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Control Theory of Descriptor Systems

2. Exercise

(Discussion on November 17, 2014)

Exercise 2.1: (Reduced System Formulation)
Prove the following Lemma:

Lemma 1. *A DAE of the form*

$$\begin{bmatrix} \hat{\mathcal{E}}_1(t) \\ 0 \\ 0 \end{bmatrix} \dot{z} = \begin{bmatrix} \hat{\mathcal{A}}_1(t) \\ \hat{\mathcal{A}}_2(t) \\ 0 \end{bmatrix} z + \begin{bmatrix} \hat{f}_1(t) \\ \hat{f}_2(t) \\ \hat{f}_3(t) \end{bmatrix}$$

with $\hat{\mathcal{E}}_1, \hat{\mathcal{A}}_1 : \mathbb{I} \to \mathbb{R}^{d,n}$, $\hat{\mathcal{A}}_2 : \mathbb{I} \to \mathbb{R}^{a,n}$, $\hat{f}_1 : \mathbb{I} \to \mathbb{R}^{d}$, $\hat{f}_2(t) : \mathbb{I} \to \mathbb{R}^{a}$, $\hat{f}_3 : \mathbb{I} \to \mathbb{R}^{v}$ *is strangeness-free if and only if the matrix* $\begin{bmatrix} \hat{\mathcal{E}}_1(t) \\ \hat{\mathcal{A}}_2(t) \end{bmatrix}$ *has pointwise full row rank* $d + a$ *for all* $t \in \mathbb{I}$.

Exercise 2.2: (Nonlinear Control Problems)
Discuss the properties of the following control problems

(a)
$$x^2 - t^2 + u = 0 \qquad (l = 1,\, n = 1,\, m = 1)$$

(b)
$$\dot{x}_1 = 0, \quad x_2 u = 0 \qquad (l = 2,\, n = 2,\, m = 1)$$

In particular, consider suitable restrictions of the domain of the associated function F.

Exercise 2.3: (Feedback Regularization)
Consider the following descriptor systems

(a)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

(b)
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(i) Does there exist a proportional state feedback such that the closed-loop system is regular and of index at most one?

(ii) Does there exist a proportional output feedback such that the closed-loop system is regular and of index at most one?