHABILITY in second-order logic.
Hint: Use a similar construction as for the Hamiltonian path problem.

Problem 2
Consider the following axioms that are valid for the natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$ with the usual addition and $\sigma(x) = x + 1$.

(NT1) $\forall x(\sigma(x) \neq 0)$

(NT2) $\forall x \forall y(\sigma(x) = \sigma(y) \Longrightarrow x = y)$

(NT3) $\forall x(x = 0 \lor \exists y \sigma(y) = x)$

(NT4) $\forall x(x + 0 = x)$

(NT5) $\forall x \forall y(x + \sigma(y) = \sigma(x + y))$

Your task is to give a model satisfying these axioms ...

a) ... with a universe that is a strict superset of $\mathbb{N}$.

b) ... that does not satisfy $\forall x \forall y(x + y = y + x)$.