Technische Universität Berlin **WS 2010/2011**
Fakultät II – Institut f. Mathematik
Lecturer: Prof. Dr. J.-D. Deuschel hand out: Wed 15.12.2010
Assistant: Simon Wasserroth due: Wed 05.01.2011

Assignment 9
„Wahrscheinlichkeitstheorie 2 - Stochastic processes 1“

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total points: 20 Points

Problem 1 **5 Points**
Find two stationary processes $\mathbb{X} = (X_1, X_2, \dots)$, $\mathbb{Y} = (Y_1, Y_2, \dots)$ such that their product $\mathbb{Z} = (X_1 Y_1, X_2 Y_2, \dots)$ is not stationary.

Problem 2 **5 Points**
Find an example or prove that it is not possible.

(i) A measure preserving map T such that T is not ergodic but T^2 is ergodic.

(ii) A measure preserving map T such that T^2 is not ergodic but T is ergodic.

Problem 3 **5 Points**
Let $\mathbb{P}$ be a probability measure on $([0,1], \mathcal{B}_{[0,1]})$. Show that $T : [0,1] \to [0,1], x \mapsto x^2$ is measure preserving iff $\mathbb{P}[\{0,1\}] = 1$.

Problem 4 **5 Points**
Let $c \in D$ be a unit root such that there exists an $n \in \mathbb{N}$ with $c^n = 1$. $D = \{z \in \mathbb{C} | |z| = 1\}$ denotes the unit sphere. Let $T_c(\omega) = c\omega$ be defined on $(D, \mathcal{B}_D, \lambda_D)$ and $\mathcal{I}$ be the sigma algebra of invariant sets under T_c.
Let X be some integrable random variable on this space. Determine $\mathbb{E}[X | \mathcal{I}]$ directly and relate that to the ergodic theorem.

Merry Christmas!