ADM III: Linear and convex optimization in game theory,
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Technische Universität Berlin
Institut für Mathematik
Dr. Britta Peis, Dr. Tobias Harks

Assignment 2

Discussion: Friday, November 29, 2008.

Exercise 1. Let Γ_a and Γ_b be two zero-sum matrix games which are defined by the following two payoff matrices of the row player (i.e., the first player).

a)

$$\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$$

Determine the optimal maxmin-strategy for the first player in the corresponding randomized matrix games.

Exercise 2. Let $A = (a_{ij}) \in \mathbb{R}^{m \times n}$ be the payoff matrix of the row player in a zero-sum matrix game. We say that

- row i_1 *dominates* row i_2 if $a_{i_1 j} \geq a_{i_2 j}$ holds for all columns j, and
- column j_1 *dominates* column j_2 if $a_{ij_1} \leq a_{ij_2}$ holds for all rows i.

a) Show that in a randomized matrix game, dominated rows and columns can be ignored when optimal maxmin-strategies are to be calculated.

b) Determine an optimal maxmin-strategy for the row player in the randomized zero-sum matrix game with payoff matrix

$$\begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 2 & 0 & 2 \\ 0 & 3 & 4 & 4 \\ 1 & 3 & 5 & 4 \end{bmatrix}.$$

Exercise 3. Consider the facility location game illustrated in Figure 1.

a) Calculate an allocation vector $x \in \mathbb{R}^3$ in the core.

b) Extend the graph by adding a new facility such that the core of the modified game has an empty core.

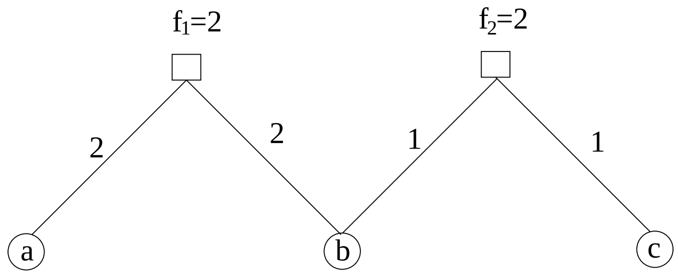

Figure 1: Facility location game with two facilities and three players $N = \{a, b, c\}$.

Exercise 4. Let (N, c) be a cooperative cost game with $c(\emptyset) = 0$. We define the corresponding *dual payoff game* by

$$v(S) := c(N) - c(N \setminus S) \quad \forall S \subseteq N.$$

Show that both games have the same core.