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Control Theory of Descriptor Systems

4. Exercise
(Discussion on January 5, 2015)

Exercise 4.1: (Controllability)
Check if the following linear descriptor systems are C-controllable, R-controllable or I-controllable.

(a)
$$0 = x_2$$
$$\dot{x}_1 = x_1 + u$$

(b)
$$\dot{x}_2 = x_1$$
$$0 = u$$

(c)
$$\dot{x}_1 = x_1$$
$$\dot{x}_3 = x_2$$
$$0 = x_3 + u$$

(d) Linearized cart-pendulum with

$$(E, A) = \left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{m_2 g}{L} & \frac{m_2 g}{L} & 0 & 0 & 0 & 0 & 0 \\ \frac{m_2 g}{L} & -\frac{m_2 g}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{m_2 g}{L} & 0 & 0 & 0 & 2L \\ 0 & 0 & -2L & 0 & 0 & 0 & 0 \end{bmatrix} \right), \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Exercise 4.2: (Controllability of Electrical Circuits)
Consider the electrical circuit given in Figure 1 consisting of a linear resistor, a linear inductor and a linear capacitor in series. The control input is given by the voltage at the voltage source.

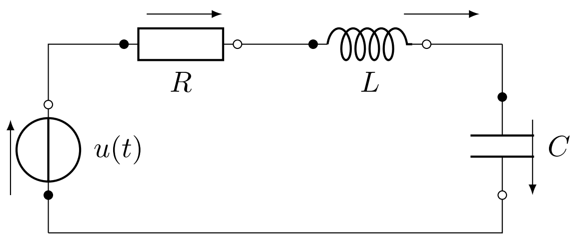

Figure 1: RLC circuit

(a) Derive the DAE that describe the dynamical behavior of the system by using Kirchhoff's voltage and current laws.

(b) Is the system C-controllable, R-controllable or I-controllable?

Hint: For linear elements the constitutive element relations are given by

$$v_R = R i_R, \quad L \frac{d}{dt} i_L = v_L, \quad C \frac{d}{dt} v_C = i_C,$$

where v_i denote the voltage across the respective element and i_i denotes the current through the respective element.

Exercise 4.3: **(Impulsive Smooth Solutions)**
Consider the nonlinear descriptor system

$$\begin{bmatrix} 0 & 0 \\ u(t) & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & u(t) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad t \in [0, 2]$$

with initial condition $x_1(0) = 0$, $x_2(0) = 1$ and input $u(t)$ defined by

$$u(t) = \begin{cases} 0 & \text{for } 0 \leq t < 1, \\ 1 & \text{for } 1 \leq t \leq 2. \end{cases}$$

Determine the solution $x(t) = [x_1(t), x_2(t)]^T$ and the impulse order $\mathsf{iord}(x)$.

Exercise 4.4: **(Impulsive Smooth Solutions)**
Analyze the system $E\dot{x} = Ax + f + Ex_0\delta$, $x = 0$ with

$$E = \begin{bmatrix} 2 & 2 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad f = \begin{bmatrix} 3\delta \\ 2\delta \\ \delta + H \\ H \end{bmatrix}.$$

(a) Characterize all the solutions for $x_0 = \begin{bmatrix} 1 & -1 & 1 & 0 \end{bmatrix}^T$.

(b) Does there exist a generalized solution x with $\mathsf{iord}(x) \leq -1$ for some x_0?