Technische Universität Berlin **Winter Semester 2012/2013**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
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Wissenschaftliches Rechnen / Scientific Computing
5. Assignment (Due date 28.01.2013)

Let $\Omega \subset \mathbb{R}^2$ be a bounded Lipschitz domain with polygonal boundary Γ. Given $f \in L^2(\Omega)$ we want to solve the boundary value problem:
Find $u \in H^1(\Omega)$ such that

$$-\Delta u = f \quad \text{in } \Omega, \tag{1}$$
$$u = 0 \quad \text{on } \Gamma, \tag{2}$$

where Γ denotes the boundary of Ω. Determine the finite element approximation u_h of u using **linear finite elements**.

- You will need data structures representing the triangulation $\mathcal{T}$ of the domain $\Omega \subset \mathbb{R}^2$.
  - `vertices.dat` containing the coordinates of all the nodes in the triangulation $\mathcal{T}$, e.g., an array such that the i-th row contains the coordinates x and y of the vertex with index i.
  - `elements.dat` containing the indices of all vertices belonging to each element $T \in \mathcal{T}$, e.g., an array such that the j-th row will contain the indices of three vertices belonging to the element with index j (remember about the right order of the vertices).
- Write the MATLAB function `lsm.m`

  ```
  function A_T=lsm(v4e)
  ```

  which determines the *local stiffness matrix* `A_T` $\in \mathbb{R}^{3\times 3}$ corresponding to the element $T \in \mathcal{T}$ based on the coordinates of its three vertices defined in `v4e`.

  3 pts.
- Write the MATLAB function `lrhs.m`

  ```
  function b_T=lrhs(v4e,f)
  ```

  which determines the *local right-hand side* `b_T` $\in \mathbb{R}^{3\times 1}$ corresponding to the element $T \in \mathcal{T}$ based on the coordinates of its three vertices defined in `v4e` and function $f \in L^2(\Omega)$. For simplicity define the function f in a separate MATLAB file `f.m` and use the MATLAB function `feval`. **Hint:** If necessary use the value of f in the center of gravity of the element $T \in \mathcal{T}$ to approximate the integrals.

  2 pts.
- Write the MATLAB M-File `solveBVP.m`

  ```
  function solveBVP(f)
  ```

  which determined the finite element approximation u_h of u.

- Load all information about the triangulation, i.e., `load vertices.dat` and `load elements.dat`.
- Assemble the global stiffness matrix A and the global right-hand side b. Use the MATLAB function `sparse` to generate sparse matrix A.
- Solve the linear system $Ax = b$ to determine the finite element approximation u_h. You can use MATLAB \.
- Use the MATLAB build in routine `trisurf` to visualize your solution. Use `help trisurf` to get more information.

3 pts.

- Test your program for the given set of data `vertices.dat`, `elements.dat` with $f = 5\pi^2 sin(\pi x)\sin(2\pi y)$, $\Omega = (0,1) \times (0,1)$ where the exact solution is given as $u = \sin(\pi x)\sin(2\pi y)$. Compare your approximation with the exact solution by looking at the error $\|u - u_h\|$. Plot your finite element approximation u_h, the exact solution u and the error $u - u_h$.

2 pts.

References:

J. Alberty, C. Carstensen and S. A. Funken, *Remarks around 50 lines of Matlab: short finite element implementation*, Numerical Algorithms 20, pp. 117–137, 1999.

M. S. Gockenbach, *Partial Differential Equations: Analytical and Numerical Methods*, SIAM, 2002.

M. S. Gockenbach, *Understanding and Implementing the Finite Element Method*, SIAM, 2006.