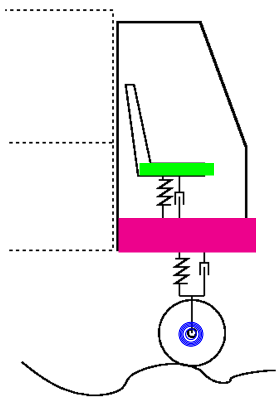
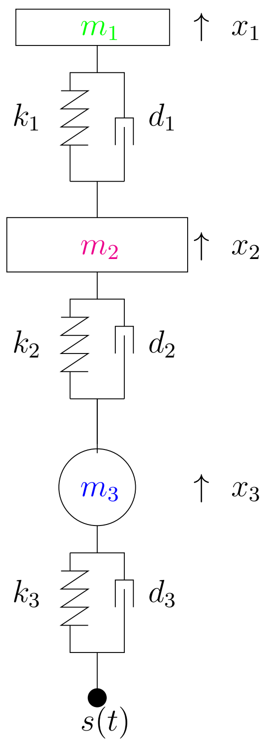
Wissenschaftliches Rechnen / Scientific Computing
2. Assignment (Due date 14.11.2012)



Consider the model problem of the truck cabin presented during the lecture. The associated balance of forces leads to a system of 2nd order ordinary differential equations for x_1, x_2, x_3 with initial values

$$\begin{aligned} x_1(0) &= -\Delta x_1 - \Delta x_2 - \Delta x_3 \\ x_2(0) &= -\Delta x_2 - \Delta x_3 \\ x_3(0) &= -\Delta x_3 \\ \dot{x}_1(0) &= \dot{x}_2(0) = \dot{x}_3(0) = 0 \end{aligned}$$



m_1	Mass of the driver and the seat
m_2	Mass of the cabin
m_3	Mass of the tire
k_1, k_2, k_3	Spring constants of the springs
d_1, d_2, d_3	Damping parameters of the dampers
x_1, x_2, x_3	Displacement of the mass points from the stationary position
$s(t)$	Street

Here, $g = 9.81$ (gravitational acceleration), m_1, m_2, m_3, $\Delta x_1, \Delta x_2, \Delta x_3$, d_1, d_2, d_3 and k_1, k_2, k_3, are the following constants.

Masses$[kg]$	$m_1 = 120,$	$m_2 = 1000,$	$m_3 = 15,$
Displacements$[m]$	$\Delta x_1 = 0.1,$	$\Delta x_2 = 0.15,$	$\Delta x_3 = 0.03,$
Damping parameters$[\frac{kg}{s}]$	$d_1 = 50,$	$d_2 = 1000,$	$d_3 = 200,$
Spring constants$[\frac{kg}{s^2}]$	$k_1 = \frac{m_1 g}{\Delta x_1},$	$k_2 = \frac{(m_1+m_2)g}{\Delta x_2},$	$k_3 = \frac{(m_1+m_2+m_3)g}{\Delta x_3}.$

The function $s(t)$ desribes the street and is given by

$$s(t) = \begin{cases} 0.1(\sin(4t) - 1) & \pi/8 < t < 2\pi + \pi/8 \\ 0 & \text{otherwise} \end{cases}$$

- First we introduce new variables $x_4 = \dot{x}_1$, $x_5 = \dot{x}_2$,$x_6 = \dot{x}_3$ and rewrite the system of 2nd order ordinary differential equations as a system of 1st order ordinary differential equations, i.e.,

$$\dot{\vec{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} F_1(t, x_1, \dots, x_6) \\ F_2(t, x_1, \dots, x_6) \\ F_3(t, x_1, \dots, x_6) \\ F_4(t, x_1, \dots, x_6) \\ F_5(t, x_1, \dots, x_6) \\ F_6(t, x_1, \dots, x_6) \end{pmatrix} = F(t, \vec{x}).$$

- The initial values are given in $\vec{x}(0)$ as

$$\vec{x}(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \\ x_5(0) \\ x_6(0) \end{pmatrix}.$$

- Write the MATLAB M-File `STREET.m`

```
function s=STREET(t)
```

such that `s`$\widehat{=}s(t)$ corresponds to the surface of the street, and the MATLAB M-File `DSTREET.m`

```
function ds=DSTREET(t)
```

such that `ds`$\widehat{=}\dot{s}(t)$ is the associated derivation.

- Write the MATLAB M-File `CABIN.m`

```
function y=CABIN(t,x)
```

t is the same parameter as in the differential equation, x is a six-dimensional vector and the output y is the the result of $F(t, x)$ (also a six-dimensional vector).

- Use the build in MATLAB function `ode45` with your `'CABIN'` function as a first parameter, the start and end point `[0, 3*pi]`, as well as the initial value `x0`$\widehat{=}\vec{x}(0)$ from above (be careful with row and column vectors!).

```
[t,x] = ode45('CABIN',[0,3*pi],x0)
```

- Test your program with the MATLAB script `script.m` which uses the MATLAB function `plotcabin(t,x,s)` for visualization (download files `script.m` and `plotcabin.m`). Press any key to see the truck driving.

5 pts.

- Write the MATLAB M-File `DxCABIN.m`

```
function J=DxCABIN(t,x)
```

t the same parameter as in the differential equation, x is a six-dimensional vector and an output J is the Jacobi matrix of the partial derivative of F with respect to x (a 6×6 matrix).

- Instead of using the build in MATLAB function `ode45` implement the Implicit Euler with MATLAB M-File `ImEULER.m`

```
function [t,x,s]=ImEULER(t,x,n)
```

With the initial value $\vec{x}(0)$ and a fixed time step size $h = \frac{3\pi - 0}{n}$ ($n = 10$, $n = 100$ and $n = 1000$) determine the approximate solution $\vec{x}_i = \vec{x}(t_i) = \vec{x}(ih)$ as follows. Define

$$G(\vec{v}) := \vec{v} - (\vec{x}(ih) + h\ F(ih + h, \vec{v}))\ , \quad \text{where } \vec{v} \approx \vec{x}_{i+1}.$$

In each step $i = 0, 1, \ldots, n-1$, determine $\vec{x}_{i+1} = \vec{x}(ih + h)$ from $\vec{x}_i = \vec{x}(ih)$ as

$$\vec{x}(ih + h) := \vec{v} + \Delta\vec{v},$$

where $\Delta\vec{v}$ is a solution of the linear system

$$\frac{dG(\vec{v})}{d\vec{v}} \Delta\vec{v} = -G(\vec{v}).$$

As a initial guess for the Newton method take

$$\vec{v} := \vec{x}(ih) + h\ F(ih, \vec{x}(ih)), \quad \text{(one step of explicit Euler method)}.$$

Use MATLAB \ to solve the linear system.

The output will be given by the vector t of length $n+1$ where $t_i = ih$, the $6 \times (n+1)$ matrix x where $x(:, i) = \vec{x}(ih)$ and vector s of length $n+1$ where $s_i = s(ih)$.

- Modify the MATLAB script `script.m` to test your program.

5 pts.

Remark: This model problem is taken from: M. Bollhöfer and V. Mehrmann, *Numerische Mathematik: Eine Projektorientierte Einführung Für Ingenieure, Mathematiker und Naturwissenschaftler*, Springer, 2004.