Lecturer: Prof. Dr. J.-D. Deuschel
Assistent: Simon Wasserroth
hand out: Wed 21.10.2010
due: Wed 28.10.2010

Assignment 1
„Wahrscheinlichkeitstheorie 2 - Stochastic processes 1“

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total points: 20 Points

Problem 1 **5 Points**

Let $\Omega = \{1, 2, 3, 4\}$ and $\Gamma = \{\emptyset, \Omega, \{1, 2\}, \{1, 3\}\}$. What is $\sigma(\Gamma)$? Find two probability measures μ, ν such that they do agree on Γ but not on $\sigma(\Gamma)$.

Problem 2 **5 Points**

For all $n \in \mathbb{N}$, let the sets $\mathcal{A}_n$ be σ-algebras generated by the sets $\mathcal{E}_n := \{\{1\}, \{2\}, \dots, \{n\}\}$ in $\Omega := \mathbb{N}$.

(i) Prove that $\mathcal{A}_n$ consists of all sets $A \subseteq \mathbb{N}$ such that: Either $A \subset \{1, \dots, n\}$ or $m \in A$ for all $m \geq n + 1$.

(ii) Show that $\cup_{n=1}^{\infty} \mathcal{A}_n$ is not a σ-algebra.

Problem 3 **5 Points**

A distribution function $F : \mathbb{R} \to [0, 1]$ is a function that is non-decreasing, right continuous and satisfies $\lim_{x\to-\infty} F(x) = 0$ and $\lim_{x\to\infty} F(x) = 1$.
Prove that the map $\mu \mapsto \mu((-\infty, x]) = F(x)$ is a bijection from the space of all probability measures on $\mathbb{R}$ onto the set of all distribution functions.

Problem 4 **5 Points**

For some algebra (or σ-algebra) $\mathcal{A}$ a nonempty set A is called atom of $\mathcal{A}$ if $A \in \mathcal{A}$ and for all $C \in \mathcal{A}$ either $A \subseteq C$ or $A \cap C = \emptyset$.
Let X be some set and $A_1, \dots, A_n$ subsets of X for $n \in \mathbb{N}$. Let $\mathcal{A}$ be the smallest algebra containing all A_i for $1 \leq i \leq n$. Take $\mathcal{I}$ to be the collection of all intersections $\bigcap_{i=1}^{n} B_i$ with $B_i = A_i$ or $B_i = A_i^c$. Show that $\mathcal{I} \setminus \{\emptyset\}$ is the set of all atoms.
Give an example for an algebra $\mathcal{A}$ which is not generated by the set of all atoms.

Reminder: Hand in your solution in groups of at most three students.