Technische Universität Berlin **WS 2010/2011**
Fakultät II – Institut f. Mathematik
Lecturer: Prof. Dr. J.-D. Deuschel hand out: Wed 02.02.2011
Assistant: Simon Wasserroth due: Wed 09.02.2011

Assignment 14
„Wahrscheinlichkeitstheorie 2 - Stochastic processes 1“

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total points: 20 Points

Problem 1 **6 Points**

(i) Let (S,d) be a metric space and $A \subseteq S$. Show that $\{\delta_a : a \in A\}$ is tight iff A is relatively compact.

(ii) Let $f, f_1, f_2, \ldots : [0,1] \to \mathbb{R}$ be continuous functions. Show that $\delta_{f_n} \circ \pi^{-1}_{t_1,\ldots,t_m} \overset{w}{\to} \delta_f \circ \pi^{-1}_{t_1,\ldots,t_m}$ for all $m \in \mathbb{N}$ and $0 \leq t_1 < t_2 < \ldots < t_m \leq 1$ iff f_n converges pointwise to f.

(iii) Find an example for f, f_n as above, such that the finite dimensional distributions of δ_{f_n} converges to the finite dimensional distributions of δ_f but $\{\delta_{f_n} : n \in \mathbb{N}\}$ is not tight.

Problem 2 **5 Points**

Let $(X, \|\cdot\|)$ be an infinite dimensional Banach space. Show that the closed unit ball $B = \{x \in X : \|x\| \leq 1\}$ is not compact. You might assume that X is even a Hilbert space with an inner product.

Problem 3 **9 Points**

Program a path generator for paths of Brownian motion W_t, use any computer language you prefer (Java, C++, Matlab,...). Simulate a path and plot a graph. Then approximate the following quantities using Monte Carlo simulations, i.e. run a lot of simulations and average the value. $\mathbb{E}[W_1]$,$\mathbb{E}[|W_1|]$,$\mathbb{E}\left[W^2_{\frac{1}{2}}\right]$, $\mathbb{E}\left[e^{W_1^2}\right]$. Comment on the values you get.
Hand in the graph, the approximations and the well-documented source code.