Technische Universität Berlin
Fakultät II – Institut f. Mathematik
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WS 2010/2011
hand out: Wed 08.12.2010
due: Wed 15.12.2010

Assignment 8
„Wahrscheinlichkeitstheorie 2 - Stochastic processes 1“

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total points: 20 Points

Problem 1 **5 Points**
Let X, Y, Z be independent random variables that are standard normally distributed. Show that then

$$\frac{X + YZ}{\sqrt{1+Z^2}}$$

is standard normally distributed. Use conditional expectation and the charakteristic function.

Problem 2 **5 Points**
Let (X, Y) be an integrable random variable in $\mathbb{R}^2$ with continuous density $f(\cdot,\cdot)$. Show that

$$\mathbb{E}\left[X\,|Y\right] = \frac{\int_{-\infty}^{\infty} x f(x,Y)\mathrm{d}x}{\int_{-\infty}^{\infty} f(x,Y)\mathrm{d}x} \quad \text{almost surely.}$$

Now assume (X, Y) to be normally distributed with zero mean and covariance $\Sigma^2 = \begin{pmatrix} \sigma^2 & \sigma^2\rho \\ \sigma^2\rho & \sigma^2 \end{pmatrix}$ with $\sigma^2 > 0$ and $\rho \in [-1, 1]$. Determine $\mathbb{E}\left[X\,|Y\right]$.

Problem 3 **5 Points**
Let (X, Y) be uniformly distributed on $B = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1\}$.

(i) Determine the distribution of Y given $X = x$.

(ii) Determine the distribution of $\arctan(Y/X)$ given $\sqrt{X^2+Y^2} = r$.

Problem 4 **5 Points**
Let $\Gamma = (\gamma_{i,j})_{i,j\in\mathbb{N}}$ be an infinite symmetric matrix. Each upper left corner $\Gamma^{(n)} = (\gamma_{i,j})_{i,j\leq n}$ is positive definit.

(i) Show that there is a stochastic process $\mathbb{X} = (X_k)_{k\in\mathbb{N}}$ such that $(X_1, \ldots, X_n)$ is distributed normally with zero mean and covariance $\Gamma^{(n)}$.

(ii) Show that $\mathbb{X}$ is stationary iff there exists a function $\phi : \mathbb{N} \to \mathbb{R}$ with $\gamma_{ij} = \phi(|i - j|)$ for all $i, j \in \mathbb{N}$.

Reminder: Hand in your solution in groups of two students.