Technische Universität Berlin **WS 2010/2011**
Fakultät II – Institut f. Mathematik
Lecturer: Prof. Dr. J.-D. Deuschel hand out: Wed 17.11.2010
Assistant: Simon Wasserroth due: Wed 24.11.2010

Assignment 5
„Wahrscheinlichkeitstheorie 2 - Stochastic processes 1“

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total points: 20 Points

Problem 1 **4 Points**
Let $(X_n)_n$ be a sequence of random variables in $\mathbb{R}$ which converges in probability to $x \in \mathbb{R}$. Let $\phi : \mathbb{R} \to \mathbb{R}$ be a bounded function with $\phi(x) = 0$ that is in x continuous. Show that $\lim_{n\to\infty} \mathbb{E}\left[\phi(X_n)\right] = 0$. Does it also hold if we do not assume that ϕ is bounded?

Problem 2 **4 Points**
Let X be a random variable in $\mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ a measurable function. Show that X and $f \circ X$ are independent if and only if $f \circ X$ is constant almost surely.

Problem 3 **7 Points**
Let $(X_n)_n$ be a sequence of random variables on $\mathbb{R}$ defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Find out whether the following events are in the terminal σ-algebra defined as $\bigcap_{m=1}^{\infty} \sigma\left(\bigcup_{n=m}^{\infty} \sigma(X_n)\right)$.

(i) $\left\{\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} X_i \text{ exists}\right\}$

(ii) $\{X_n = 0 \text{ for all } n \geq 10\}$

(iii) $\{\text{The series } \sum_{i=1}^{\infty} X_i \text{ converges}\}$

(iv) $\{\text{The series } \sum_{i=1}^{\infty} X_i \text{ converges and the limit is greater that } 1\}$

(v) $\{\sum_{i=1}^{n} X_i = 1 \text{ infinitely often}\}$

(vi) $\{\sum_{i=1}^{n} X_i = 0 \text{ for some } n\}$

(vii) $\left\{\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} X_i = 0\right\}$

Problem 4 **5 Points**
Let $(X_n)_n$ be iid random variables with $\mathbb{E}\left[X_1\right] = 0$ and $0 < \mathbb{E}\left[X_1^2\right] < \infty$. Let $S_n = \sum_{i=1}^{n} X_i$. Use Kolmogorovs 0-1 law to show that $\limsup_{n\to\infty} \frac{S_n}{\sqrt{n}} = \infty$ almost surely.

Reminder: Hand in your solution in groups of two students.