Technische Universität Berlin
Institut für Mathematik
Lena Scholz

Winter term 2014/15

Control Theory of Descriptor Systems

3. Exercise
(Discussion on December 1, 2014)

Exercise 3.1: (Local equivalence of inflated pairs)
Consider two pairs $(\mathcal{E}, \mathcal{A})$, $(\tilde{\mathcal{E}}, \tilde{\mathcal{A}})$ of sufficiently smooth matrix functions $\mathcal{E}, \mathcal{A}, \tilde{\mathcal{E}}, \tilde{\mathcal{A}} : \mathbb{I} \to \mathbb{R}^{m,n}$ that are globally equivalent, i.e., there exists pointwise nonsingular and sufficiently smooth matrix functions $P : \mathbb{I} \to \mathbb{R}^{m,m}$ and $Q : \mathbb{I} \to \mathbb{R}^{n,n}$ such that

$$\tilde{\mathcal{E}} = P\mathcal{E}Q, \quad \tilde{\mathcal{A}} = P\mathcal{A}Q - PE\dot{Q}.$$

Let $(\mathcal{M}_l, \mathcal{N}_l)$ and $(\tilde{\mathcal{M}_l}, \tilde{\mathcal{N}_l})$, $l \in \mathbb{N}_0$, be the corresponding inflated matrix pairs. Then the matrix pairs $(\mathcal{M}_l, \mathcal{N}_l)$ and $(\tilde{\mathcal{M}_l}, \tilde{\mathcal{N}_l})$ are locally equivalent, i.e., for every $t \in \mathbb{I}$ there exists nonsingular matrices Π_l and Θ_l and a matrix Ψ_l such that

$$\tilde{\mathcal{M}_l} = \Pi_l \mathcal{M}_l \Theta_l, \quad \tilde{\mathcal{N}_l} = \Pi_l \mathcal{N}_l \Theta_l - \Pi_l \mathcal{M}_l \Psi_l.$$

.

Hint: Consider matrices Π_l, Θ_l and Ψ_l defined as follows

$$\begin{aligned}
(\Pi_l)_{i,j} &= \binom{i}{j} P^{(i-j)}, \\
(\Theta_l)_{i,j} &= \binom{i+1}{j+1} Q^{(i-j)}, \\
(\Psi_l)_{i,j} &= \begin{cases} Q^{(i+1)} & \text{for } i = 0, \dots, l, \ j = 0, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

Exercise 3.2: (Matrix exponential)
Let $A \in \mathbb{R}^{n,n}$. Show, that there exist analytic functions $\beta_j(t)$, for $j = 0, \dots, n-1$, such that e^{At} can be expressed as a polynomial in A with coefficients $\beta_j(t)$, i.e.,

$$e^{At} = \beta_0(t)I + \beta_1(t)A + \cdots + \beta_{n-1}(t)A^{n-1}.$$