Technische Universität Berlin
Institut für Mathematik
Dr. Britta Peis, Dr. Tobias Harks

Assignment 3

Discussion: Tuesday, December 16, 2008.

Exercise 1. Let $\Gamma = (N, v)$ be a cooperative payoff game with $v : 2^N \to \mathbb{R}_+$. If v is not superadditive, a coalition might achieve a better payoff if it splits into disjoint subsets. Thus, it makes sense to consider the extended game $\tilde{\Gamma} = (N, \tilde{v})$ with

$$\tilde{v}(S) = \max\{\sum_i v(S_i) \mid S_i \subseteq S \text{ and } S_i \cap S_j = \emptyset \text{ if } i \neq j\} \quad \forall S \subseteq N.$$

a) Show that $\tilde{v}$ is monotone increasing and superadditive.

b) Show that if $v(N) = \tilde{v}(N)$, then $\text{core}(v) = \text{core}(\tilde{v})$.

Exercise 2. Consider the game $\Gamma = (N, v)$ on three players $N = \{1, 2, 3\}$ whose payoff function is defined by $v(\{\}) = v(\{1\}) = v(\{2\}) = v(\{3\}) = 0, v(\{1, 2\}) = v(\{2, 3\}) = 1, v(\{1, 3\}) = 2$ and $v(\{1, 2, 3\}) = 4$. Determine all marginal vectors and the core of Γ.

Exercise 3. Let $N = \{1, 2, 3\}$ and $v : 2^N \to \mathbb{R}_+$ be a payoff function with values

$$v(S) = \left\{ \begin{array}{ll} 0 & \text{if } |S| \leq 1 \\ 60 & \text{if } |S| = 2 \\ 72 & \text{if } |S| = 3. \end{array} \right\}$$

Determine the excesses with respect to the full allocations $x = (30, 30, 12)^T$ and $x' = (24, 24, 24)^T$. Is $l(x)$ lexicographically smaller than $l(x')$? Determine the nucleolus of $\Gamma(N, v)$.

Exercise 4. Consider a parliament with three parties $\{P_1, P_2, P_3\}$, where the parties P_1 and P_2 own 10 seats each, while party P_3 owns 21 seats. A majority of seats is necessary for a coalition to form the regime. How much power has each party? I.e., what is the Shapley value for the game in which each coalition has value 1 if it is able to form the regime, and zero otherwise?