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Control Theory of Descriptor Systems

1. Exercise
(Discussion on November 3, 2014)

Exercise 1.1: (Consistency and Regularity)
Consider the following descriptor systems

(a)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

(b)

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Check consistency and regularity for these systems.

Exercise 1.2: (Linear DAEs with constant coefficients)
Show that the matrix pair

$$(E, A) = \left(\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right)$$

with $E, A \in \mathbb{R}^{\ell,n}$ and $r < \min\{\ell, n\}$, is regular and of index $\nu = ind(E, A) = 1$ if and only if A_{22} is square and nonsingular.

Exercise 1.3: (Linear DAEs with variable coefficients)
Consider the linear descriptor system given by

$$\begin{bmatrix} 0 & 0 \\ 1 & \eta t \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -\eta t \\ 0 & -(1+\eta) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

for a given parameter $\eta \in \mathbb{R}$.

(a) Compute the characteristic values μ, a, d and v for the DAE with given input function $u(t)$.

(b) Compute the characteristic values $\hat{\mu}$, $\hat{a}$, $\hat{d}$ and $\hat{v}$ for the descriptor system in behavior form.

(c) Compute the reduced system formulations corresponding to the results from (a) and (b).