Mathematical Tools for Engineering and Management

Lecture 1

19 Oct 2011

- ▷ Introduction
- ▷ Organisational issues
  - Lectures
  - Exercises
  - Computers
  - Book
  - Exam
- ▷ Topics
  - Why mathematics?
  - Models and Data Sets
  - Two Examples

- Responsible for the Mathematics Section of GPE:
  Prof. Dr. Dr. h.c. mult. Martin Grötschel
  Zuse Institute Berlin, Chair for Combinatorial Optimization at TU Berlin (Mathematics Department)

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- Lectures and Exercises:

  Dr. Rüdiger Stephan

  Zuse Institute Berlin and TU Berlin, Room MA 307 at TU

  `http://page.math.tu-berlin.de/~stephan/`

  e-mail: `stephan@math.tu-berlin.de`

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- **Lecture Website:**
  `http://www3.math.tu-berlin.de/Vorlesungen/WS11/gpe/`

- **Lecture:** Wednesday, 10–12

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- **Exercise Sessions:** Wednesday, 12–14

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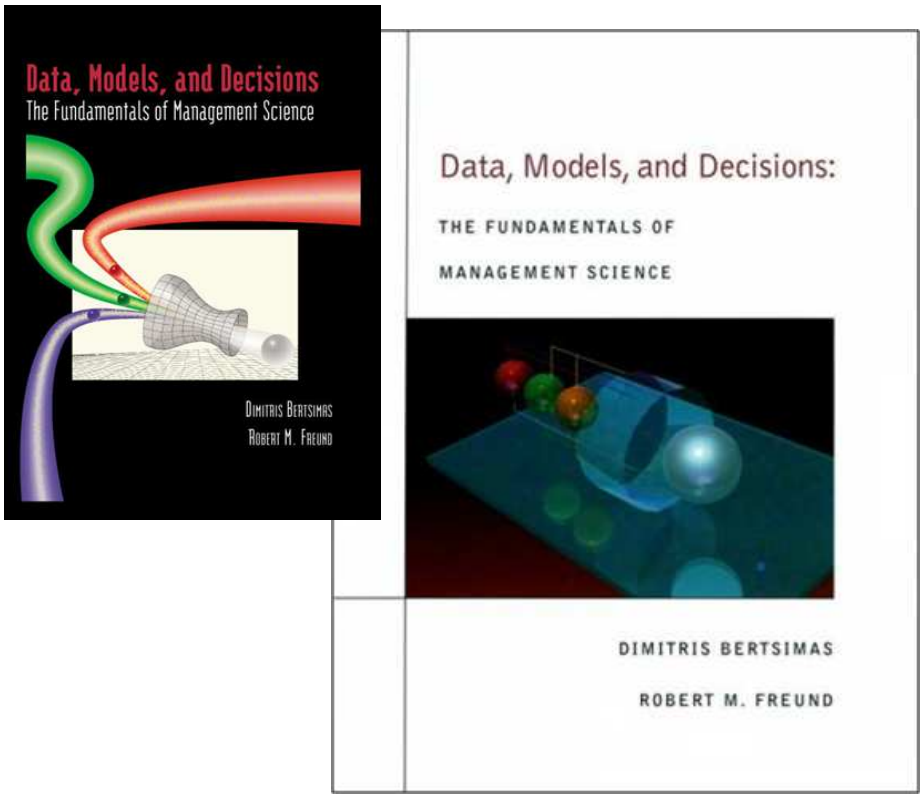
- **Exam:** Oral exam in the last two weeks of semester
- **Grade:** Computed from results in homeworks (50%) and exam (50%)

- Dimitris Bertsimas & Robert M. Freund, *Data, Models, and Decisions – The Fundamentals of Management Science*, 1st ed South-Western College Publishing 2000, 2nd ed Dynamic Ideas 2004



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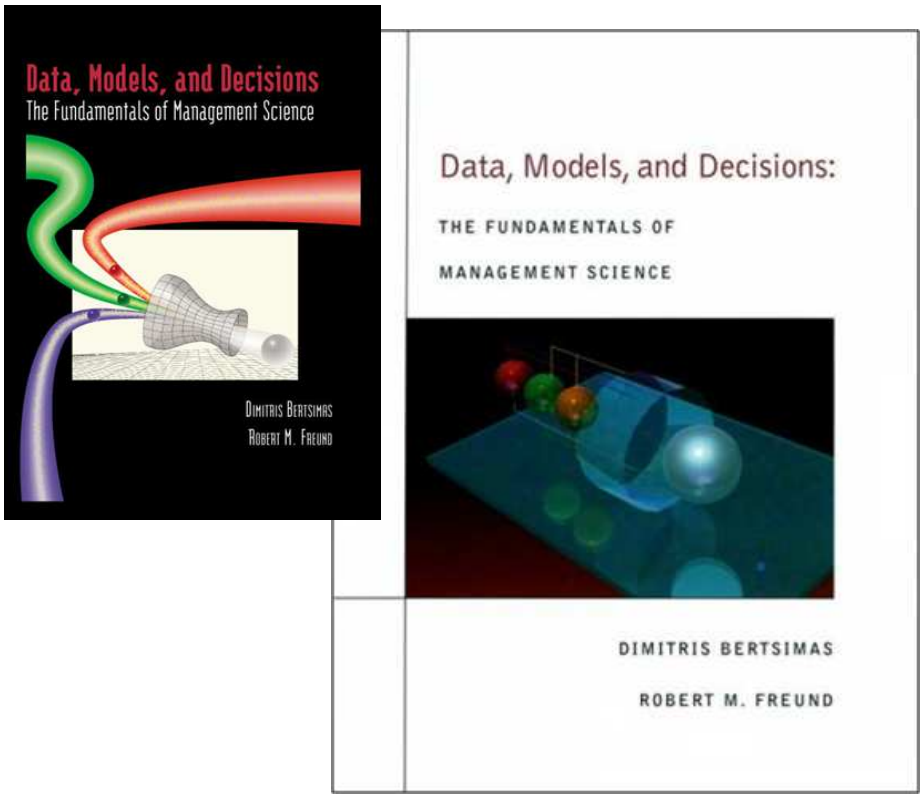

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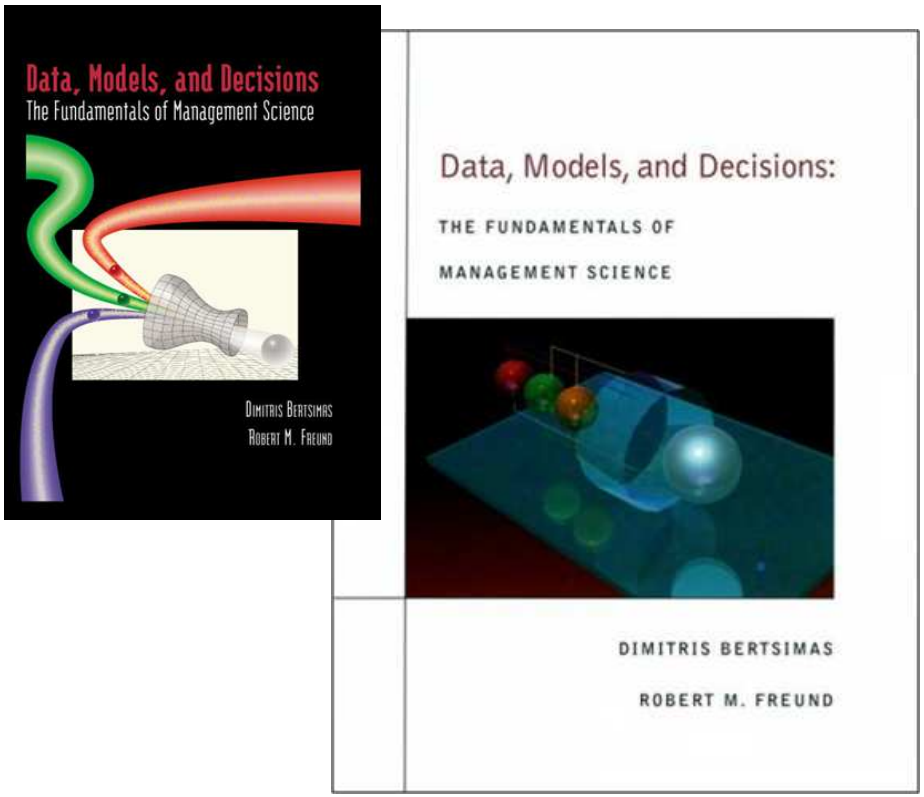

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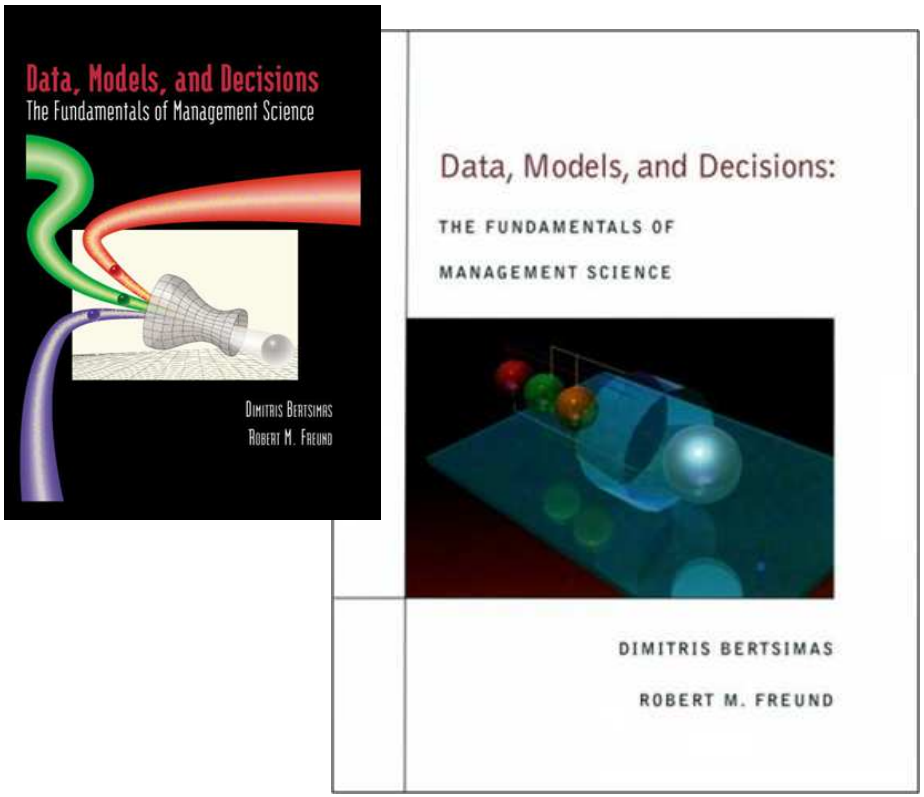

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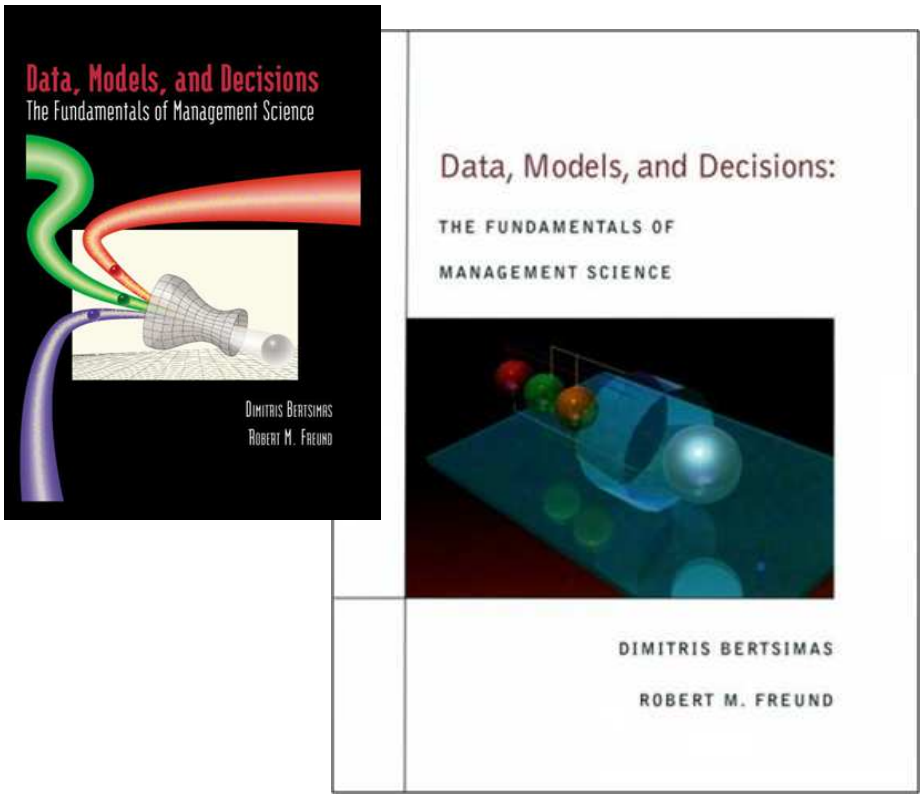

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  - ZIB Optimization Suite
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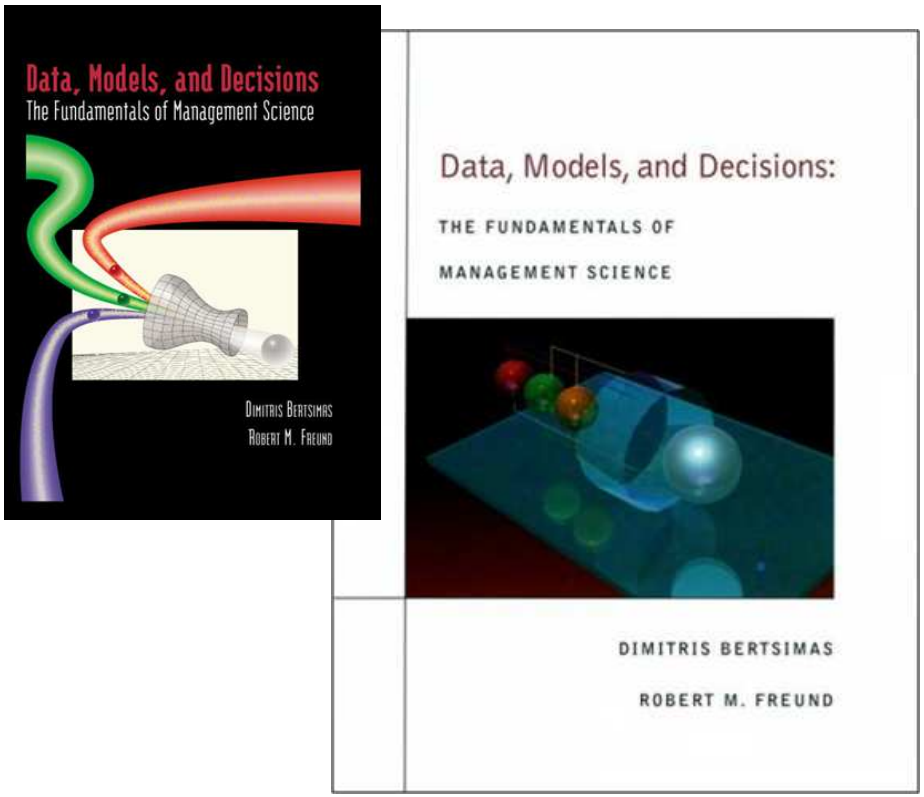

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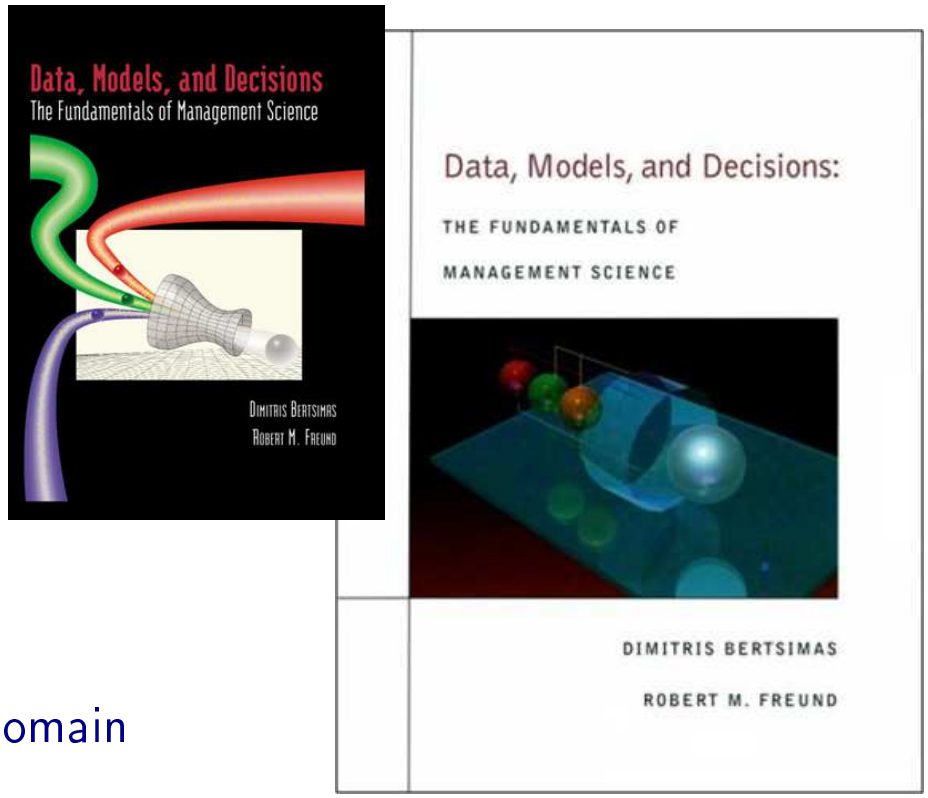

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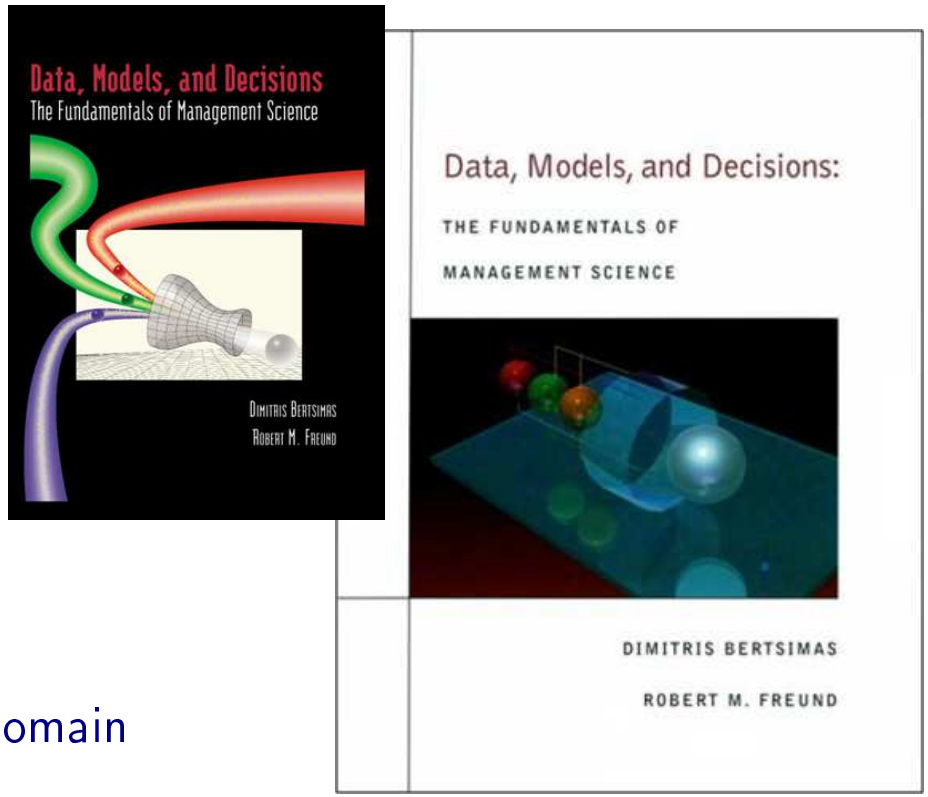

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  - More commercial solvers (most notably CPLEX, Gurobi) and modelling languages (AMPL, GAMS)

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling; Mathematical Background: Branch & Bound
- Branch & Bound, Cutting Planes; More Examples; Combinatorial Optimization
- Combinatorial Optimization: Examples, Graphs, Algorithms
- Complexity Theory
- Nonlinear Optimization
- Scheduling
- Lot Sizing
- Multicriteria Optimization
- Oral exam

▷ Mathematical Tools

- How can mathematics support technical/management decisions?

Lectures

▷ **Mathematical Tools**

- How can mathematics support technical/management decisions?

Lectures

▷ **Mathematical Modelling**

- How can a technical/management problem be modelled as a mathematical problem?

Lectures
Exercises

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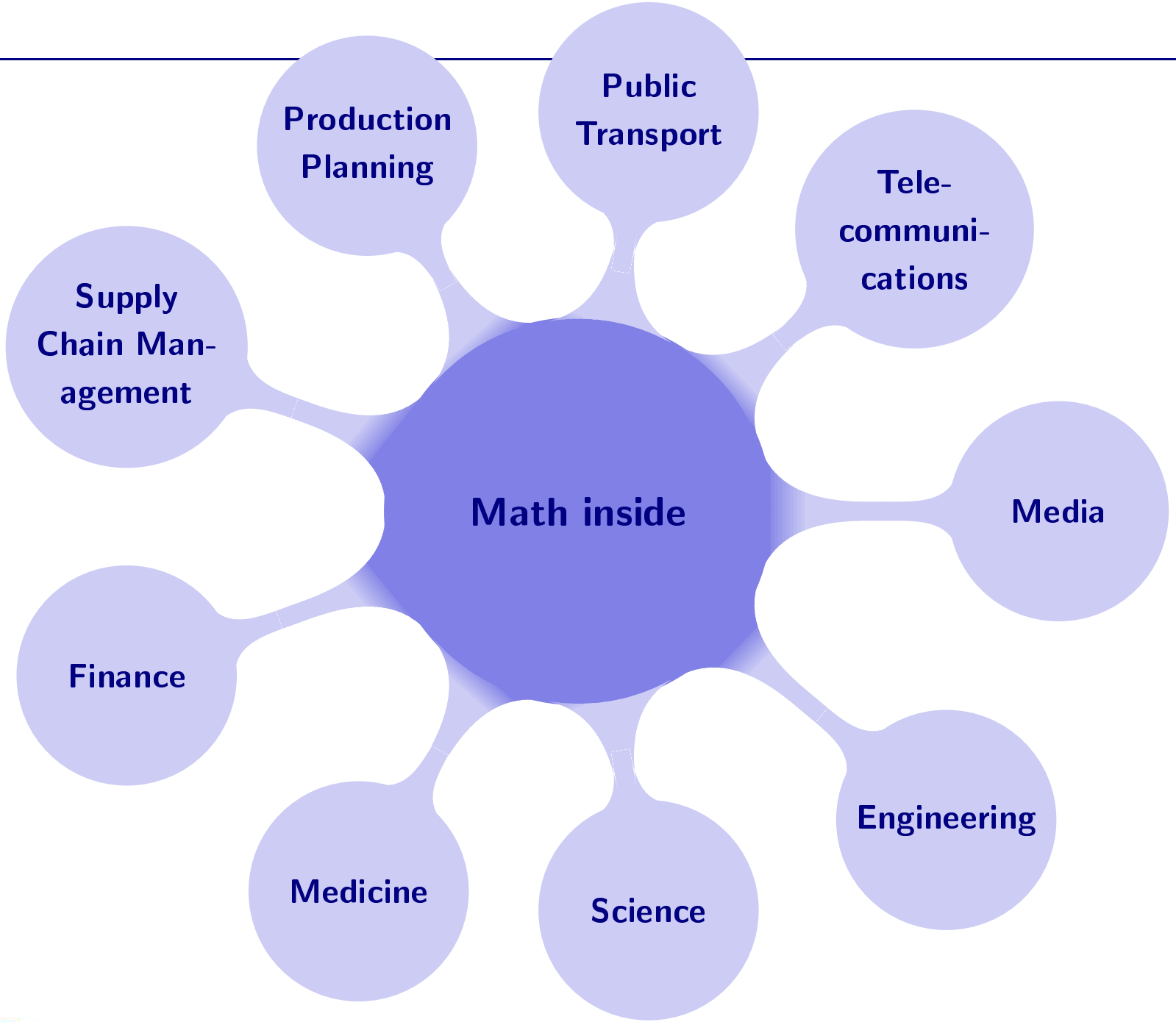
- Why does it work?

Lectures

- **Mathematical Tools** — Lectures
  - How can mathematics support technical/management decisions?
- **Mathematical Modelling** — Lectures, Exercises
  - How can a technical/management problem be modelled as a mathematical problem?
- **Mathematical Solutions** — Lectures, Exercises
  - How can I solve my mathematical problem?
- **Mathematical Background** — Lectures
  - Why does it work?
- **Practise** — Exercises
  - How do I use computers and my own wits to solve problems?


Production Planning
Public Transport
Tele-communi-cations
Supply Chain Man-agement
Math inside
Media
Finance
Engineering
Medicine
Science

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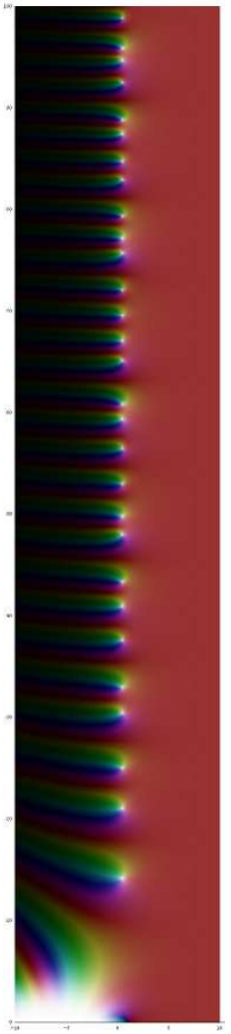
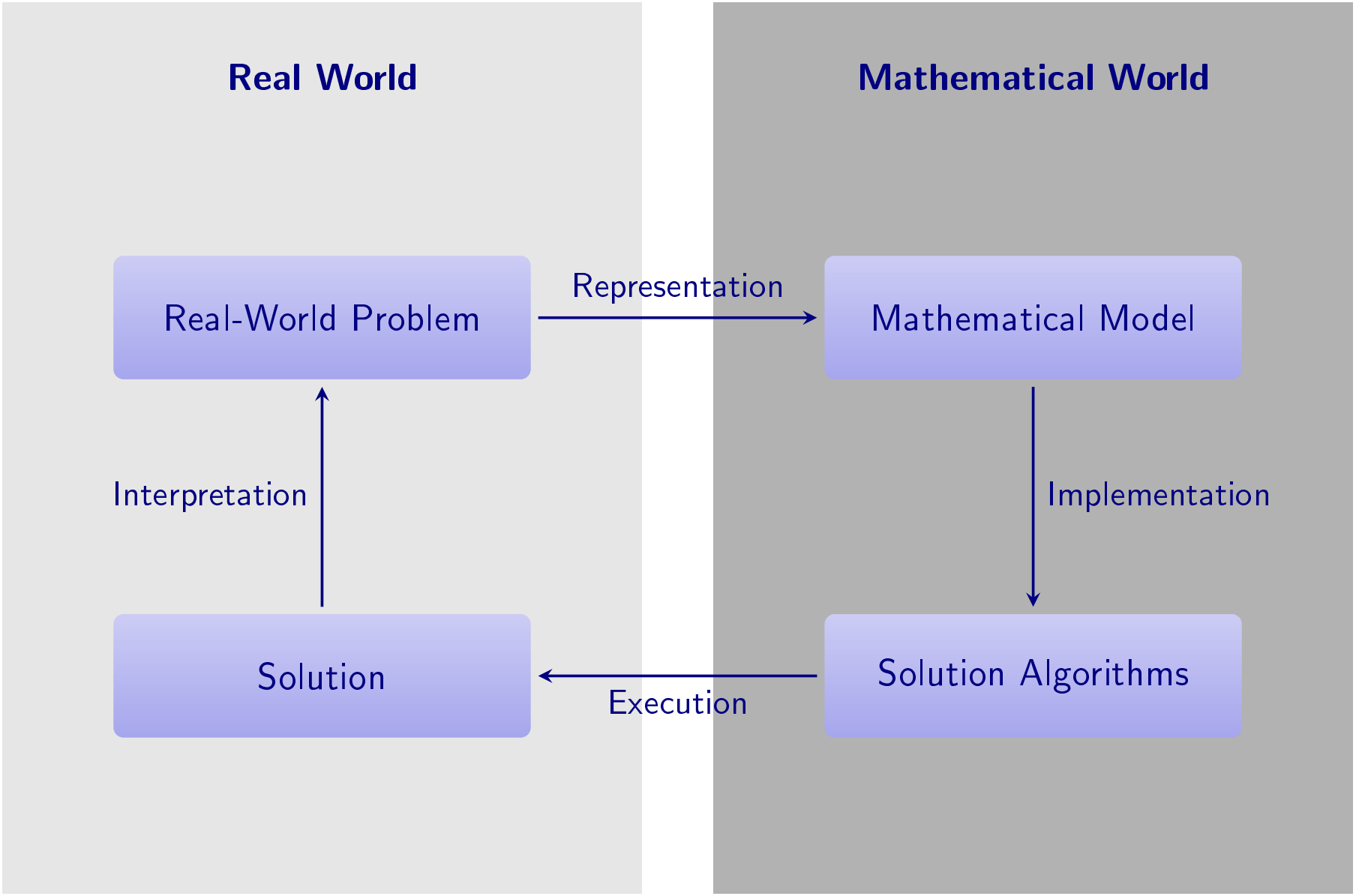
  (which might be a personal viewpoint...)

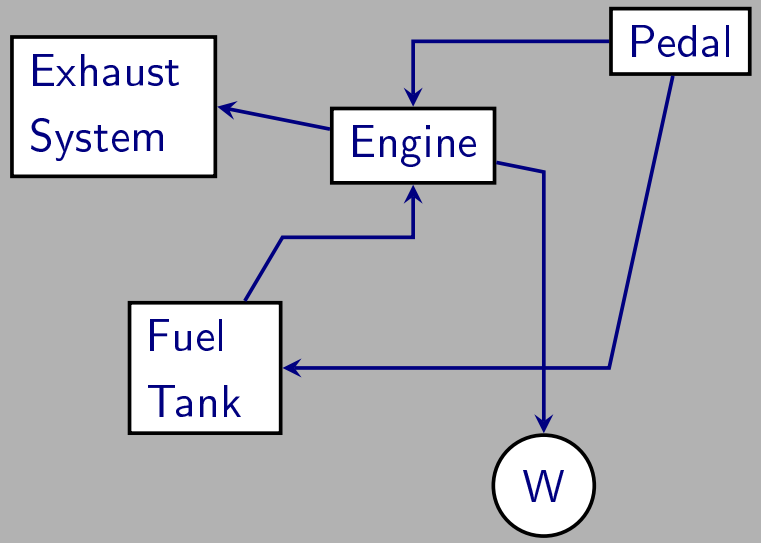



Real World
Mathematical World
Real-World Problem
Representation
Mathematical Model
Interpretation
Implementation
Solution
Execution
Solution Algorithms

Concrete Models

Used for experiments leaving out some functionality

Abstract Models



Objects representing real-world elements, properties etc

Concrete Models

Used for experiments leaving out some functionality

Mathematical Models

$$
\begin{aligned}
\max \quad & \sum_{(m,i,j)\in I'} \left(\frac{\tilde{w}_i^{(\pi)} - \tilde{w}_j^{(\pi)}}{l_m} \right) x_{mij} & \\
\text{s.t.} \quad & P_{\min} x_{mij} \leq p_{mij} \leq P_{\max} x_{mij} & \forall\ (m,i,j) \in I' \\
& \sum_{j\in S} \sum_{m\in M} (x_{mij} + x_{mji}) \leq 1 & \forall\ i \in S \\
& \gamma_{ij} p_{mij} + \xi_m SG_{\infty}^{(m,i,j)} (1 - x_{mij}) \geq & \\
& \xi_m \left(\nu + \sum_{\substack{(n,s,r)\in I' \\ s \| (m,i,j)}} \gamma_{si} p_{nsr} \right) & \forall\ (m,i,j) \in I' \\
& x_{mij} + x_{nsr} \leq 1 & \forall\ (m,i,j),(n,s,r) \in I' : \\
& & (m,i,j) \nparallel (n,s,r) \\
& x_{mij} \in \{0,1\} & \forall\ (m,i,j) \in I' \\
& p_{mij} \in [0, P_{\max}] & \forall\ (m,i,j) \in I'
\end{aligned}
$$

Symbols representing real-world objects, decisions etc

- Automobile assembly line:

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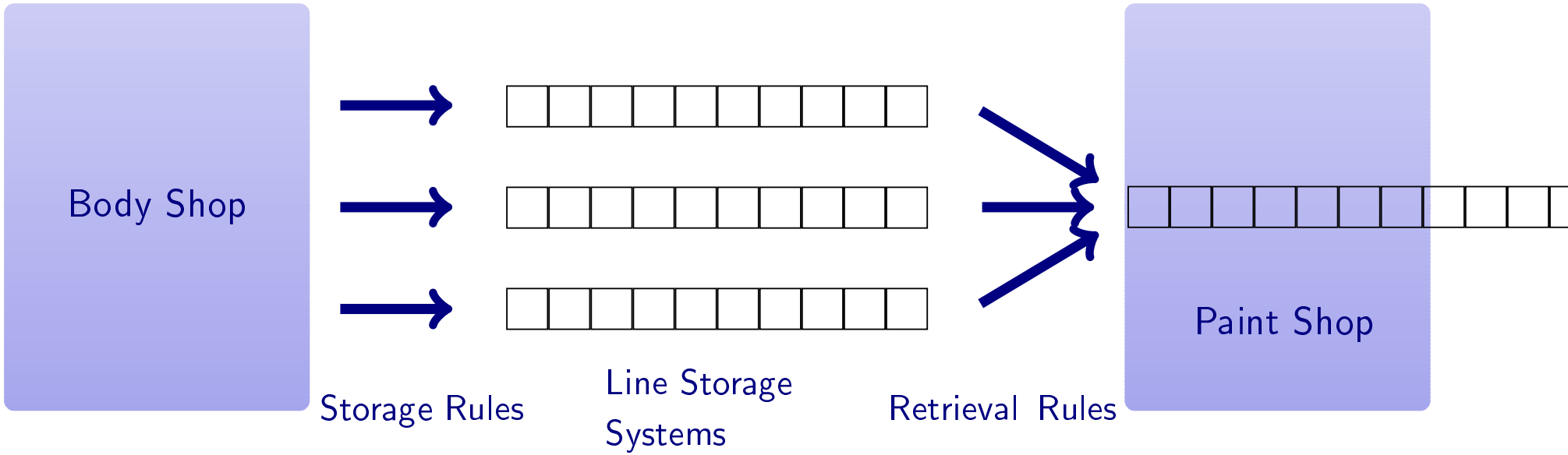
- Body to Paint Shop:



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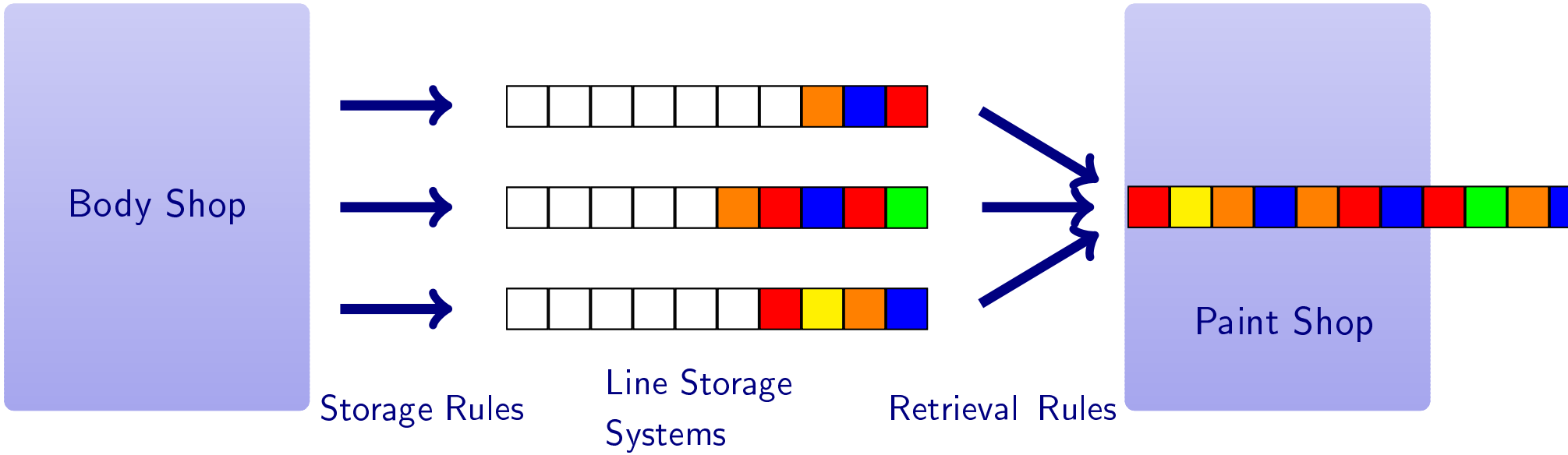

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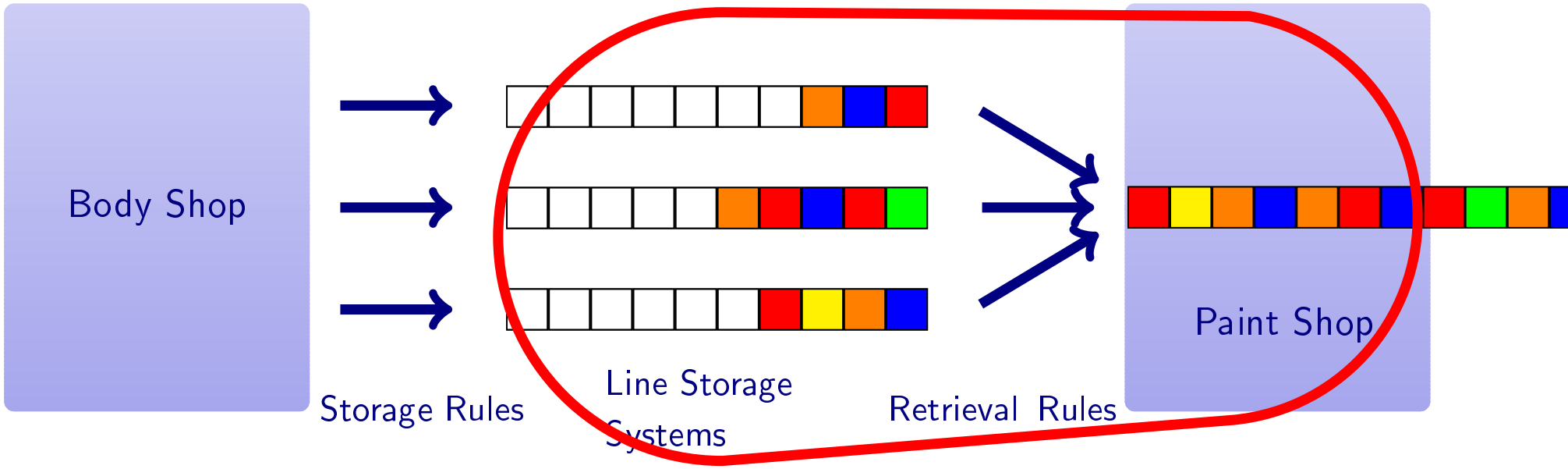

- Problem to solve: Design good retrieval rules!

- What is a ‘Retrieval Rule’?
- What does ‘good’ mean?

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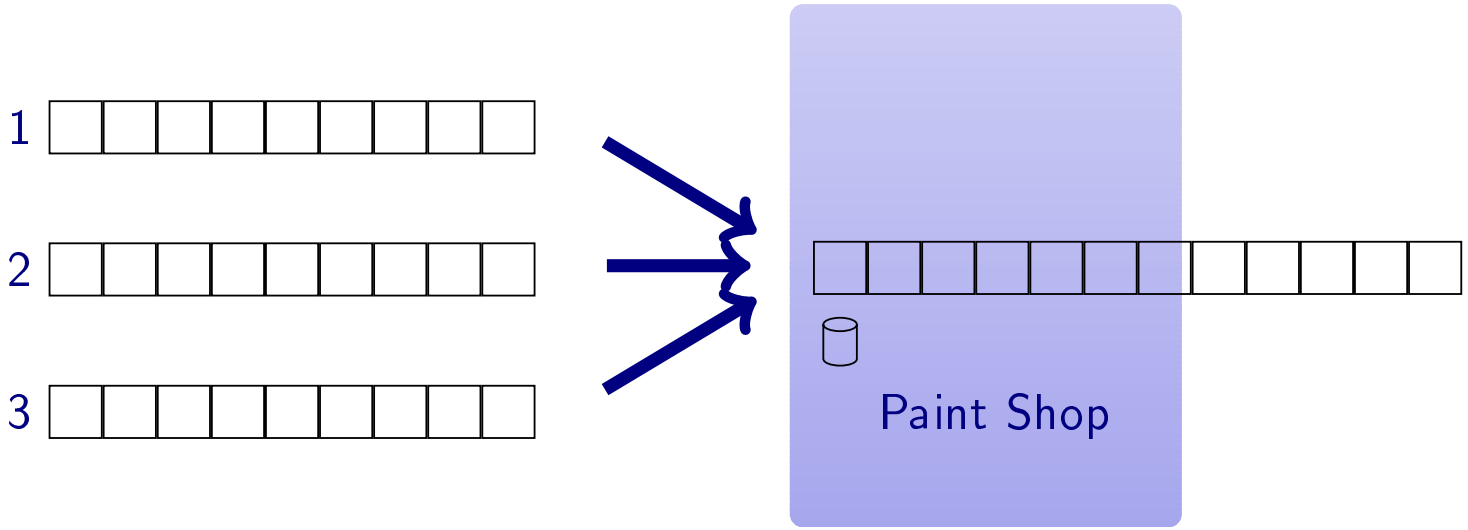

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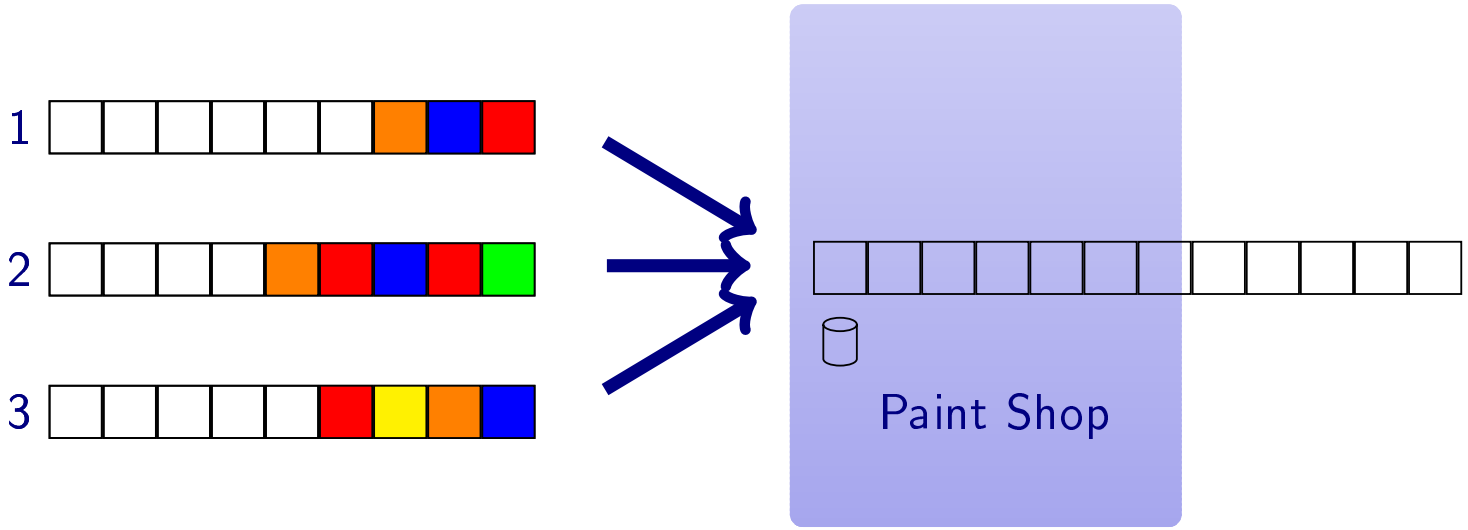

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  - ➡ Instructions to produce a sequence of storage line numbers from incoming colour requests
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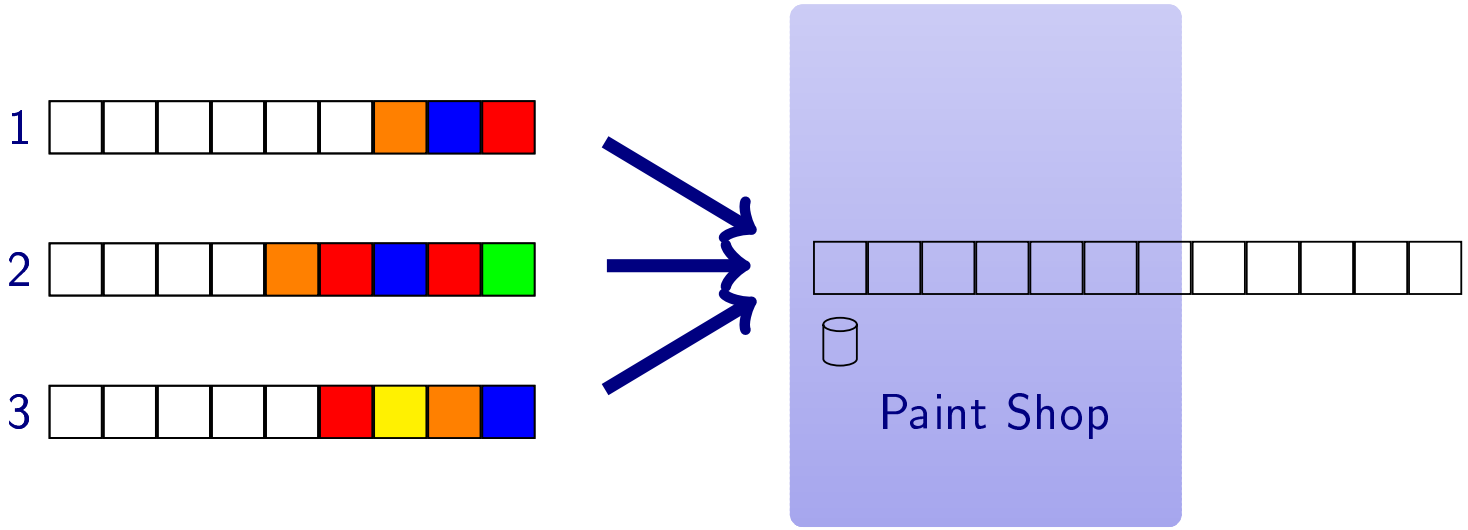

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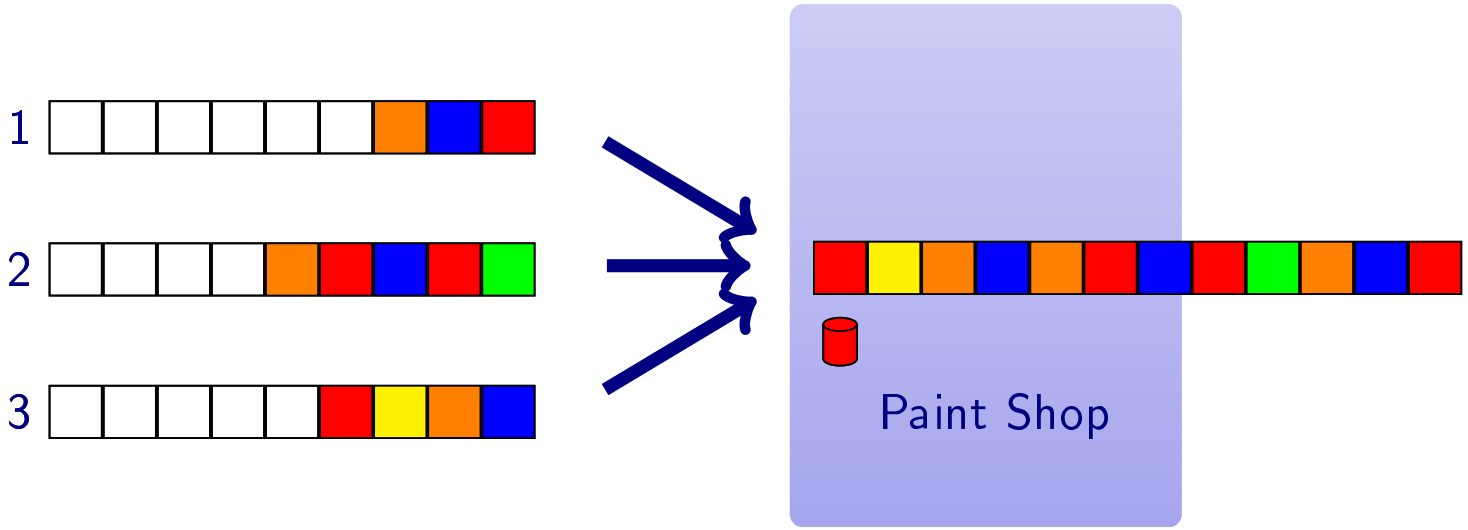

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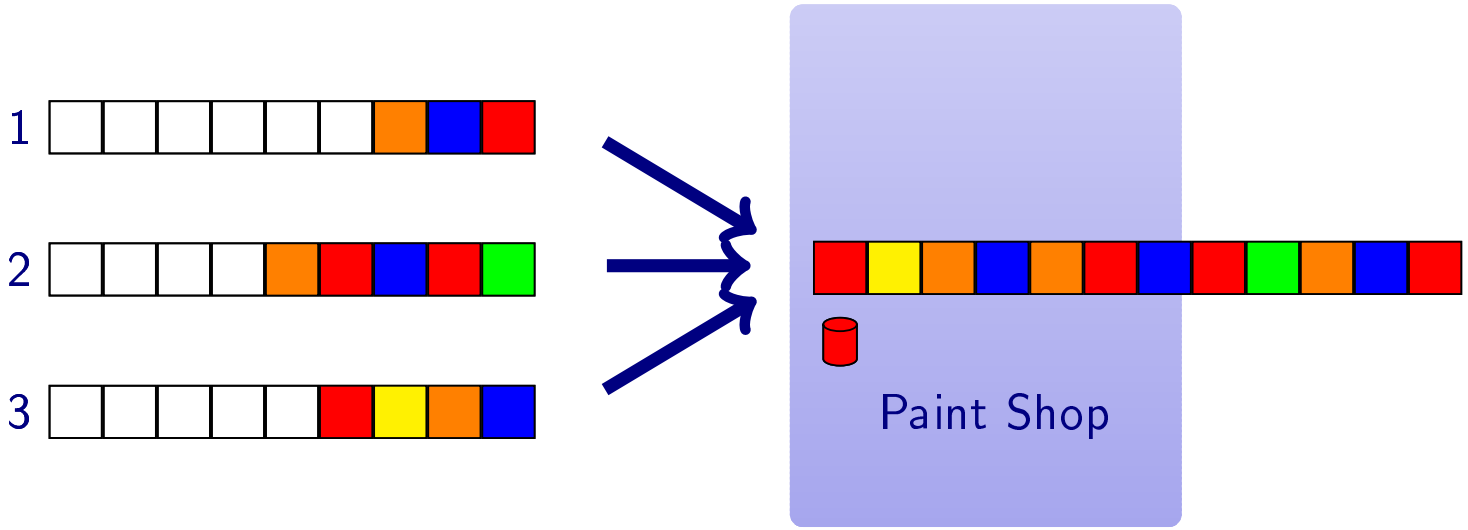

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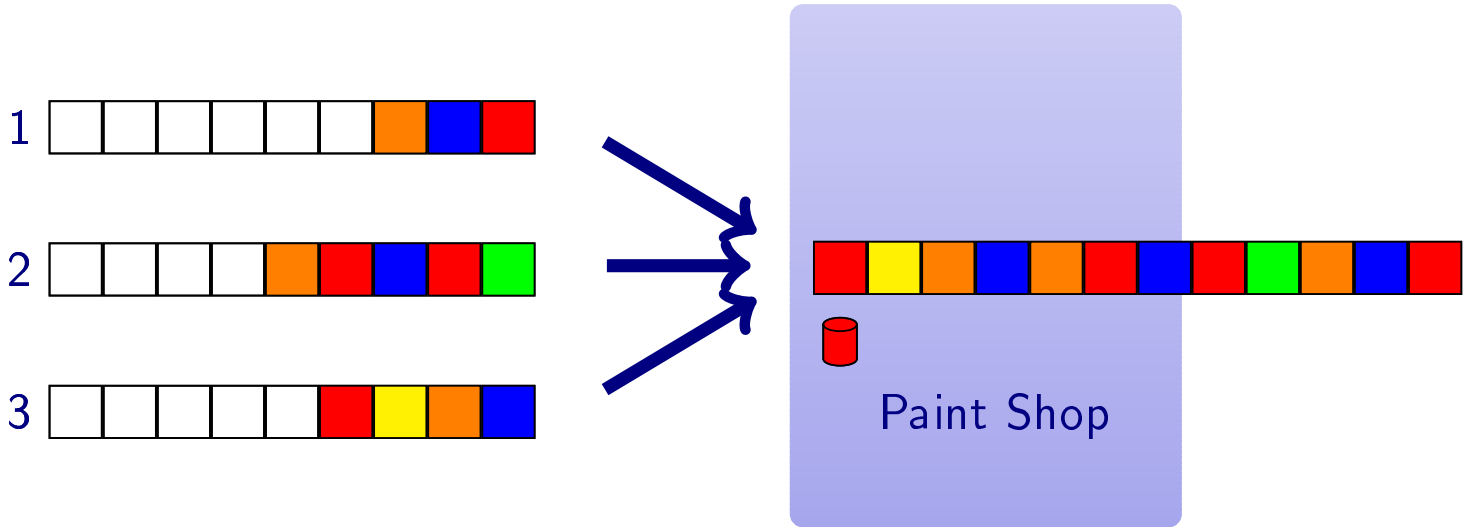

- Example: Retrieval Sequence: 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3
  - ➡ Number of colour changes: 11

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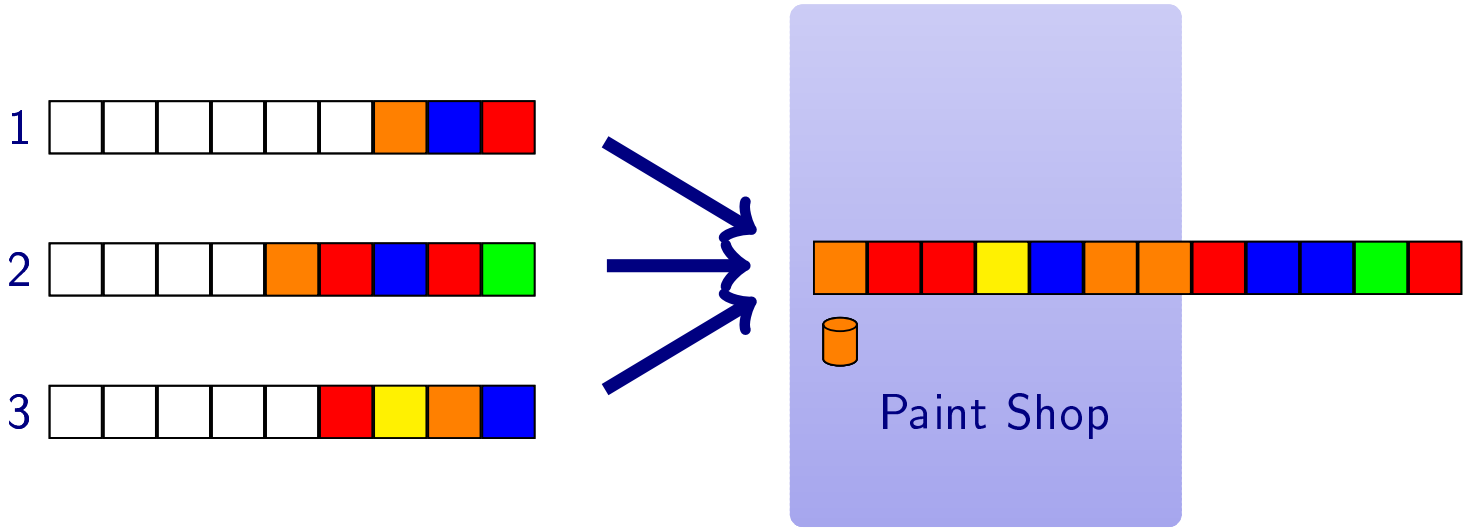

- Example: Retrieval Sequence: 1, 2, 3, 1, 2, 3, 1, 2, 3, 2, 3, 2
  - ➡ Number of colour changes: 8

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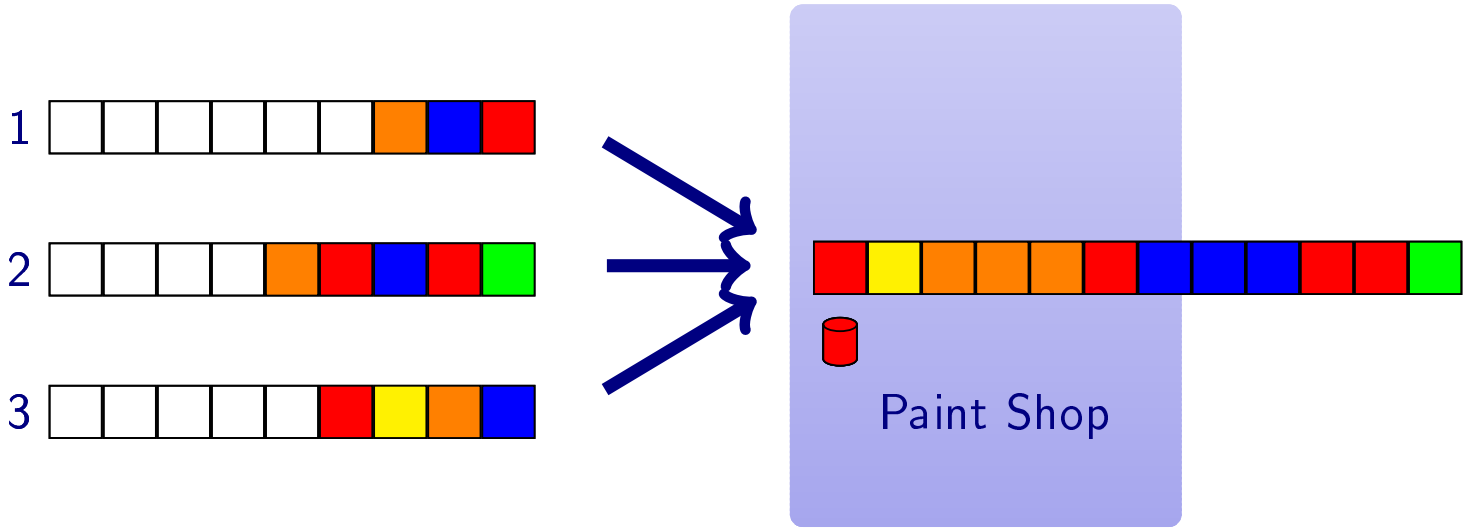

- Example: Retrieval Sequence: 2, 1, 2, 2, 1, 3, 2, 2, 1, 3, 3, 3
  - ➡ Number of colour changes: 6 (optimal!)

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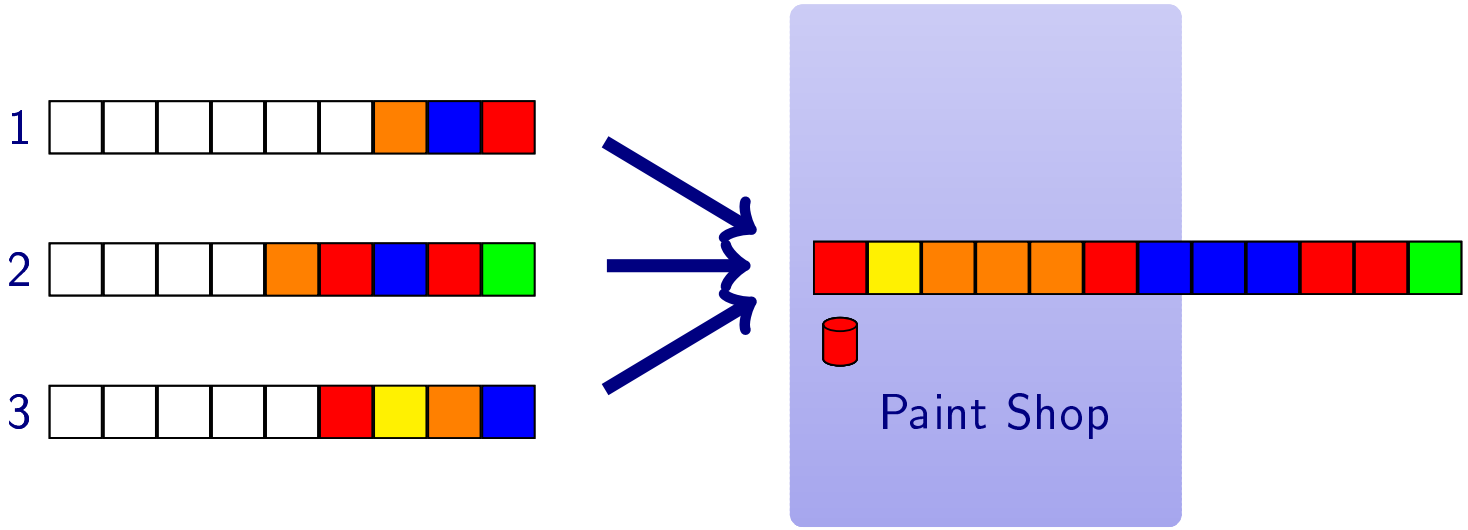

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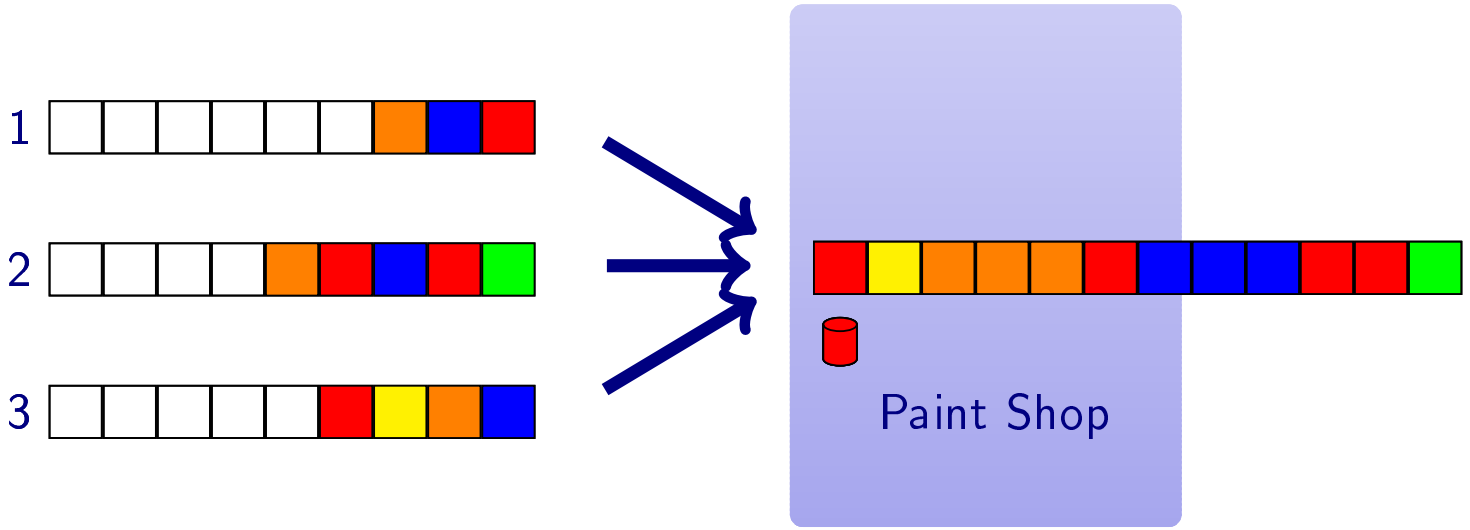

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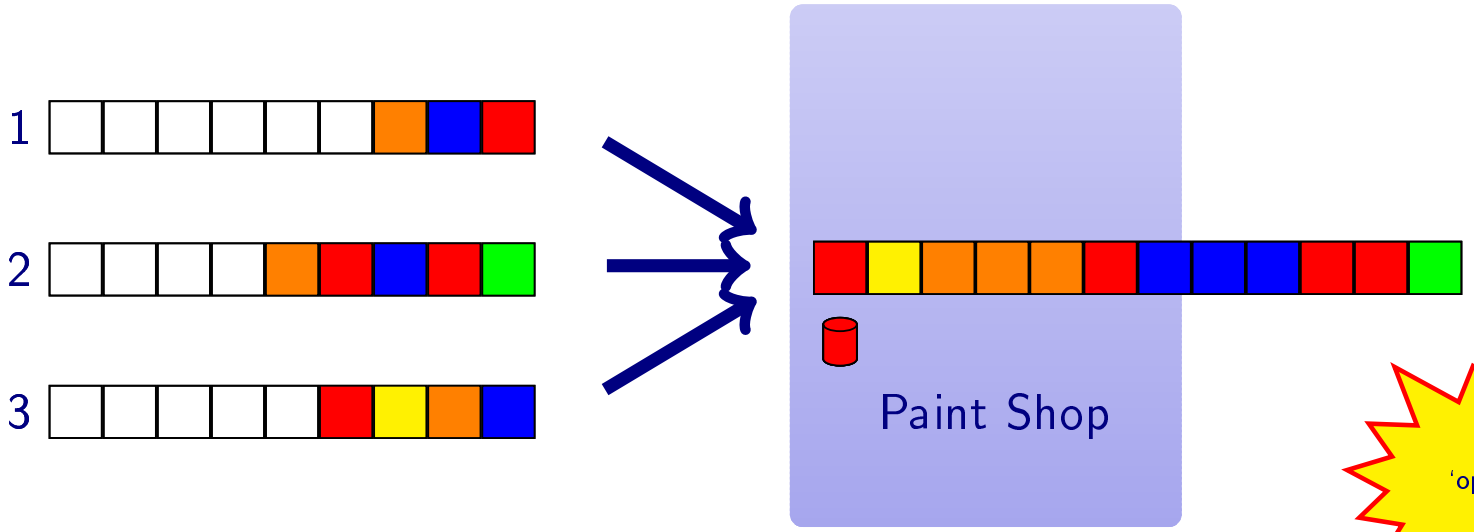

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what does 'optimal' mean anyway?!

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Abstract Model:

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Precise mathematical model:

Abstract Model: Given a list of colour requests for each of n storage lines, find a sequence of storage line numbers (1 to n), such that the resulting colour list has minimal possible colour changes.

▷ **NOTE:**

- Solving the model means presenting an **Algorithm** that leads to an optimal solution for every possible input!

  ➡ Algorithms are independent of the actual input colour lists!

- It is well possible that there is no such algorithm!

  ➡ Instead solve a (hopefully) easier problem: Give an **approximation algorithm** to find a 'good' resulting colour list.

Precise mathematical model: Given n sequences of colours $c_{i,j}$ $(1 \le i \le n,\ j \in \mathbb{N})$ find a mapping $f : \mathbb{N} \to \{1, \ldots, n\}$ such that $\#\{k \mid c_{f(k),j(k)} \neq c_{f(k+1),j(k+1)}\}$ is minimised, where

$$j(k) := \begin{cases} 1 & \text{if } k{=}1 \text{ or } f(k'){\neq}f(k)\ \forall k'{<}k \\ j(k'){+}1 & \text{otherwise, where } k' \text{ is maximal with } k'{<}k, f(k'){=}f(k) \end{cases}$$

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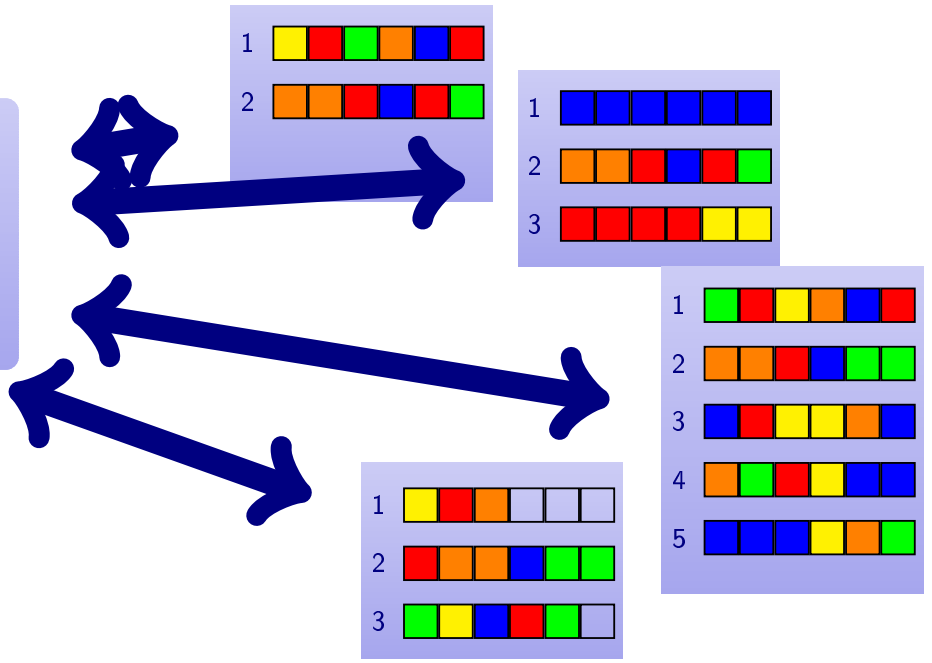

Model

Data Sets

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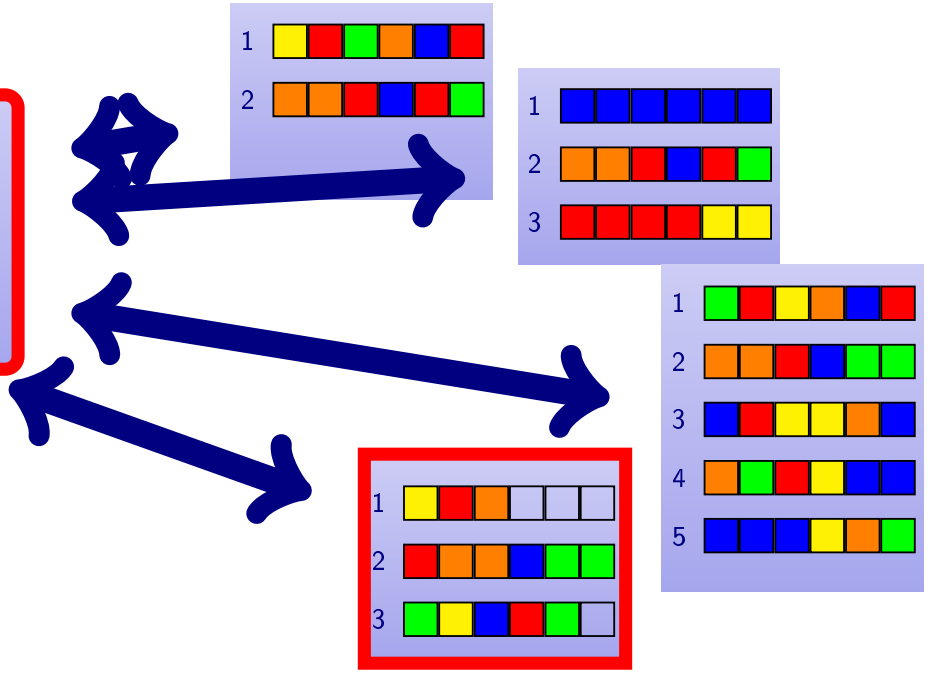

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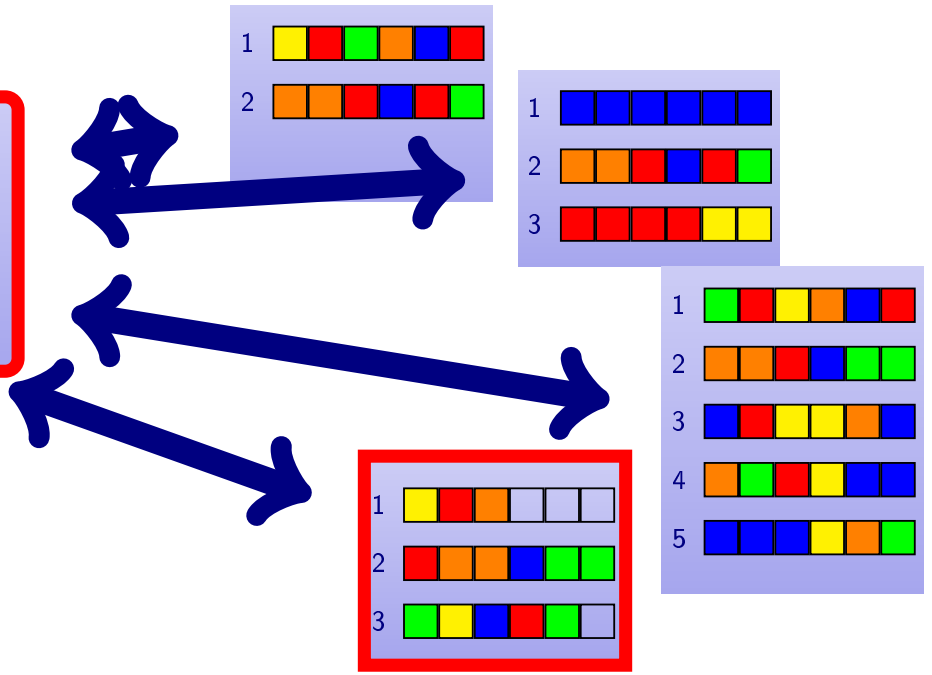

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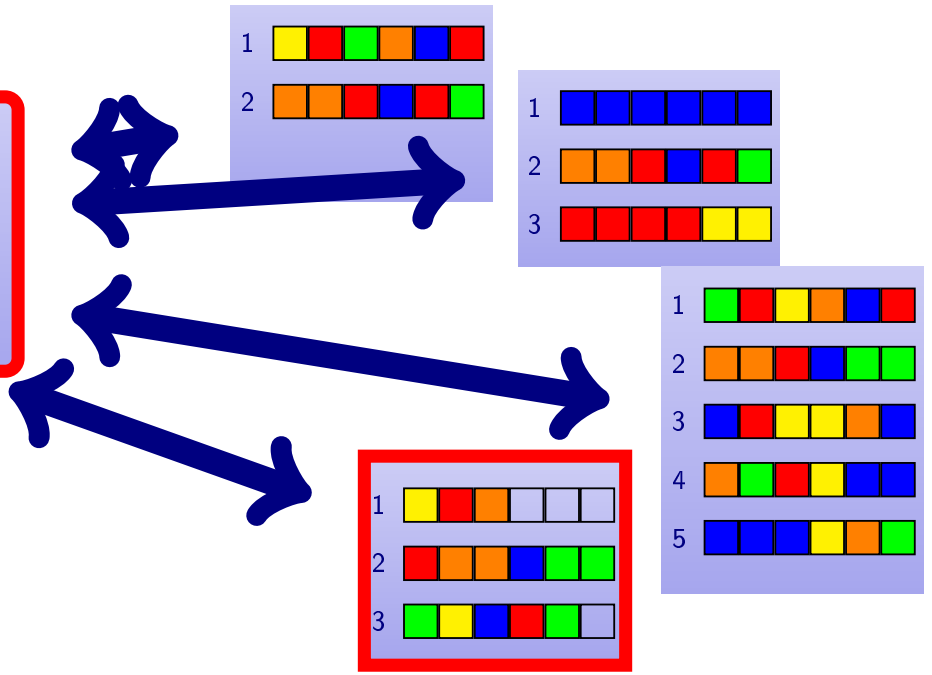

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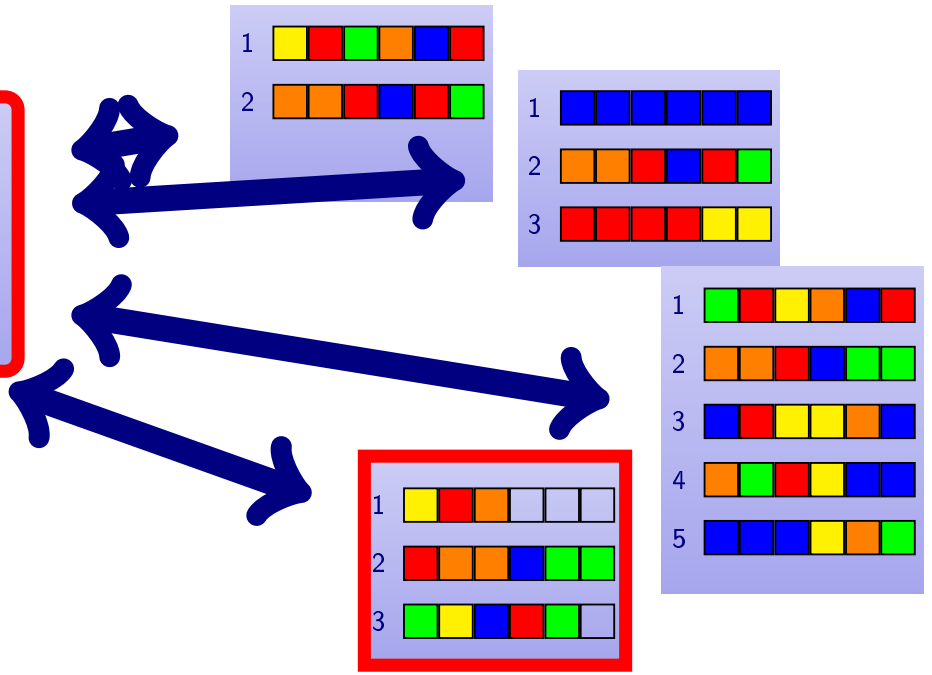

Model

Data Sets

- An instance is a mathematical model, together with one associated data set
- Worst case: An instance where a given algorithm performs worst
- Average case analysis: Average of the algorithm performance over all instances
- The Objective of an optimization problem is the quantity to be minimized/maximized

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- An **Approximation algorithm** guarantees that for all (specified) instances the produced solution has objective value not worse than a given factor times the optimum

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling; Mathematical Background: Branch & Bound
- Branch & Bound, Cutting Planes; More Examples; Combinatorial Optimization
- Combinatorial Optimization: Examples, Graphs, Algorithms
- Complexity Theory
- Nonlinear Optimization
- Scheduling
- Lot Sizing
- Multicriteria Optimization
- Oral exam