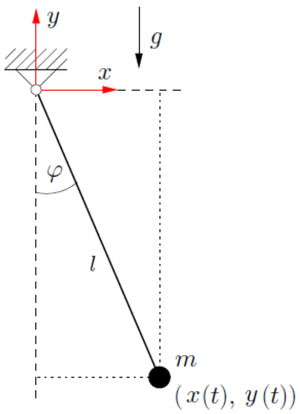
Wissenschaftliches Rechnen / Scientific Computing
3. Assignment (Due date 28.11.2012)



Consider the model problem of the mathematical pendulum presented during the lecture. Modeling via Lagrange equation leads to a quasi-linear DAE of the form

$$\mathbf{E}\dot{\mathbf{q}}(t) = f(t, \mathbf{q}(t))$$

with $\mathbf{E}$, $\mathbf{q}(t)$ given by

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{q}(t) = \begin{bmatrix} x(t) \\ y(t) \\ v_x(t) \\ v_y(t) \\ \lambda(t) \end{bmatrix}.$$

We have introduced three different formulation of the problem, namely

$$d\text{-index} = 3 \quad \text{with} \quad f(t, \mathbf{q}(t)) = \begin{bmatrix} v_x(t) \\ v_y(t) \\ -2x(t)\lambda(t) \\ -mg - 2y(t)\lambda(t) \\ x(t)^2 + y(t)^2 - l^2 \end{bmatrix},$$

$$d\text{-index} = 2 \quad \text{with} \quad f(t, \mathbf{q}(t)) = \begin{bmatrix} v_x(t) \\ v_y(t) \\ -2x(t)\lambda(t) \\ -mg - 2y(t)\lambda(t) \\ x(t)v_x(t) + y(t)v_y(t) \end{bmatrix},$$

$$d\text{-index} = 1 \quad \text{with} \quad f(t, \mathbf{q}(t)) = \begin{bmatrix} v_x(t) \\ v_y(t) \\ -2x\lambda(t) \\ -mg - 2y(t)\lambda(t) \\ v_x(t)^2 + v_y(t)^2 - \frac{2}{m}(x(t)^2 + y(t)^2)\lambda - gy(t) \end{bmatrix}.$$

Here, the mass $m = 1$ [kg], the pendulum length $l = 0.994$ [m], and the gravitational acceleration $g = 9.81$ [m/s^2] are given. $(x(t), y(t))$ describe the position of the pendulum at the time t, $v_x(t)$, $v_y(t)$ are the velocity components in x, y direction, respectively, and $\lambda(t)$ is the Lagrange multiplier.

- Write the MATLAB M-Files `f1.m`, `f2.m` and `f3.m` which implement the function $f(t, \mathbf{q}(t))$ for d-index $= 1, 2,$ and 3.

1 pts.

- Write the MATLAB M-File `DAEImpEuler.m`

```
function [Q,t]=DAEImpEuler(E,f,q0,T,h,tol)
```

which implements the implicit Euler method for each of the formulation above. The initial vector is given by `q0`, `T` denotes the end of the time interval $[0, \texttt{T}]$, the size of the time steps is given by `h` and `tol` is the tolerance value which should be used for the stopping criteria of the Newton Method. An argument `f` will be given as a string `'f1'`, `'f2'`, and `'f3'` depending on the chosen DAE. For the function evaluation use MATLAB function `feval` (use `help feval` for more details). The i-th columns of `Q` contain a solution vector $\mathbf{q}(t_i)$ for time point $t_i \in [0, T]$ such that $t_0 = 0$, $t_n = T$ and $t_i = i * \texttt{h}$.

5 pts.

- Write the MATLAB script `pendulum.m` where you will test your pendulum model. Make the process of testing different formulation automatic. All the plots should be generated here. Use MATLAB `subplot` to avoid generating too many figures.

2 pts.

- Test your program for initial vector $\texttt{q0} = [1, 0, 0, 0, 0]^T$, $\texttt{T} = 2s, 20s, 50s$ and time steps $\texttt{h} = 0.1, 0.01, 0.001, 0.0001$.
- Plot components of vector $\mathbf{q}(t_i)$ for each time point $t_i \in [0, T]$ as follows:
  (a) x, y with respect to time t,
  (b) v_x, v_y with respect to time t,
  (c) λ with respect to time t.
- Try to answer the following questions:
  (a) What is the position (x, y) of the pendulum after $2s$?
  (b) What is the period of this pendulum?
  (c) Plot x against y for d-index $= 1, 2$. What curve should you get? What effect do you observe?
  (d) What behavior of the Lagrange multiplier do you observe for the d-index $= 3$ formulation?

2 pts.

Remark: You may read more on DAEs in the book: V. Mehrmann and P. Kunkel, *Differential-Algebraic Equations: Analysis and Numerical Solution*, European Mathematical Society, 2006.