Technische Universität Berlin **WS 2010/2011**
Fakultät II – Institut f. Mathematik
Lecturer: Prof. Dr. J.-D. Deuschel hand out: Wed 10.11.2010
Assistant: Simon Wasserroth due: Wed 17.11.2010

Assignment 4
„Wahrscheinlichkeitstheorie 2 - Stochastic processes 1“

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total points: 20 Points

Problem 1 **5 Points**
Let X and Y be random variables taking values in the interval $[a, b]$ with $\mathbb{E}[X^n] = \mathbb{E}[Y^n]$ for all $n \in \mathbb{N}$. Thus, all the moments of X and Y agree. Show that X and Y are identically distributed. You can use that the characteristic function determines the distribution uniquely.

Problem 2 **5 Points**
Let $X_n : \Omega \to \mathbb{R}$ be a sequence of non negative integrable random variables that converges in probability to some integrable random variable $X : \Omega \to \mathbb{R}$. Furthermore $\mathbb{E}[X_n] \to \mathbb{E}[X]$.
Show that then $X_n \to X$ in L_1, i.e. $\mathbb{E}[|X_n - X|] \to 0$.

Problem 3 **5 Points**
For the following problems find an example or prove that it is not possible. Let the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ be given.

(i) $X : \Omega \to \mathbb{R}$ is a finite random variable with infinite expectation.

(ii) $X : \Omega \to \bar{\mathbb{R}}$ has finite expectation but takes the value infinity on a set of positive measure.

(iii) A random variable has a finite first but infinite second moment.

(iv) A random variable has a finite second but infinite first moment.

Problem 4 **5 Points**
Let X_n be a sequence of random variables that are standard normally distributed. Assume that the X_n converge in L^1 to some random variable X. Show that also X is standard normally distributed.

Reminder: Hand in your solution in groups of two students.