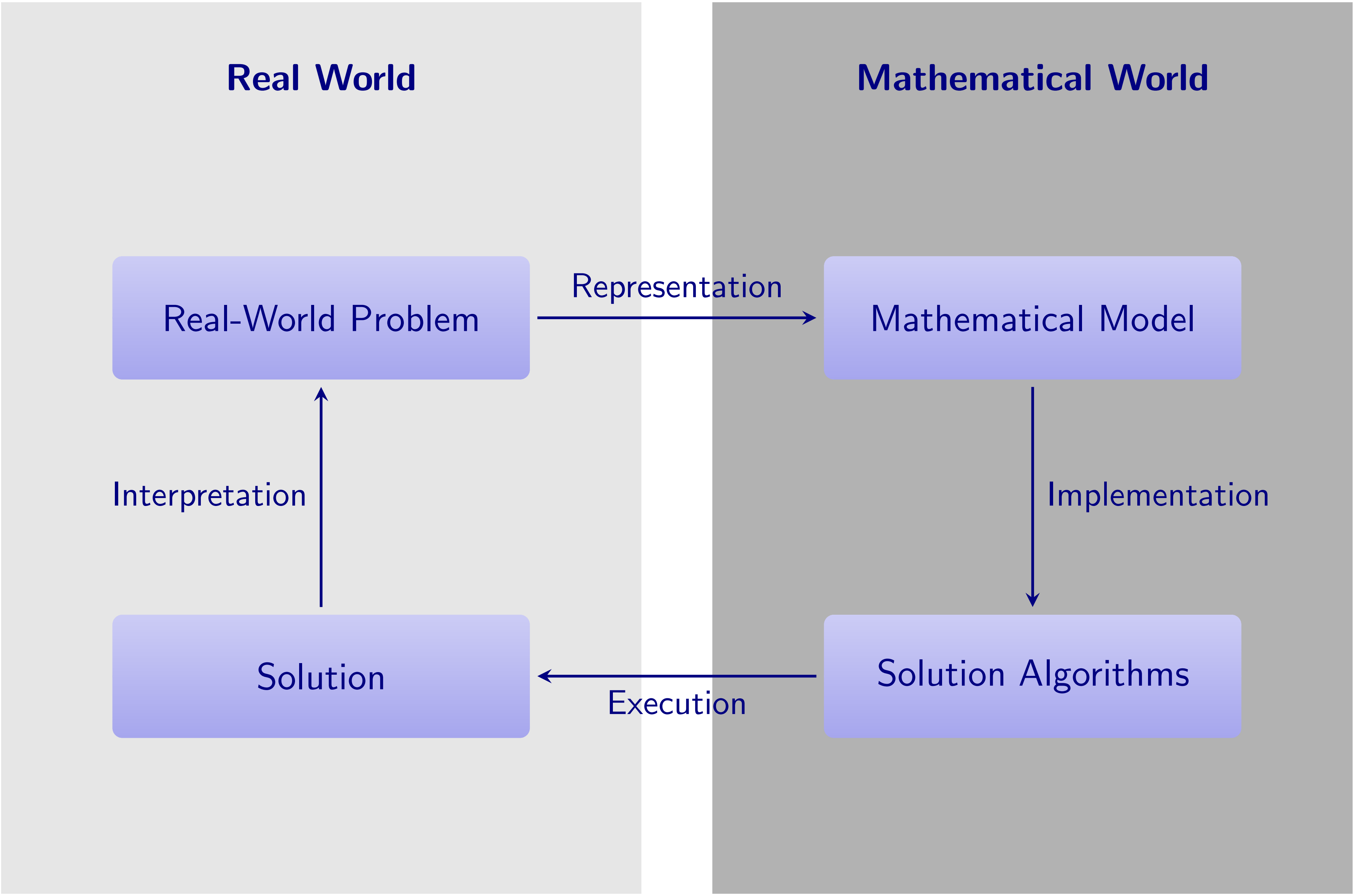
Mathematical Tools for Engineering and Management

Lecture 2

26 Oct 2011

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling; Mathematical Background: Branch & Bound
- Branch & Bound, Cutting Planes; More Examples; Combinatorial Optimization
- Combinatorial Optimization: Examples, Graphs, Algorithms
- Complexity Theory
- Nonlinear Optimization
- Scheduling
- Lot Sizing
- Multicriteria Optimization
- Oral exam


Real World
Mathematical World
Real-World Problem
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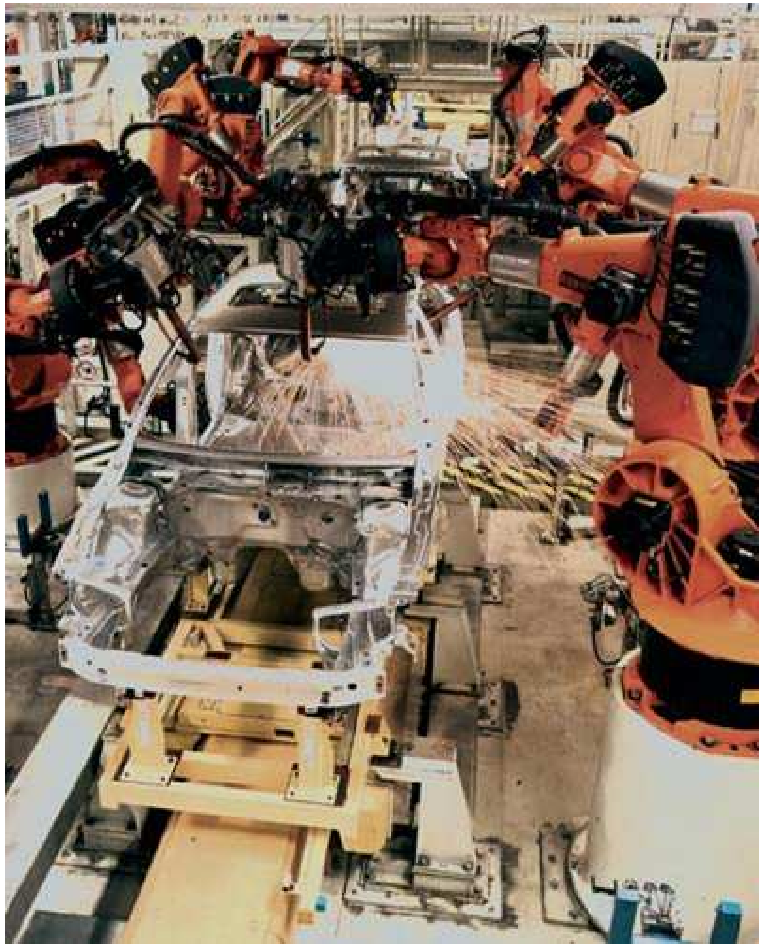
Example: Robotics



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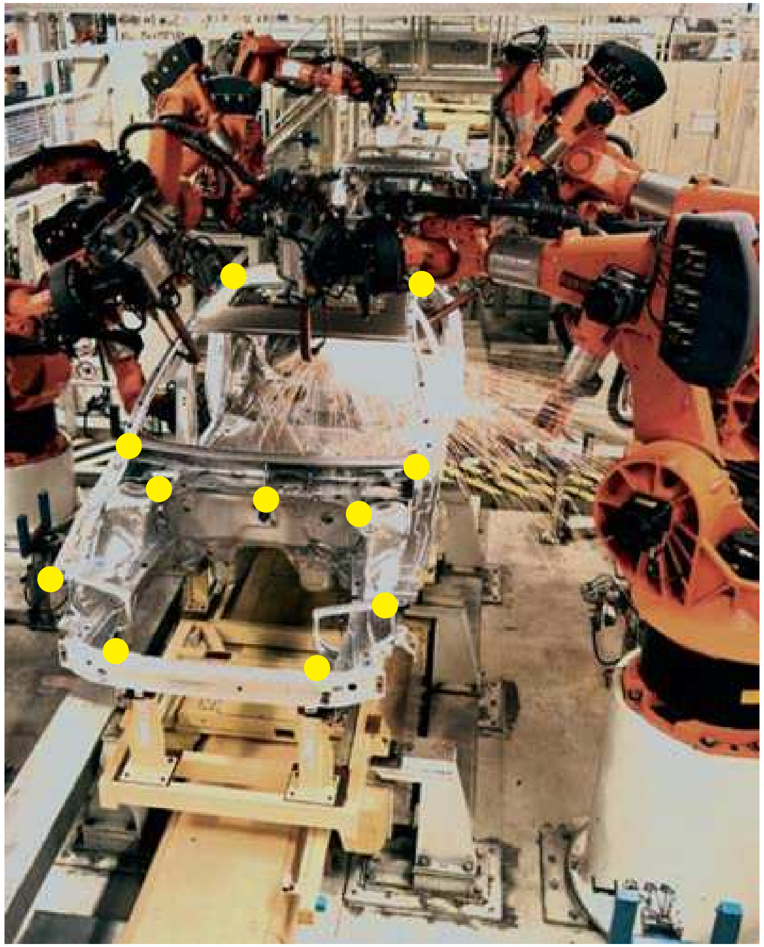

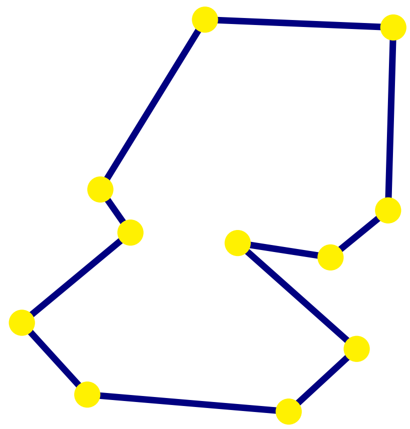

▷ Problem to solve: find an optimal order of welding points!

Abstract Model: Given a set of welding points together with processing times for each point and travel times to travel from every point to every other, find an order of the points such that processing the points in this order takes minimal time.

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▷ Reformulation as abstract (mathematical) problem enables us to use results and solutions previously encountered!

➡ Know your examples!

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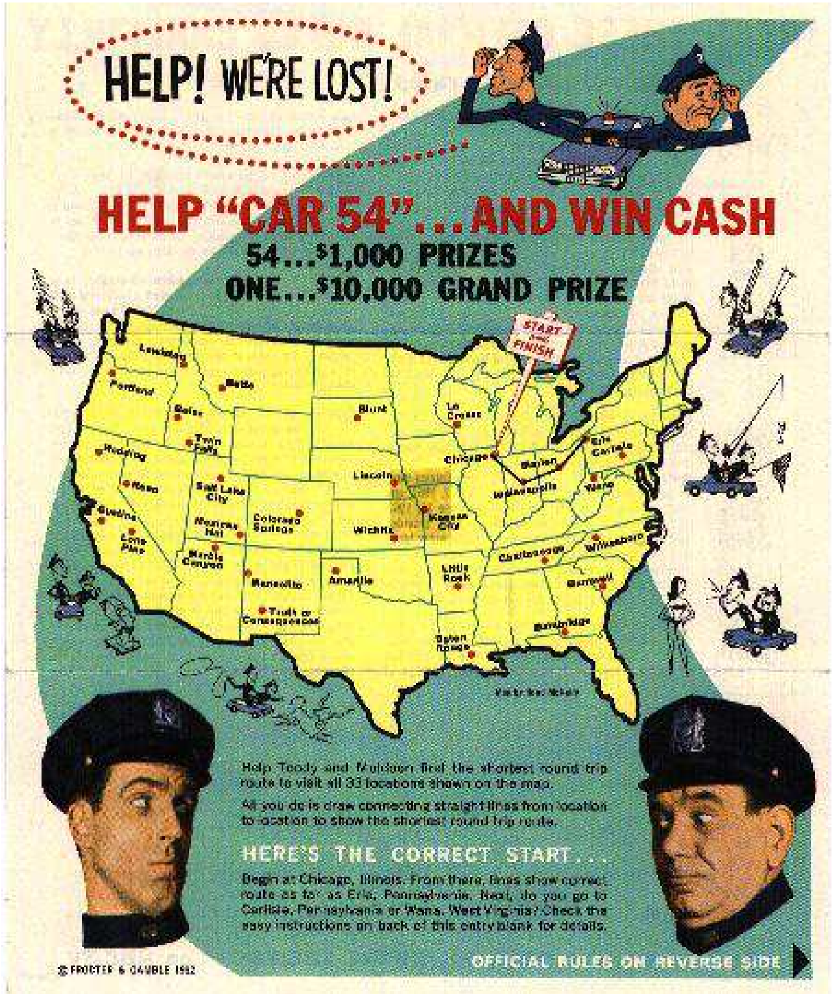
▷ Well-studied problem since the 20th century



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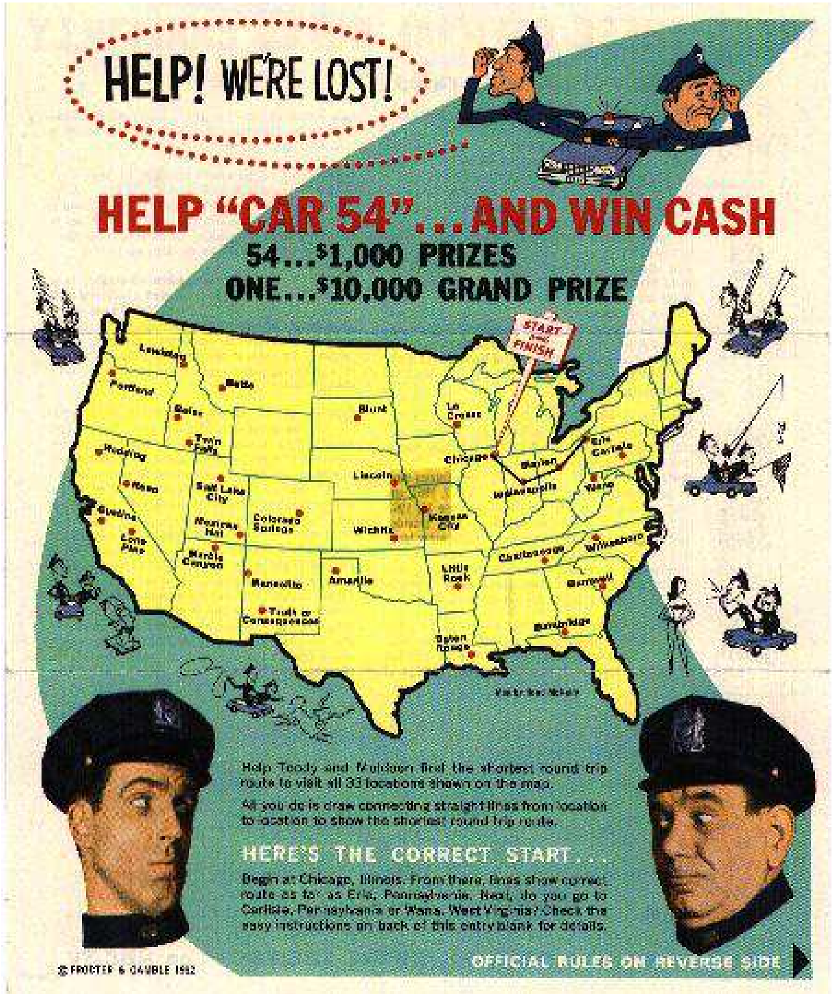
- ▷ Well-studied problem since the 20th century
- ▷ Algorithms and solution approaches from different areas in optimization



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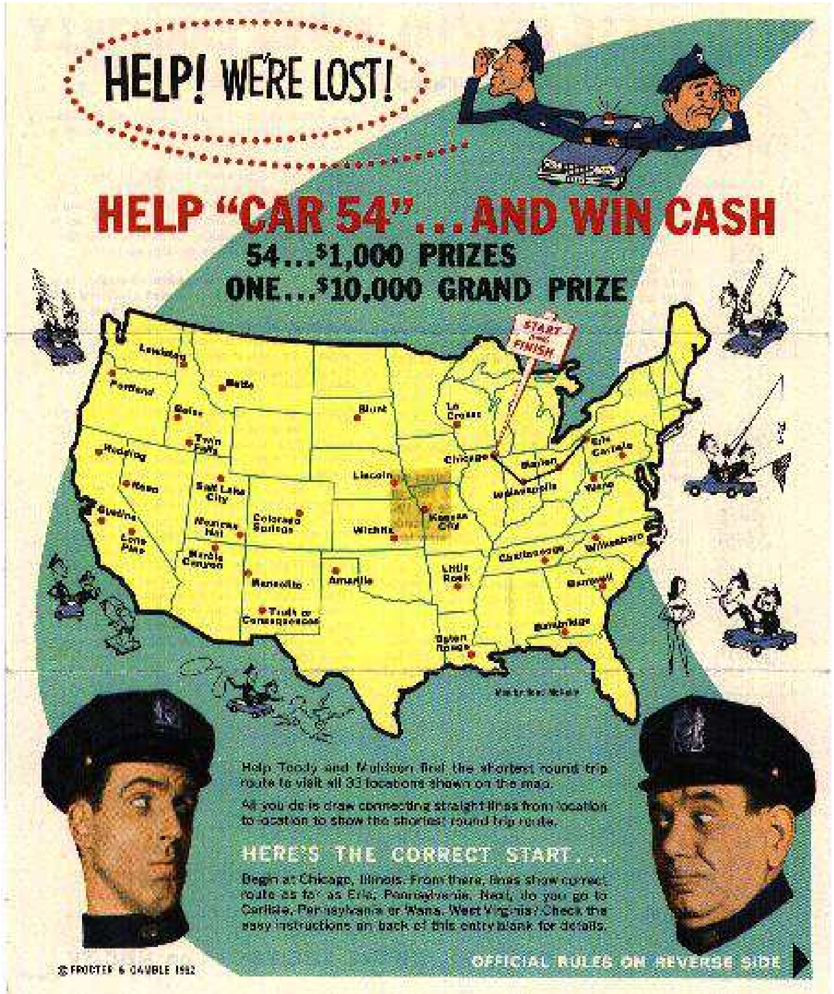

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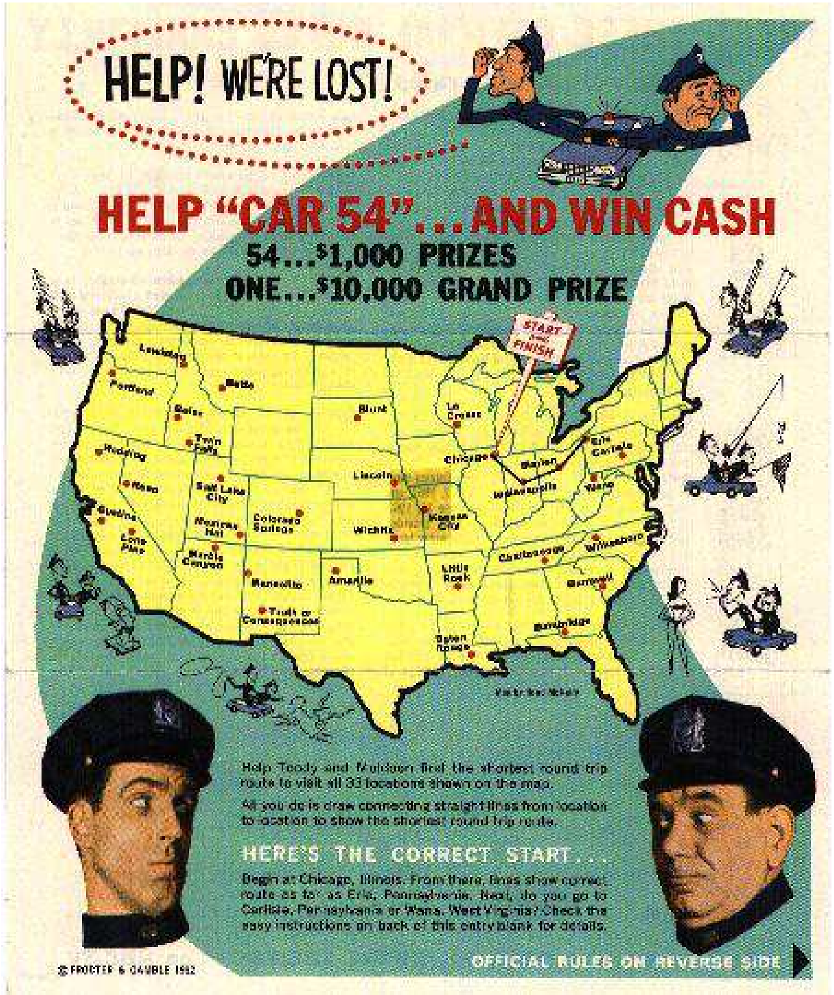

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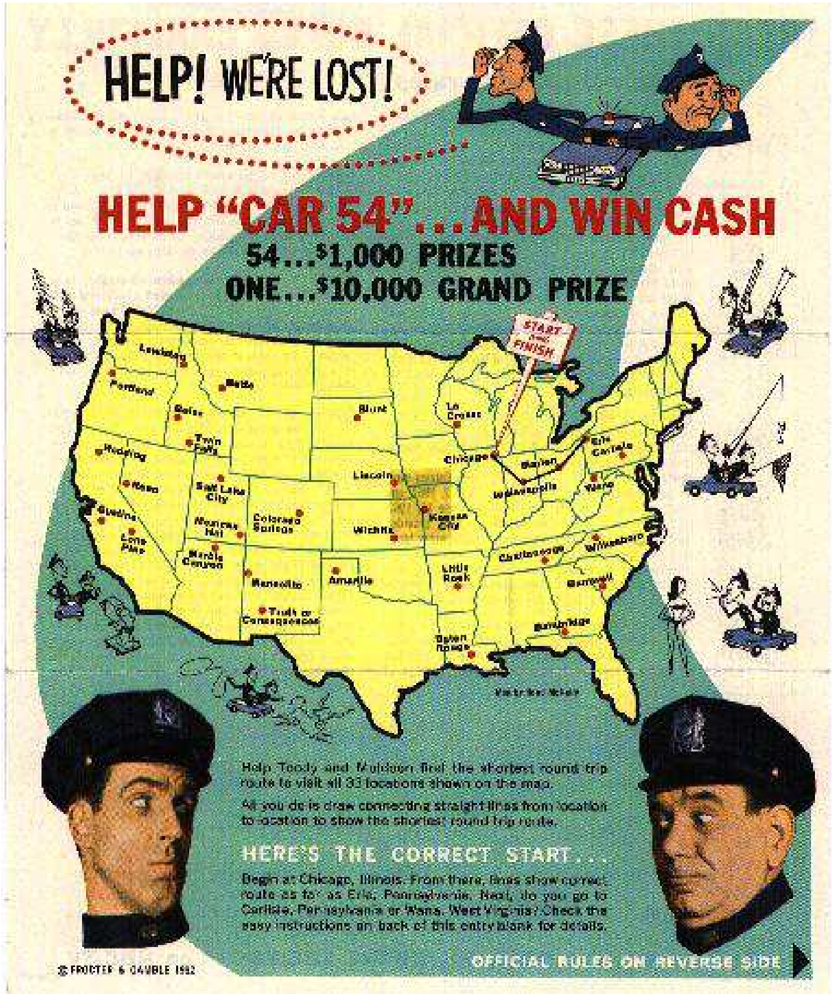

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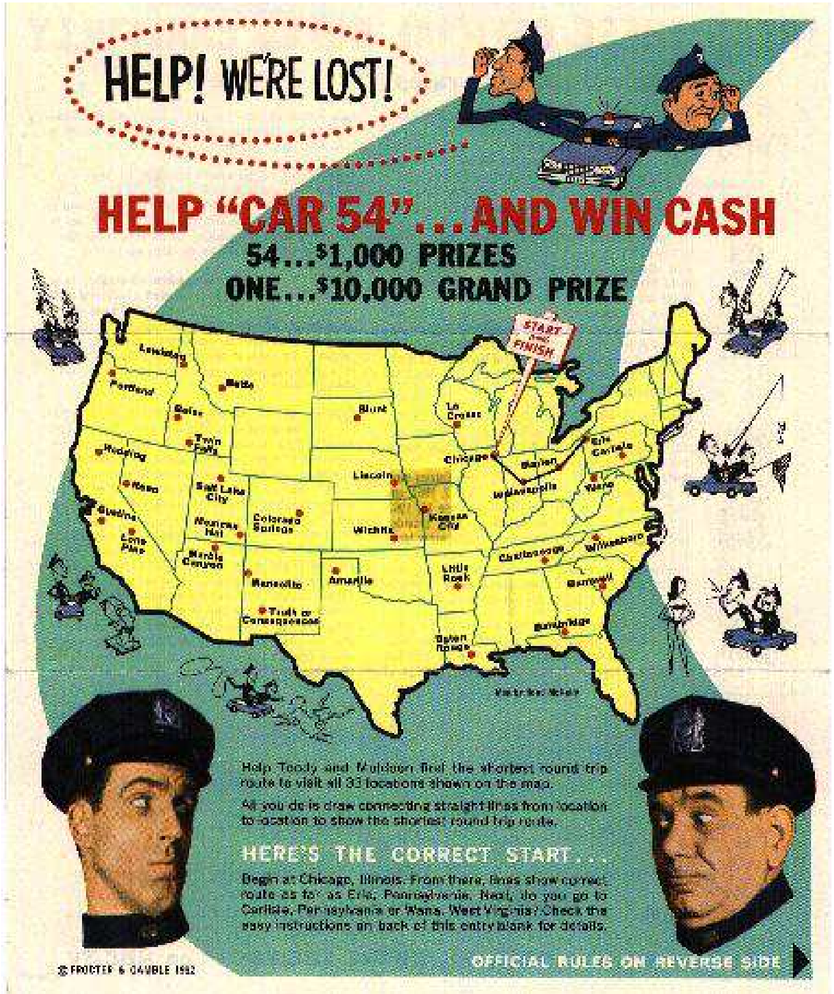
- ▷ Typical problem in discrete optimization:
  - Combinatorial explosion (vast number of feasible solutions already for moderate input sizes)
  - NP-complete (➡ P-vs-NP Problem)
  - Optimality of a given tour is hard to prove (➡ Heuristics for suboptimal results)



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- Mathematical Background: Cuts, Branch & Bound
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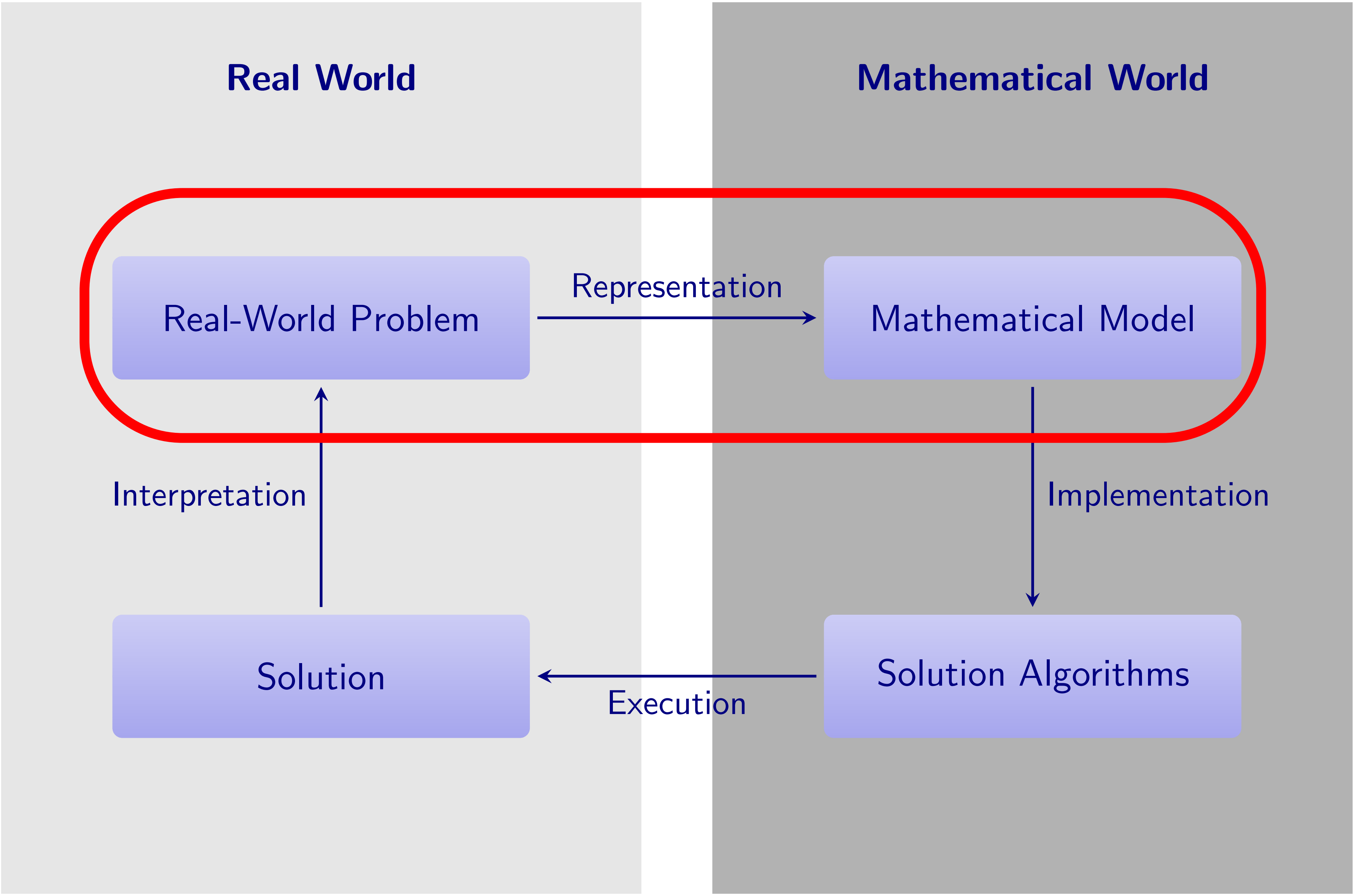
- Exam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Example: Production planning

- Production Planning in Automobile Industry

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Product	Beetle	Cabrio
Revenue	\$10000	\$20000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

▷ Production Planning in Automobile Industry

Product	**Beetle**	**Cabrio**
Revenue	$10000	$20000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 4500kg

▷ Production Planning in Automobile Industry

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➡ Question: How many cars of each type should be produced to maximize the profit?

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
	5	6				
		available capacities:		50	70	4500

Production planning – some “solutions”

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
	5	6	170	43	62	4400
			available capacities:	50	70	4500

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
			available capacities:	50	70	4500

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
	4	7	180			
		available capacities:		50	70	4500

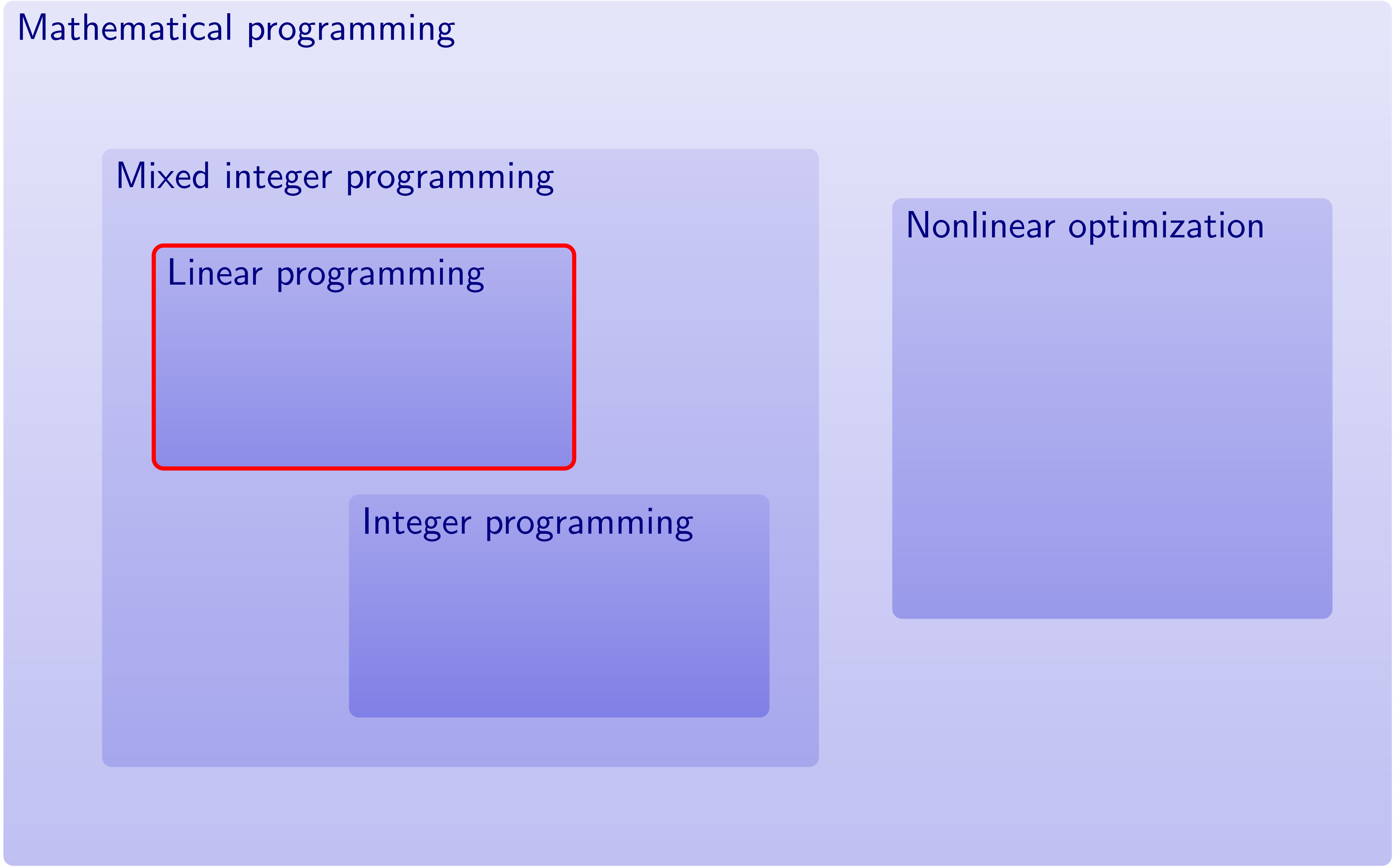
	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
another feasible solution	4	7	180	41	65	4400
		available capacities:		50	70	4500

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
another feasible solution	4	7	180	41	65	4400
	2	9				
			available capacities:	50	70	4500

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
another feasible solution	4	7	180	41	65	4400
	2	9	200			
		available capacities:		50	70	4500

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
another feasible solution	4	7	180	41	65	4400
infeasible solution	2	9	200	37	71	4400
		available capacities:		50	70	4500

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
another feasible solution	4	7	180	41	65	4400
infeasible solution	2	9	200	37	71	4400
optimal solution	0	10	200	30	70	4000
			available capacities:	50	70	4500


Mathematical programming
Mixed integer programming
Linear programming
Integer programming
Nonlinear optimization

Mathematical Programming

$\neq$

Computer Programming

Mathematical Programming

$\neq$

Computer Programming

Mathematical Programming

$=$

Planning with Mathematics

Mathematical Programming
$\neq$
Computer Programming

Mathematical Programming
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Mathematical Programming
uses
Computing Power of Computers

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- Mathematical Programming always involves
  optimization (i.e. either minimization or maximization) of some quantity
  subject to certain restrictions concerning this quantity

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(for example: products, cities, machines, types of raw material, ...)

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C Constraints: Relationships that have to hold between variables and parameters
(for example: maximal number of items that can be produced by a machine, minimal number of shops to open, budget for buying raw material, ...)

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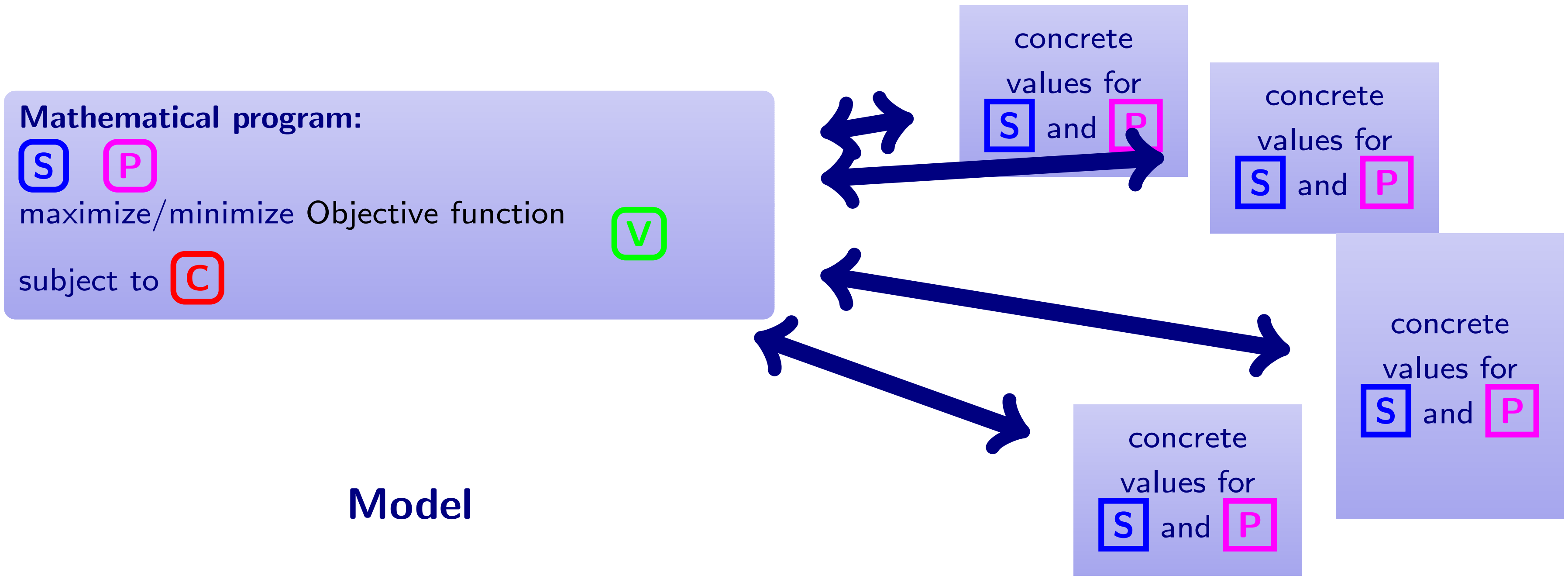
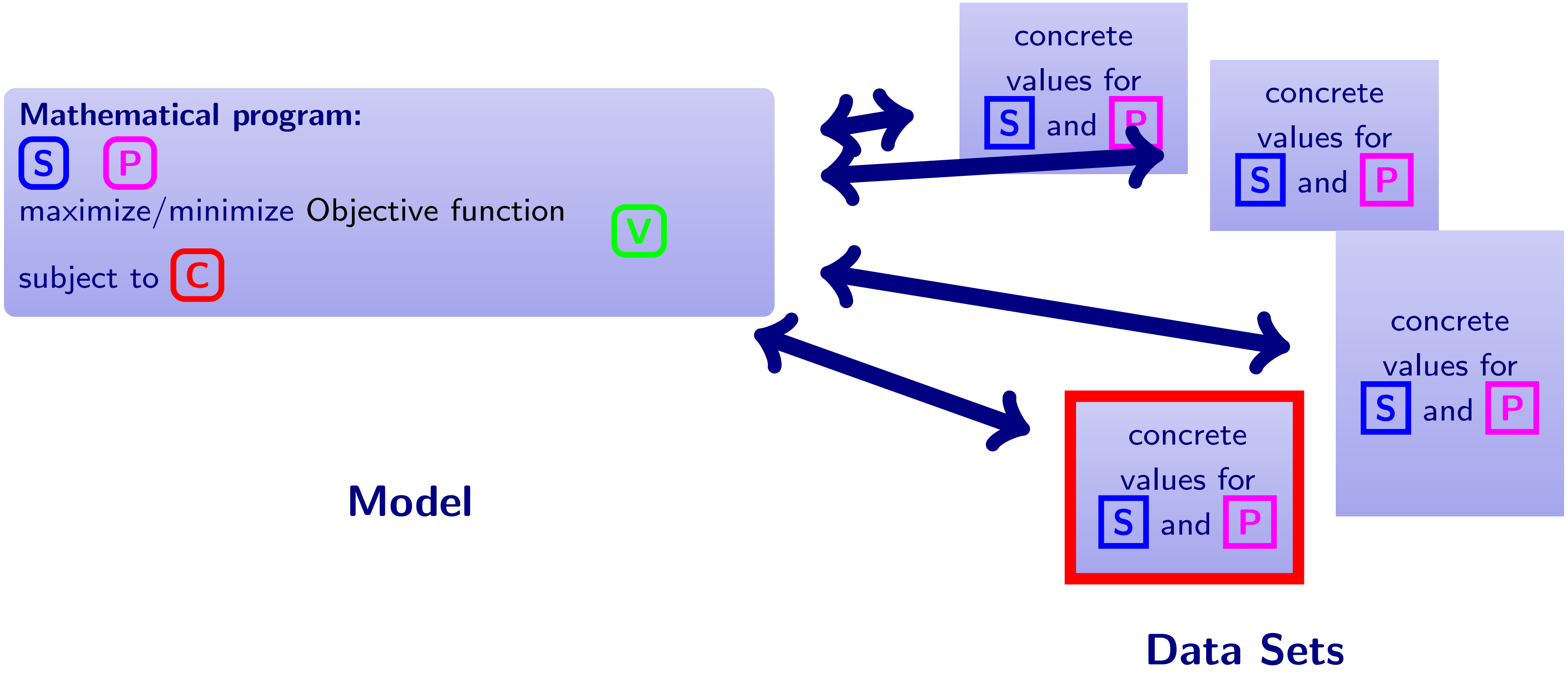
▷ Mathematical Program: Collection of constraints and variables together with an Objective function to be maximized/minimized

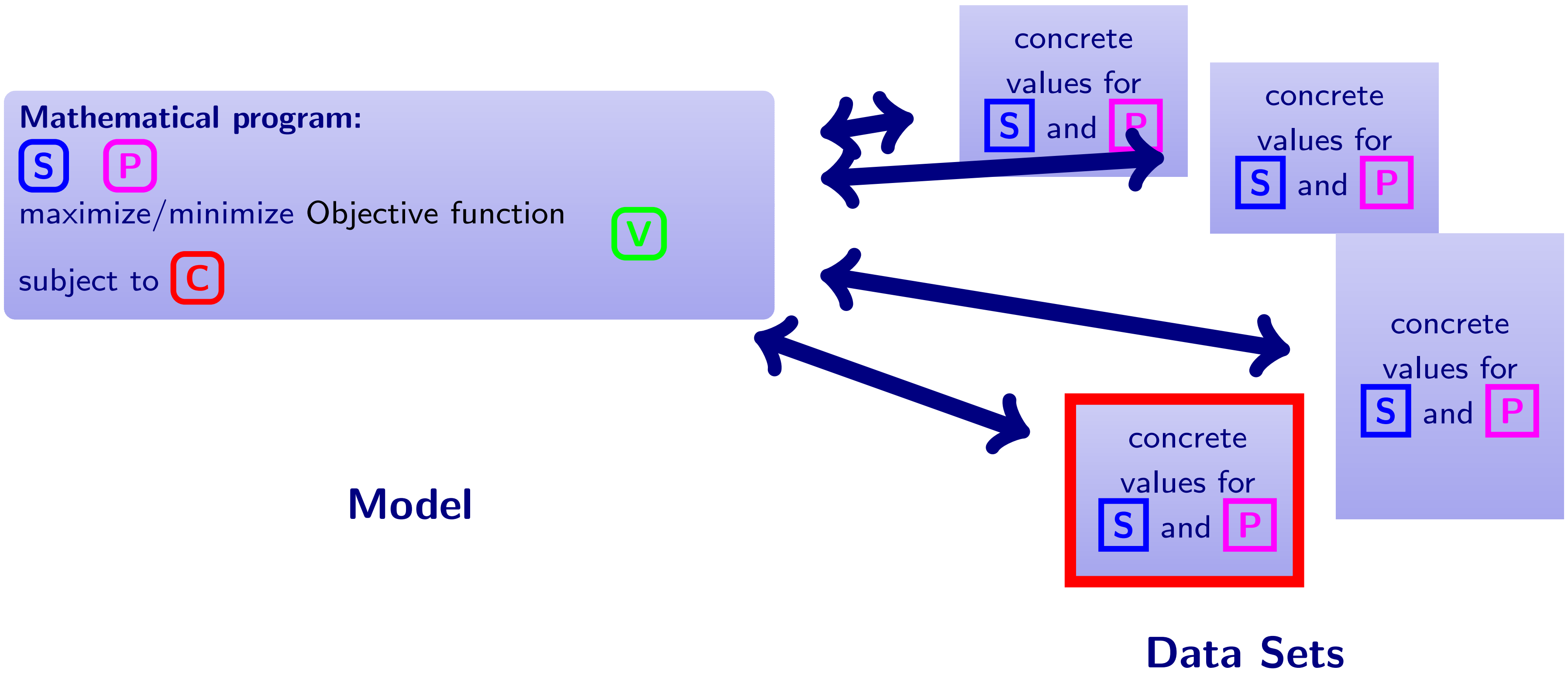


Model

Data Sets



▷ Input: Values for sets and parameters.



- ▷ Input: Values for sets and parameters.
- ▷ Output: Values for all variables such that the objective function value is maximal/minimal and the constraints are respected

Back to the production planning example

▷ Production Planning in Automobile Industry

Product	**Beetle**	**Cabrio**
Revenue	$10000	$20000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 4500kg

➡ Question: How many cars of each type should be produced to maximize the profit?

V Variables: x_{beetle}, x_{cabrio} ➡ number of cars of respective type to produce

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S Sets: $C = \{\text{beetle}, \text{cabrio}\}$ ➡ set of car types

V Variables: x_{beetle}, x_{cabrio} ➡ number of cars of respective type to produce

$x_c \geq 0$ for all $c \in C$

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P Parameters: r_c ➡ revenue for car type c

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$x_c \geq 0$ for all $c \in C$

S Sets: $C = \{\text{beetle}, \text{cabrio}\}$ ➡ set of car types

P Parameters: r_c ➡ revenue for car type c

$r_{\text{beetle}} = 10000$, $r_{\text{cabrio}} = 20000$

V Variables: x_{beetle}, x_{cabrio} ➡ number of cars of respective type to produce

$x_c \geq 0$ for all $c \in C$

S Sets: $C = \{\text{beetle}, \text{cabrio}\}$ ➡ set of car types

P Parameters: r_c ➡ revenue for car type c

$r_{\text{beetle}} = 10000$, $r_{\text{cabrio}} = 20000$

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ρ_c ➡ raw material needed for car type c

$\rho_{\text{beetle}} = 400$, $\rho_{\text{cabrio}} = 400$

V Variables: $x_{\text{beetle}}, x_{\text{cabrio}}$ ➡ number of cars of respective type to produce

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ρ_c ➡ raw material needed for car type c

$\rho_{\text{beetle}} = 400,\ \rho_{\text{cabrio}} = 400$

R ➡ total raw material available ($R = 4500$)

V Variables: x_{beetle}, x_{cabrio} ➡ number of cars of respective type to produce

$x_c \geq 0$ for all $c \in C$

S Sets: $C = \{\text{beetle}, \text{cabrio}\}$ ➡ set of car types

$D = \{\text{manufacturing}, \text{assembly}\}$ ➡ set of departments

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R ➡ total raw material available ($R = 4500$)

T_d ➡ time capacity for department $d \in D$

V Variables: x_{beetle}, x_{cabrio} ➡ number of cars of respective type to produce

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R ➡ total raw material available ($R = 4500$)

T_d ➡ time capacity for department $d \in D$

$T_{\text{manufacturing}} = 50$, $T_{\text{assembly}} = 70$

V Variables: x_{beetle}, x_{cabrio} ➡ number of cars of respective type to produce

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R ➡ total raw material available ($R = 4500$)

T_d ➡ time capacity for department $d \in D$

$T_{\text{manufacturing}} = 50$, $T_{\text{assembly}} = 70$

$t_{c,d}$ ➡ time needed for car type c in department d

V Variables: x_{beetle}, x_{cabrio} ➡ number of cars of respective type to produce

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R ➡ total raw material available ($R = 4500$)

T_d ➡ time capacity for department $d \in D$

$T_{\text{manufacturing}} = 50$, $T_{\text{assembly}} = 70$

$t_{c,d}$ ➡ time needed for car type c in department d

$t_{\text{beetle,manufacturing}} = 5$, $t_{\text{cabrio,manufacturing}} = 3$

$t_{\text{beetle,assembly}} = 4$, $t_{\text{cabrio,assembly}} = 7$

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maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

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C Constraints:

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maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C Constraints:

(total raw material available)

▷ Objective function:

maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C Constraints:

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

▷ Objective function:

maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C Constraints:

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department)

▷ Objective function:

maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C Constraints:

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio,d}} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

▷ Objective function:

maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C Constraints:

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio,d}} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

(non-negativity of variables)

▷ Objective function:

maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C Constraints:

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio,d}} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

(non-negativity of variables) $x_c \geq 0$ for all $c \in C$

▷ Objective function:

maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C Constraints:

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio,d}} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

(non-negativity of variables) $x_c \geq 0$ for all $c \in C$

▷ Data Set: set of car types: $C = \{\text{beetle}, \text{cabrio}\}$

set of department: $D = \{\text{manufacturing}, \text{assembly}\}$

$r_{\text{beetle}} = 10000,\ r_{\text{cabrio}} = 20000$

$\rho_{\text{beetle}} = 400,\ \rho_{\text{cabrio}} = 400, \quad R = 4500$

$T_{\text{manufacturing}} = 50,\ T_{\text{assembly}} = 70$

$t_{\text{beetle,manufacturing}} = 5,\ t_{\text{cabrio,manufacturing}} = 3$

$t_{\text{beetle,assembly}} = 4,\ t_{\text{cabrio,assembly}} = 7$

▷ Model (Linear Program):

$$
\begin{array}{llrcl}
\text{maximize} & \text{(total revenue)} & \displaystyle\sum_{c\in C} r_c \cdot x_c & & \\
\text{subject to} & \text{(total raw material)} & \displaystyle\sum_{c\in C} \rho_c x_c & \leq & R \\
 & \text{(time in departments)} & \displaystyle\sum_{c\in C} t_{c,d} x_c & \leq & T_d \ \text{ for all } d \in D \\
 & \text{(non-negativity)} & x_c & \geq & 0 \ \text{ for all } c \in C
\end{array}
$$

- Model (Linear Program):

$$
\begin{array}{llrcl}
\text{maximize} & \text{(total revenue)} & \sum_{c\in C} r_c \cdot x_c & & \\
\text{subject to} & \text{(total raw material)} & \sum_{c\in C} \rho_c x_c & \leq & R \\
& \text{(time in departments)} & \sum_{c\in C} t_{c,d} x_c & \leq & T_d \ \text{ for all } d \in D \\
& \text{(non-negativity)} & x_c & \geq & 0 \ \text{ for all } c \in C
\end{array}
$$

- Data Set: set of car types: $C = \{\text{beetle}, \text{cabrio}\}$

  set of department: $D = \{\text{manufacturing}, \text{assembly}\}$

  $r_{\text{beetle}} = 10000,\ r_{\text{cabrio}} = 20000$

  $\rho_{\text{beetle}} = 400,\ \rho_{\text{cabrio}} = 400, \quad R = 4500$

  $T_{\text{manufacturing}} = 50,\ T_{\text{assembly}} = 70$

  $t_{\text{beetle,manufacturing}} = 5,\ t_{\text{cabrio,manufacturing}} = 3$

  $t_{\text{beetle,assembly}} = 4,\ t_{\text{cabrio,assembly}} = 7$

▷ Model (Linear Program):

maximize	(total revenue)	$\sum_{c\in C} r_c \cdot x_c$	**Objective**
subject to	(total raw material)	$\sum_{c\in C} \rho_c x_c \leq R$	
	(time in departments)	$\sum_{c\in C} t_{c,d} x_c \leq T_d$ for all $d \in D$	
	(non-negativity)	$x_c \geq 0$ for all $c \in C$	

▷ Data Set: set of car types: $C = \{\text{beetle}, \text{cabrio}\}$

set of department: $D = \{\text{manufacturing}, \text{assembly}\}$

$r_{\text{beetle}} = 10000$, $r_{\text{cabrio}} = 20000$

$\rho_{\text{beetle}} = 400$, $\rho_{\text{cabrio}} = 400$, $R = 4500$

$T_{\text{manufacturing}} = 50$, $T_{\text{assembly}} = 70$

$t_{\text{beetle,manufacturing}} = 5$, $t_{\text{cabrio,manufacturing}} = 3$

$t_{\text{beetle,assembly}} = 4$, $t_{\text{cabrio,assembly}} = 7$

- Model (Linear Program):

Objective

C

maximize	(total revenue)	$\sum_{c \in C} r_c \cdot x_c$		
subject to	(total raw material)	$\sum_{c \in C} \rho_c x_c$	$\leq$	R
	(time in departments)	$\sum_{c \in C} t_{c,d} x_c$	$\leq$	T_d for all $d \in D$
	(non-negativity)	x_c	$\geq$	0 for all $c \in C$

- Data Set: set of car types: $C = \{\text{beetle}, \text{cabrio}\}$
  set of department: $D = \{\text{manufacturing}, \text{assembly}\}$
  $r_{\text{beetle}} = 10000$, $r_{\text{cabrio}} = 20000$
  $\rho_{\text{beetle}} = 400$, $\rho_{\text{cabrio}} = 400$, $R = 4500$
  $T_{\text{manufacturing}} = 50$, $T_{\text{assembly}} = 70$
  $t_{\text{beetle,manufacturing}} = 5$, $t_{\text{cabrio,manufacturing}} = 3$
  $t_{\text{beetle,assembly}} = 4$, $t_{\text{cabrio,assembly}} = 7$

- Model (Linear Program):

maximize	(total revenue)	$\sum_{c \in C} r_c \cdot x_c$	**Objective**
C subject to	(total raw material)	$\sum_{c \in C} \rho_c x_c \leq R$	
	(time in departments)	$\sum_{c \in C} t_{c,d} x_c \leq T_d$ for all $d \in D$	
V	(non-negativity)	$x_c \geq 0$ for all $c \in C$	

- Data Set: set of car types: $C = \{\text{beetle}, \text{cabrio}\}$

  set of department: $D = \{\text{manufacturing}, \text{assembly}\}$

  $r_{\text{beetle}} = 10000,\ r_{\text{cabrio}} = 20000$

  $\rho_{\text{beetle}} = 400,\ \rho_{\text{cabrio}} = 400,\quad R = 4500$

  $T_{\text{manufacturing}} = 50,\ T_{\text{assembly}} = 70$

  $t_{\text{beetle,manufacturing}} = 5,\ t_{\text{cabrio,manufacturing}} = 3$

  $t_{\text{beetle,assembly}} = 4,\ t_{\text{cabrio,assembly}} = 7$

▷ Model (Linear Program):

maximize	(total revenue)	$\sum_{c\in C} r_c \cdot x_c$	**Objective**
subject to	(total raw material)	$\sum_{c\in C} \rho_c x_c \leq R$	
	(time in departments)	$\sum_{c\in C} t_{c,d} x_c \leq T_d$ for all $d \in D$	
	(non-negativity)	$x_c \geq 0$ for all $c \in C$	

C

V

▷ Data Set: set of car types: $C = \{\text{beetle}, \text{cabrio}\}$

S

set of department: $D = \{\text{manufacturing}, \text{assembly}\}$

$r_{\text{beetle}} = 10000,\ r_{\text{cabrio}} = 20000$

$\rho_{\text{beetle}} = 400,\ \rho_{\text{cabrio}} = 400, \quad R = 4500$

$T_{\text{manufacturing}} = 50,\ T_{\text{assembly}} = 70$

$t_{\text{beetle,manufacturing}} = 5,\ t_{\text{cabrio,manufacturing}} = 3$

$t_{\text{beetle,assembly}} = 4,\ t_{\text{cabrio,assembly}} = 7$

- Model (Linear Program):

maximize	(total revenue)	$\sum_{c \in C} r_c \cdot x_c$	**Objective**
subject to	(total raw material)	$\sum_{c \in C} \rho_c x_c \leq R$	
	(time in departments)	$\sum_{c \in C} t_{c,d} x_c \leq T_d$ for all $d \in D$	
	(non-negativity)	$x_c \geq 0$ for all $c \in C$	

C

V

- Data Set: set of car types: $C = \{\text{beetle}, \text{cabrio}\}$

  set of department: $D = \{\text{manufacturing}, \text{assembly}\}$

  $r_{\text{beetle}} = 10000,\ r_{\text{cabrio}} = 20000$

  $\rho_{\text{beetle}} = 400,\ \rho_{\text{cabrio}} = 400, \quad R = 4500$

  $T_{\text{manufacturing}} = 50,\ T_{\text{assembly}} = 70$

  $t_{\text{beetle,manufacturing}} = 5,\ t_{\text{cabrio,manufacturing}} = 3$

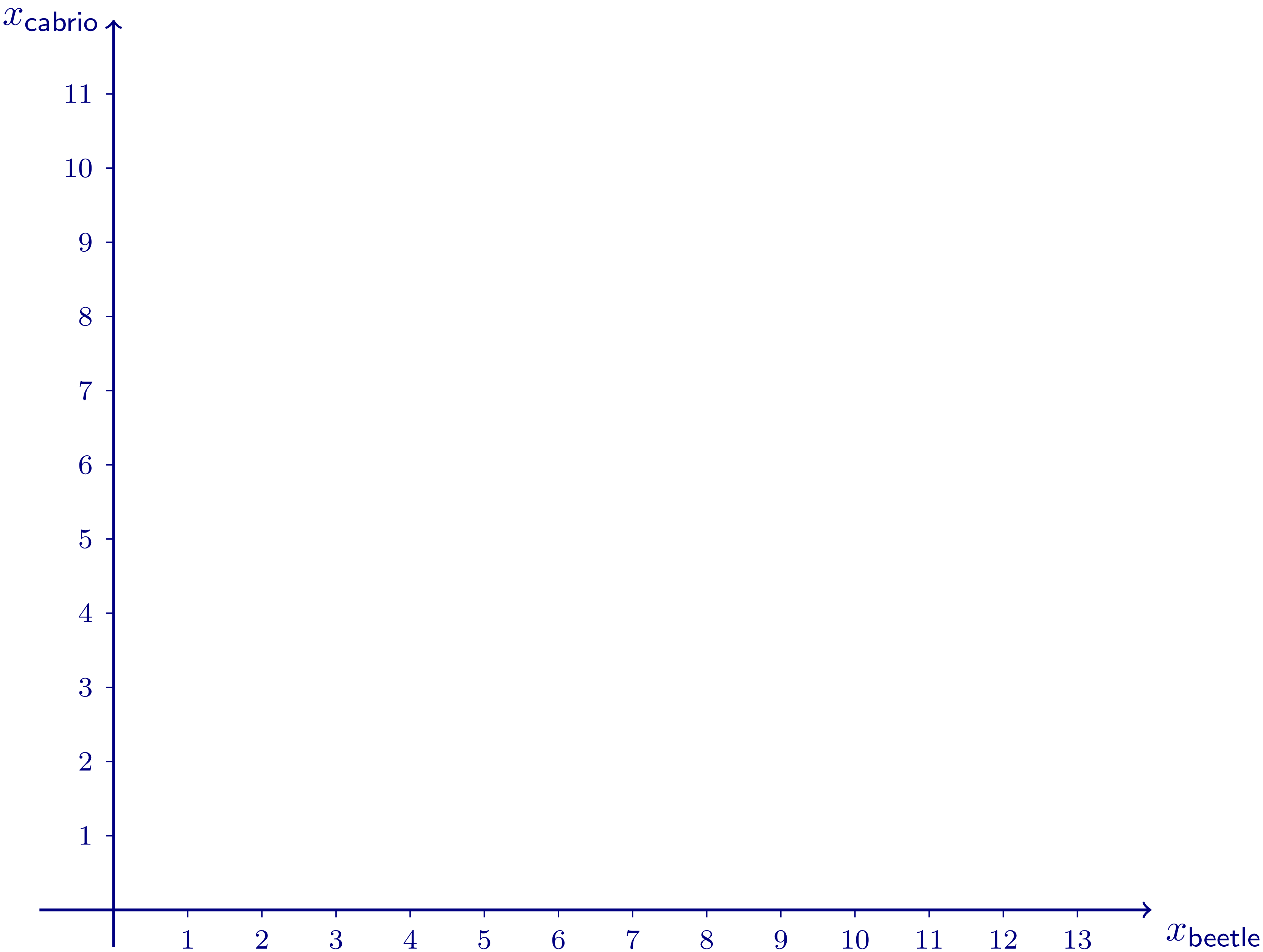
  $t_{\text{beetle,assembly}} = 4,\ t_{\text{cabrio,assembly}} = 7$

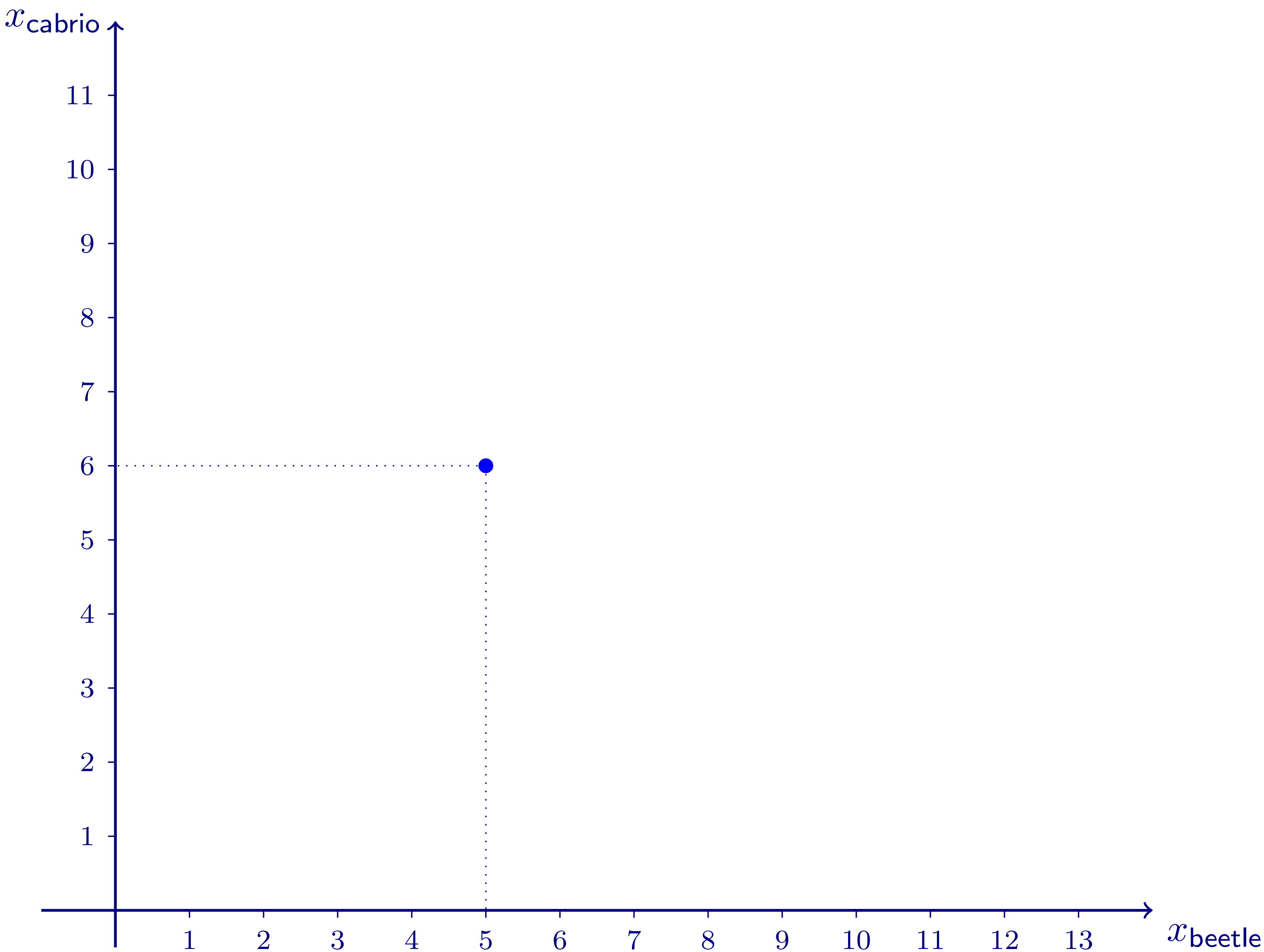
S

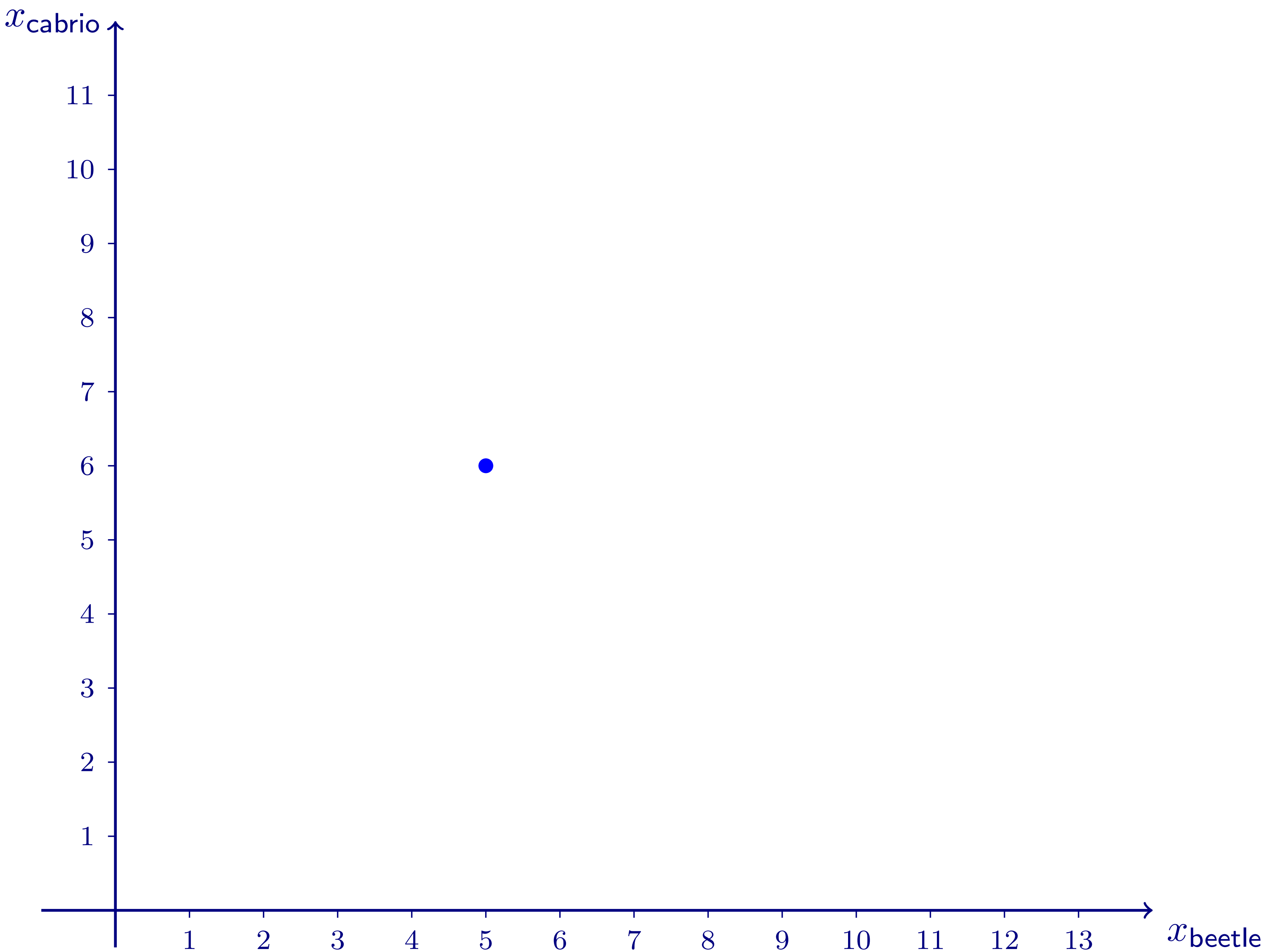
P

	# beetles	# cabrios	revenue	manufact. h	assembly h	raw material
feasible solution	5	6	170	43	62	4400
another feasible solution	4	7	180	41	65	4400
infeasible solution	2	9	200	37	71	4400
optimal solution	0	10	200	30	70	4000
		available capacities:		50	70	4500

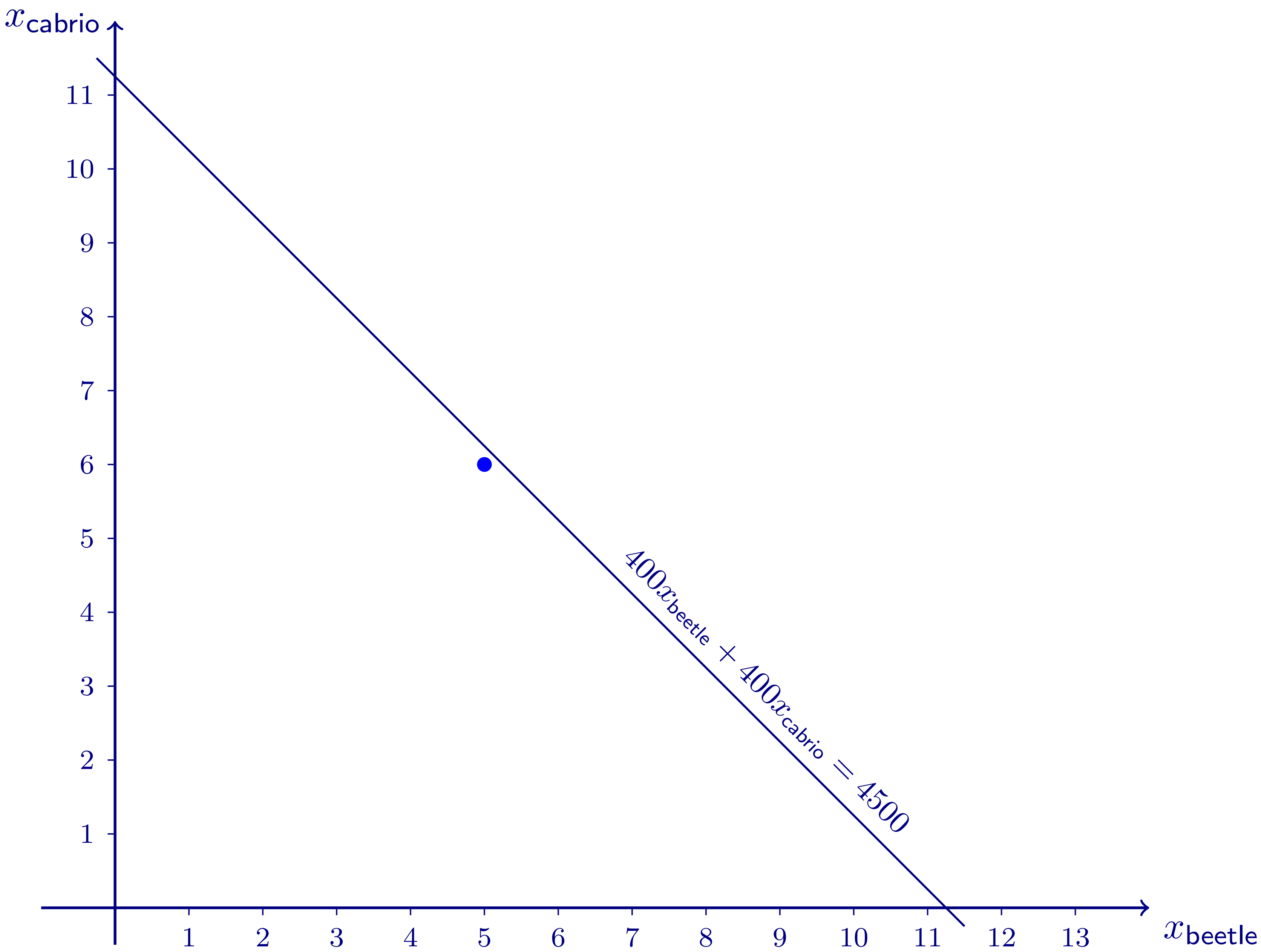
Production planning – geometric interpretation

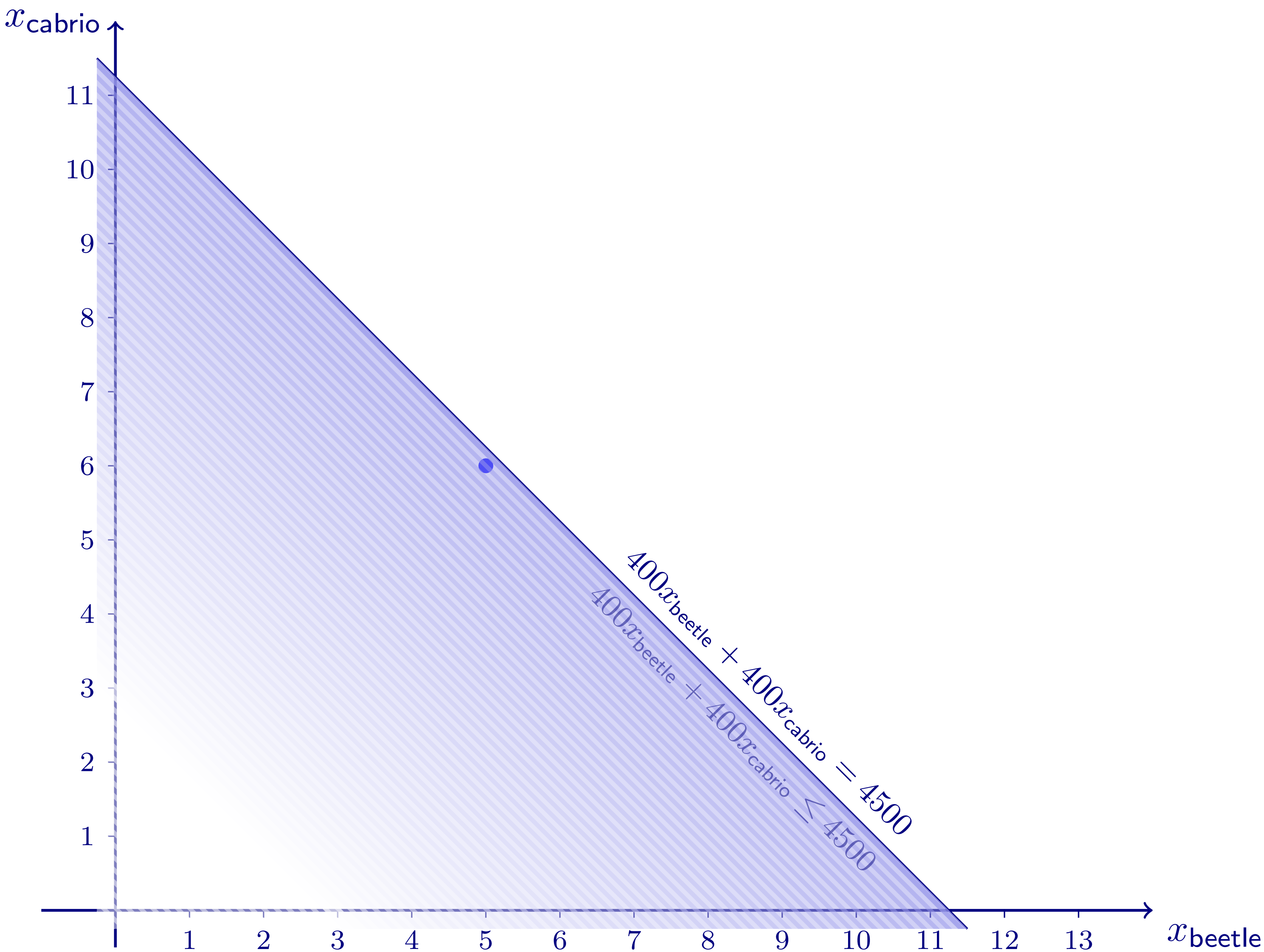


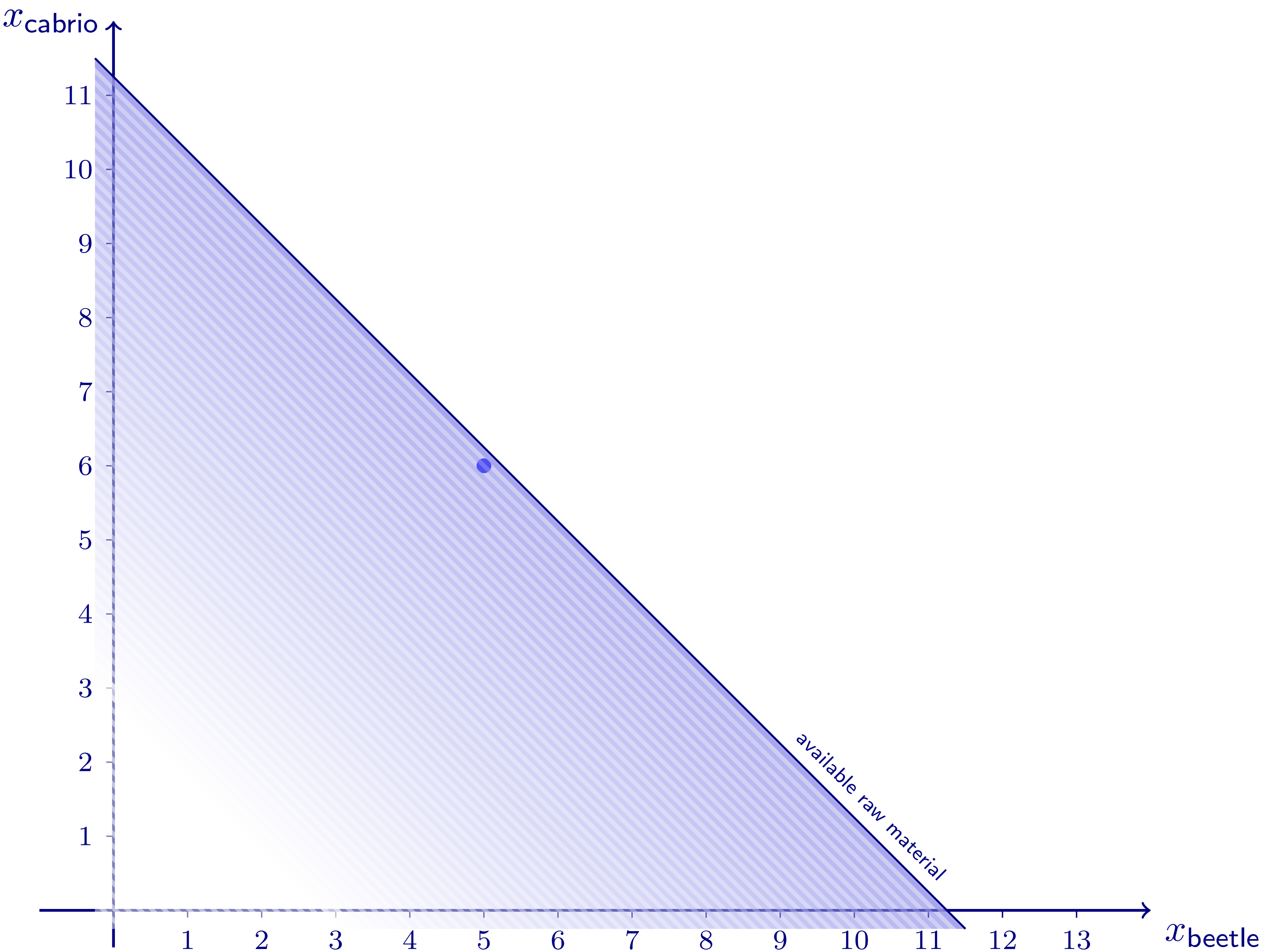

x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}



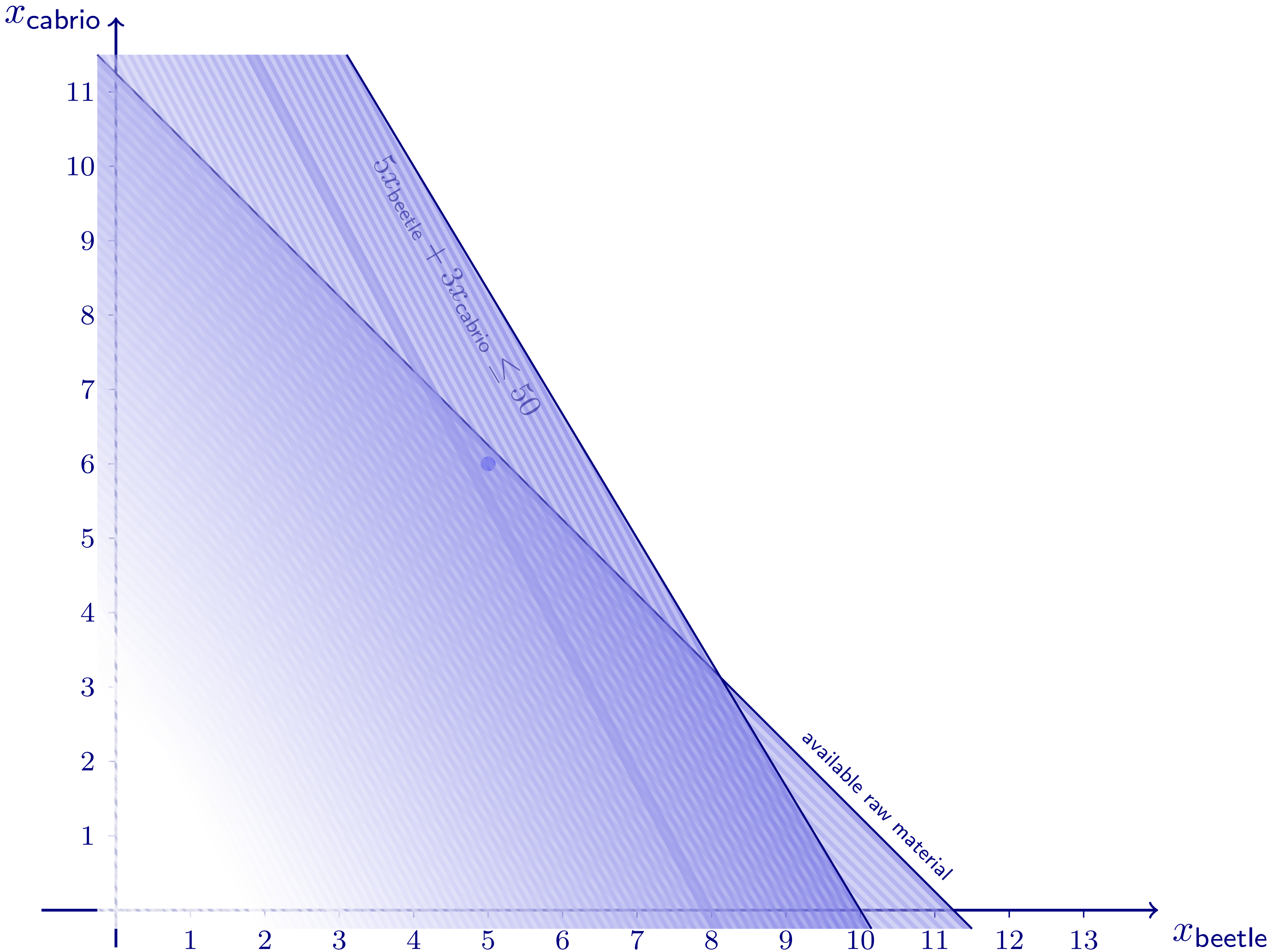
x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}

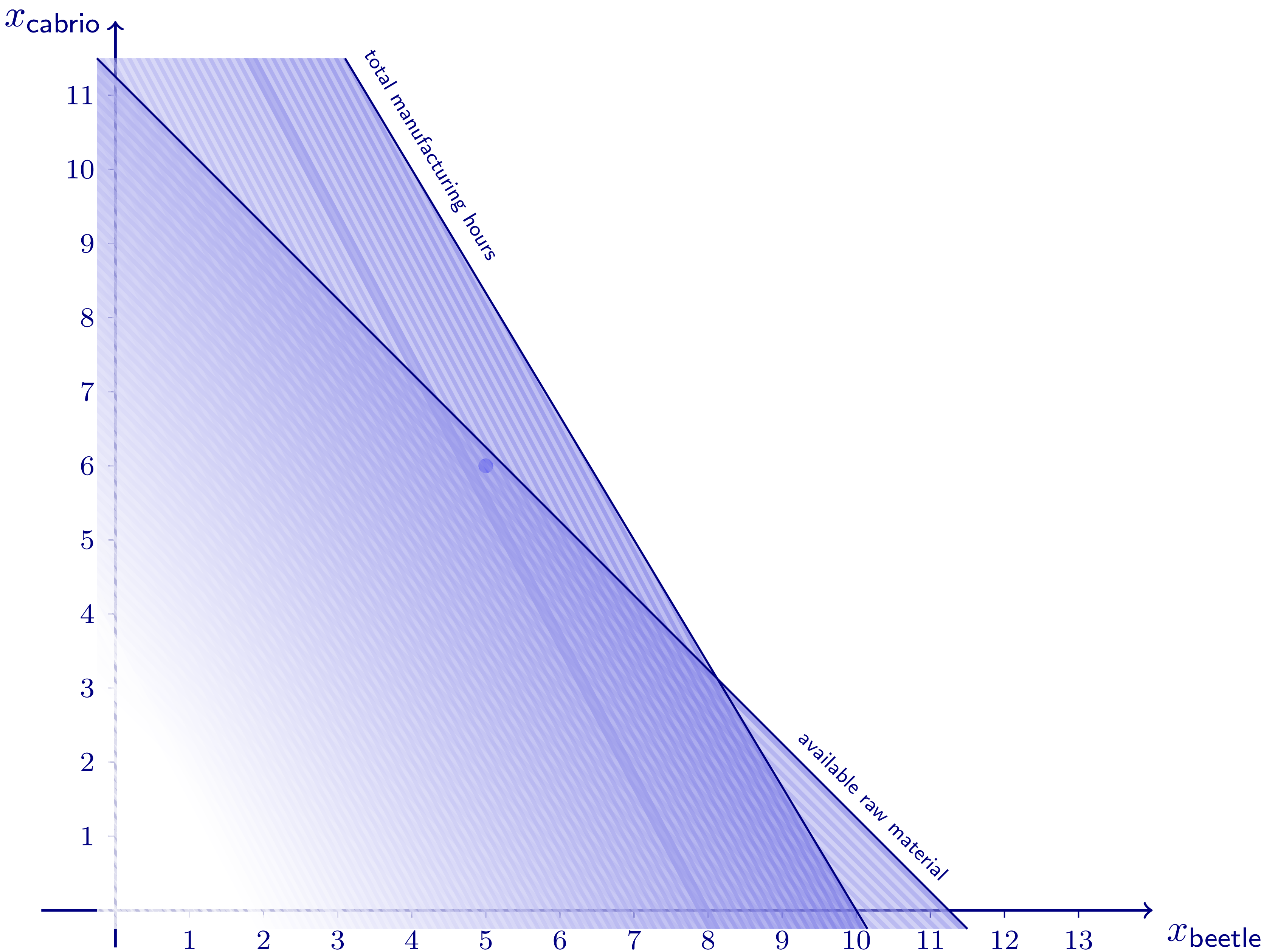

x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}
$400x_{\text{beetle}} + 400x_{\text{cabrio}} = 4500$


x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10
11
12
13
x_{beetle}
$400x_{\text{beetle}} + 400x_{\text{cabrio}} = 4500$
$400x_{\text{beetle}} + 400x_{\text{cabrio}} \leq 4500$

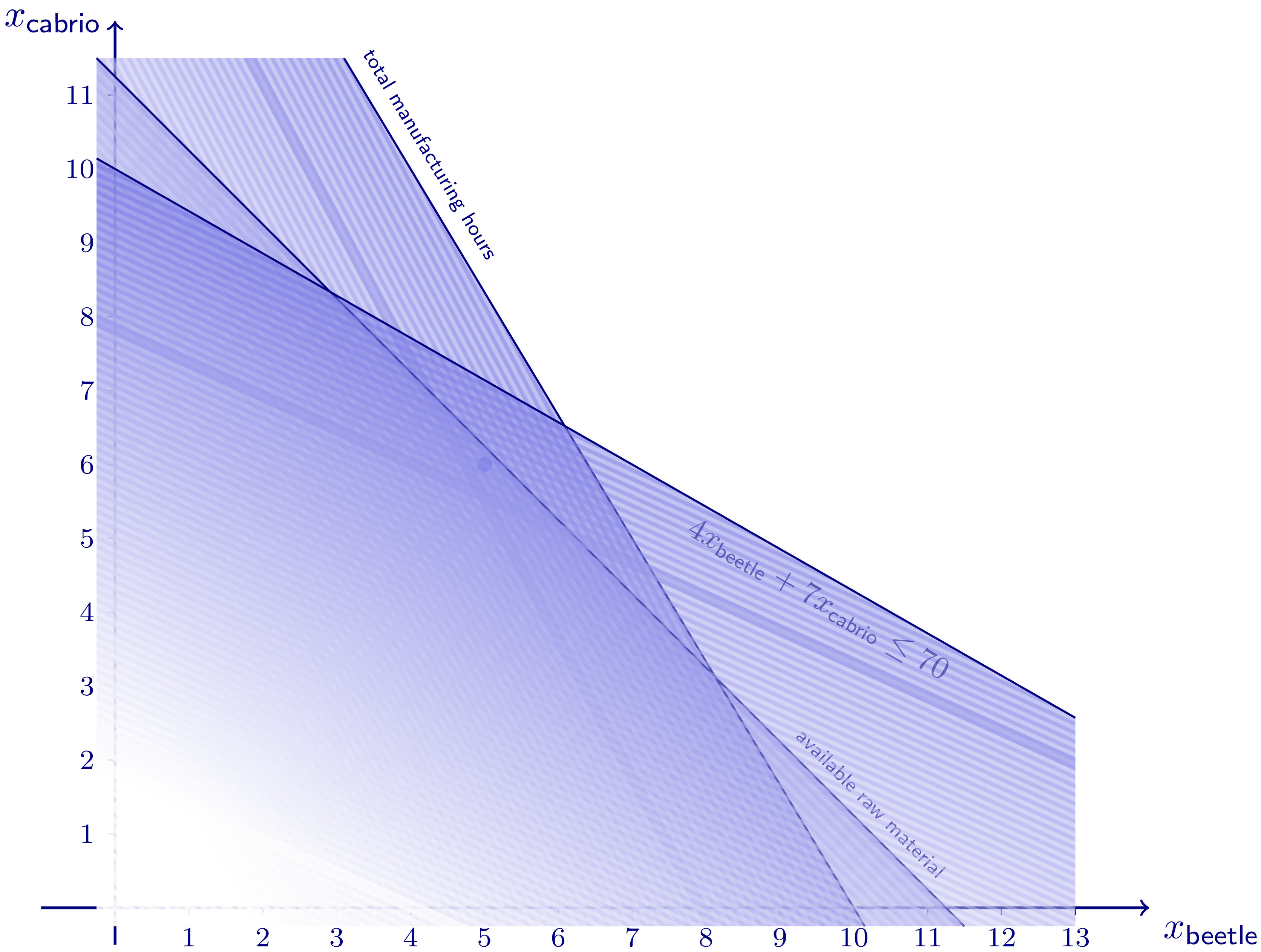

x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
available raw material
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}

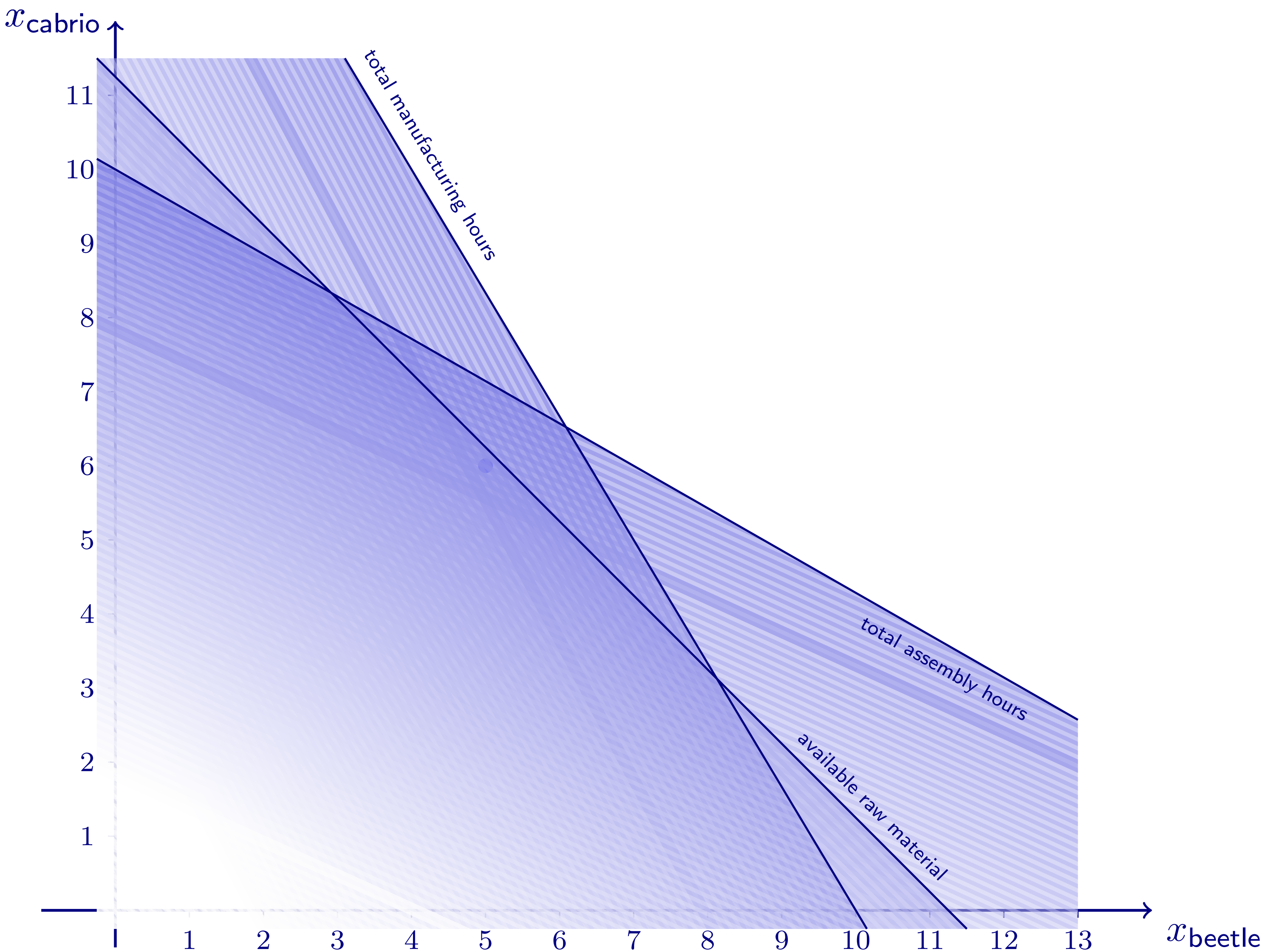
Production planning – geometric interpretation

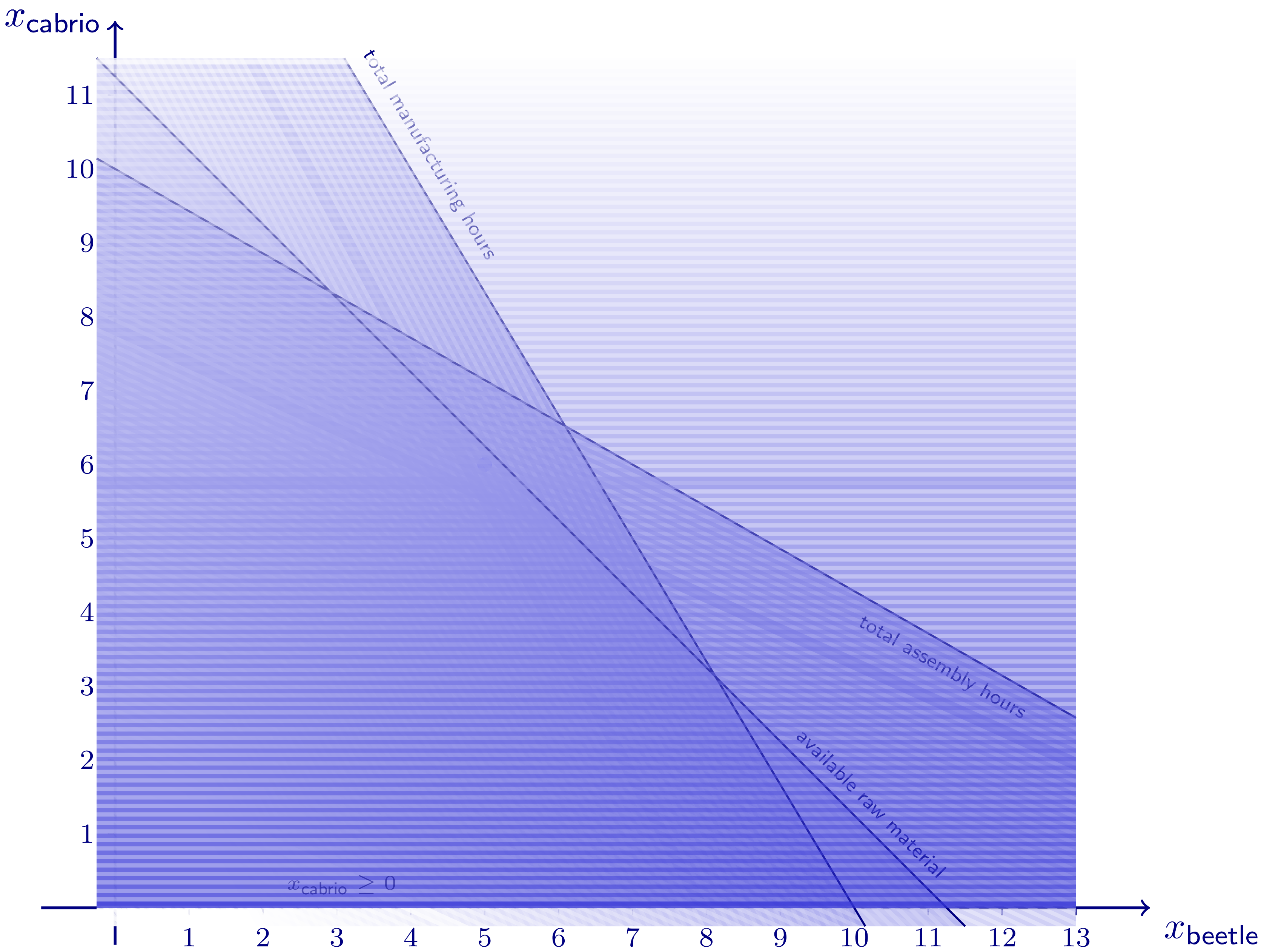


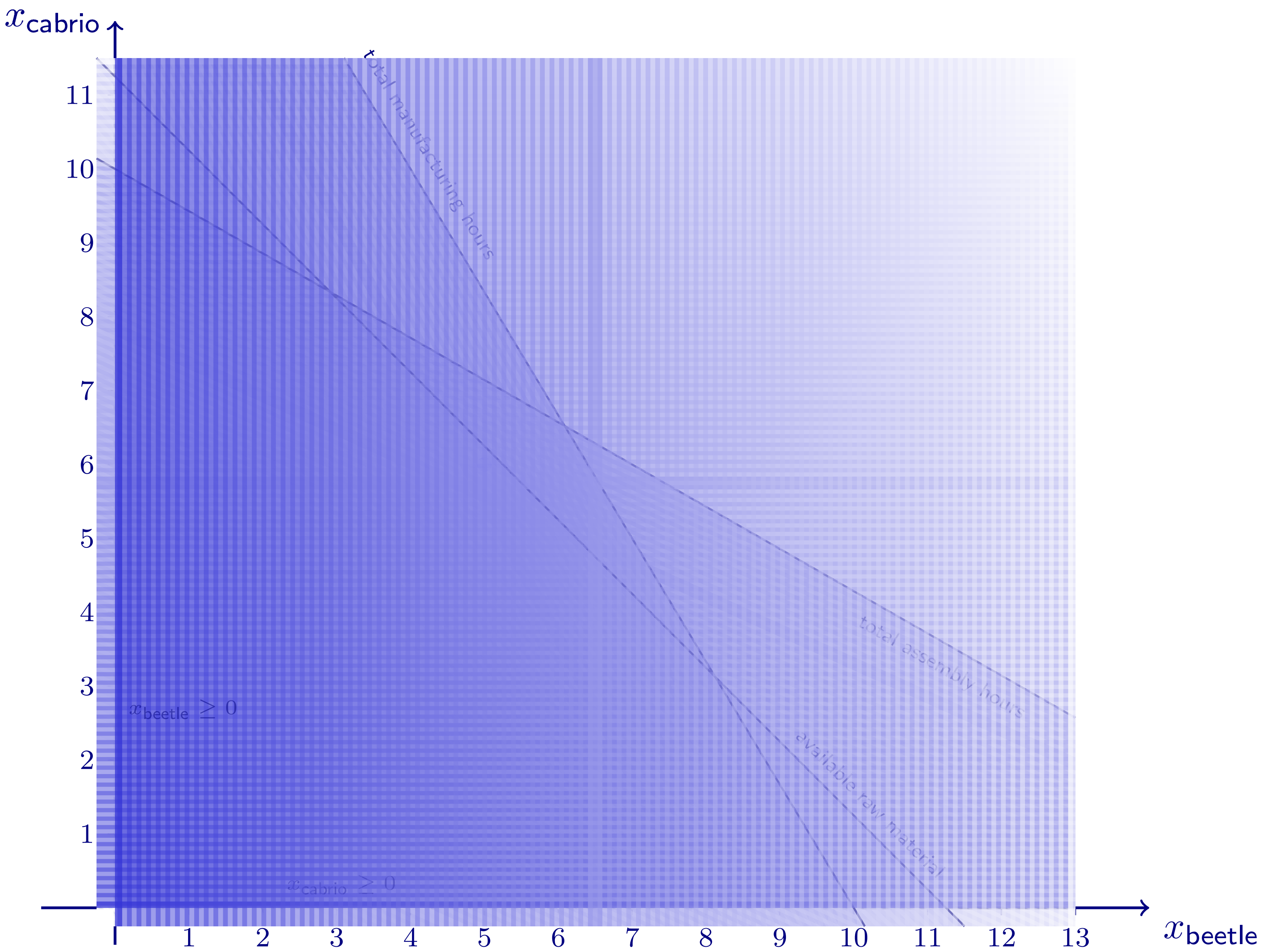


x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}
total manufacturing hours
available raw material

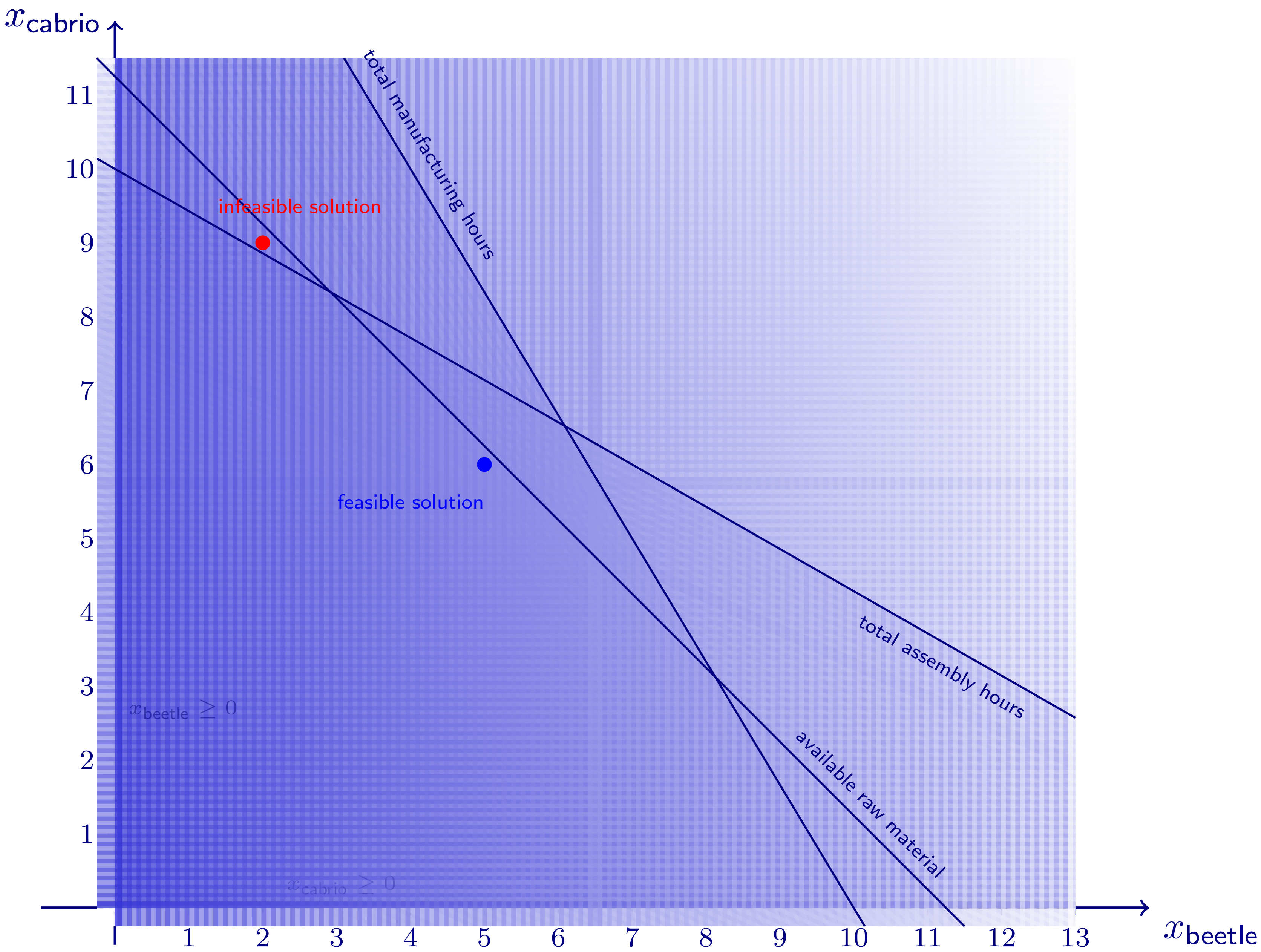

x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}
total manufacturing hours
$4x_{\text{beetle}} + 7x_{\text{cabrio}} \leq 70$
available raw material

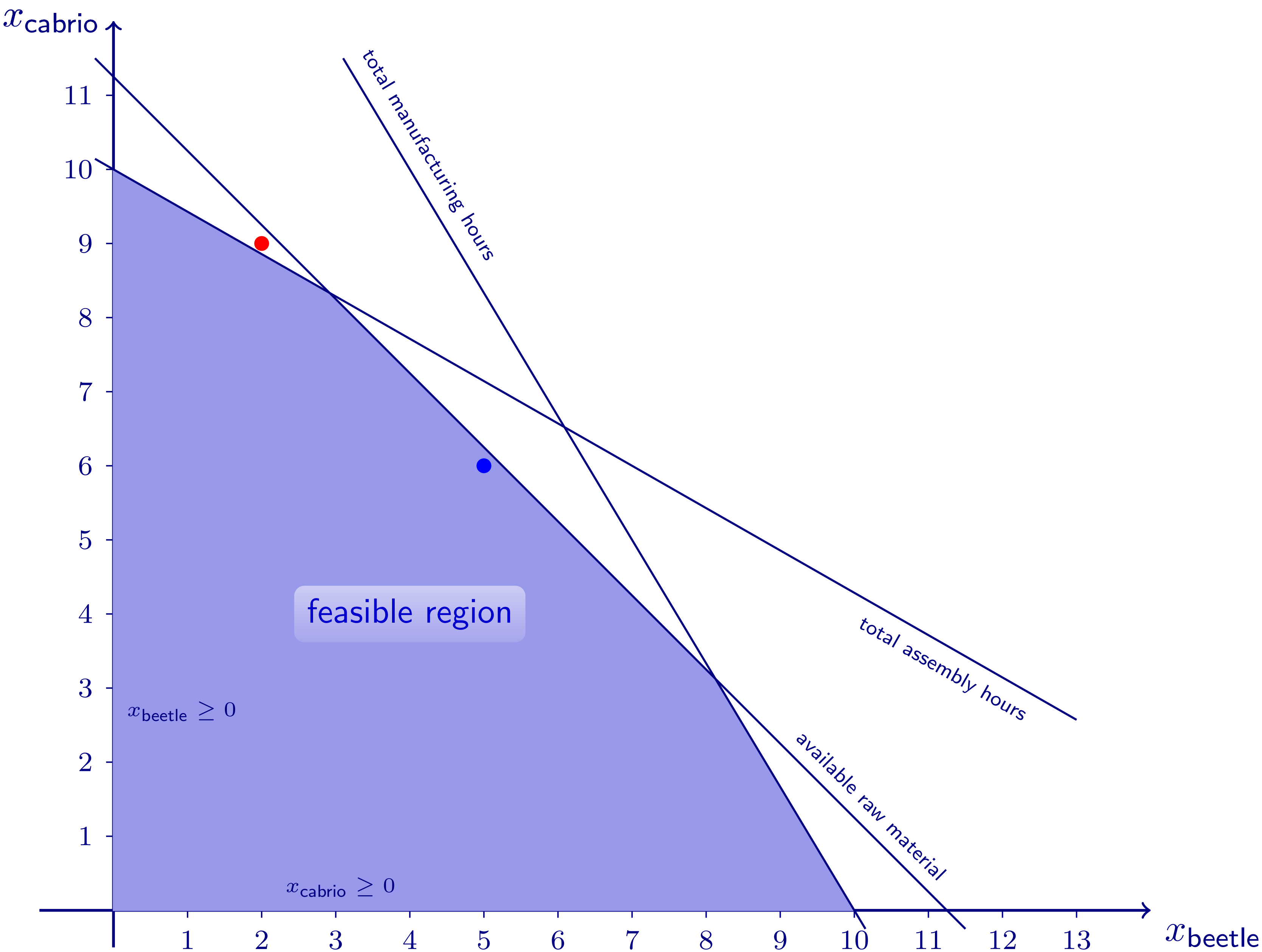

x_{cabrio}
x_{beetle}
total manufacturing hours
total assembly hours
available raw material
1 2 3 4 5 6 7 8 9 10 11 12 13
1 2 3 4 5 6 7 8 9 10 11


x_{cabrio}
x_{beetle}
11
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10
11
12
13
total manufacturing hours
total assembly hours
available raw material
$x_{\text{cabrio}} \geq 0$

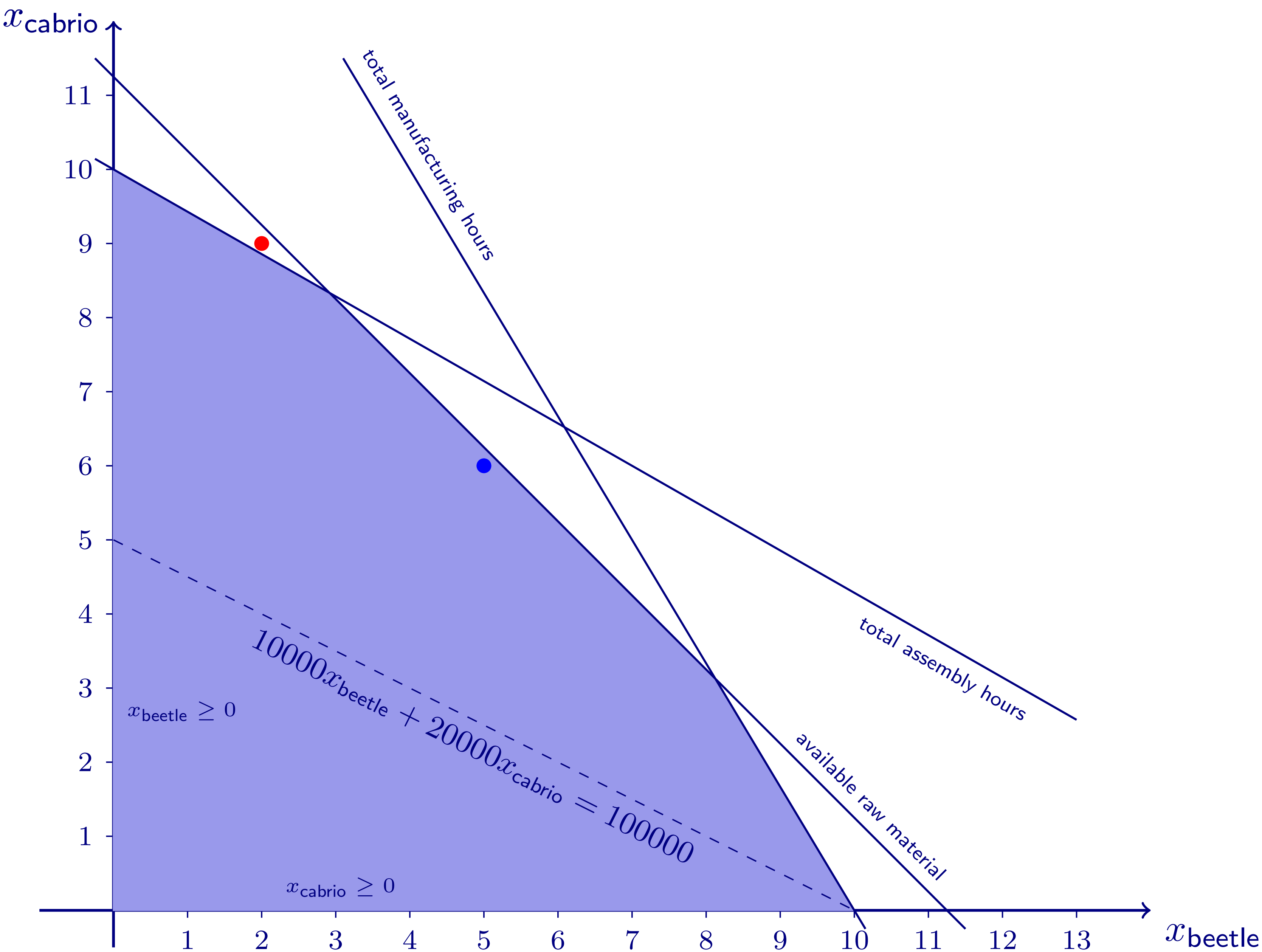

x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
total manufacturing hours
total assembly hours
available raw material
$x_{\text{beetle}} \geq 0$
$x_{\text{cabrio}} \geq 0$
1
2
3
4
5
6
7
8
9
10
11
12
13
x_{beetle}

Production planning – geometric interpretation



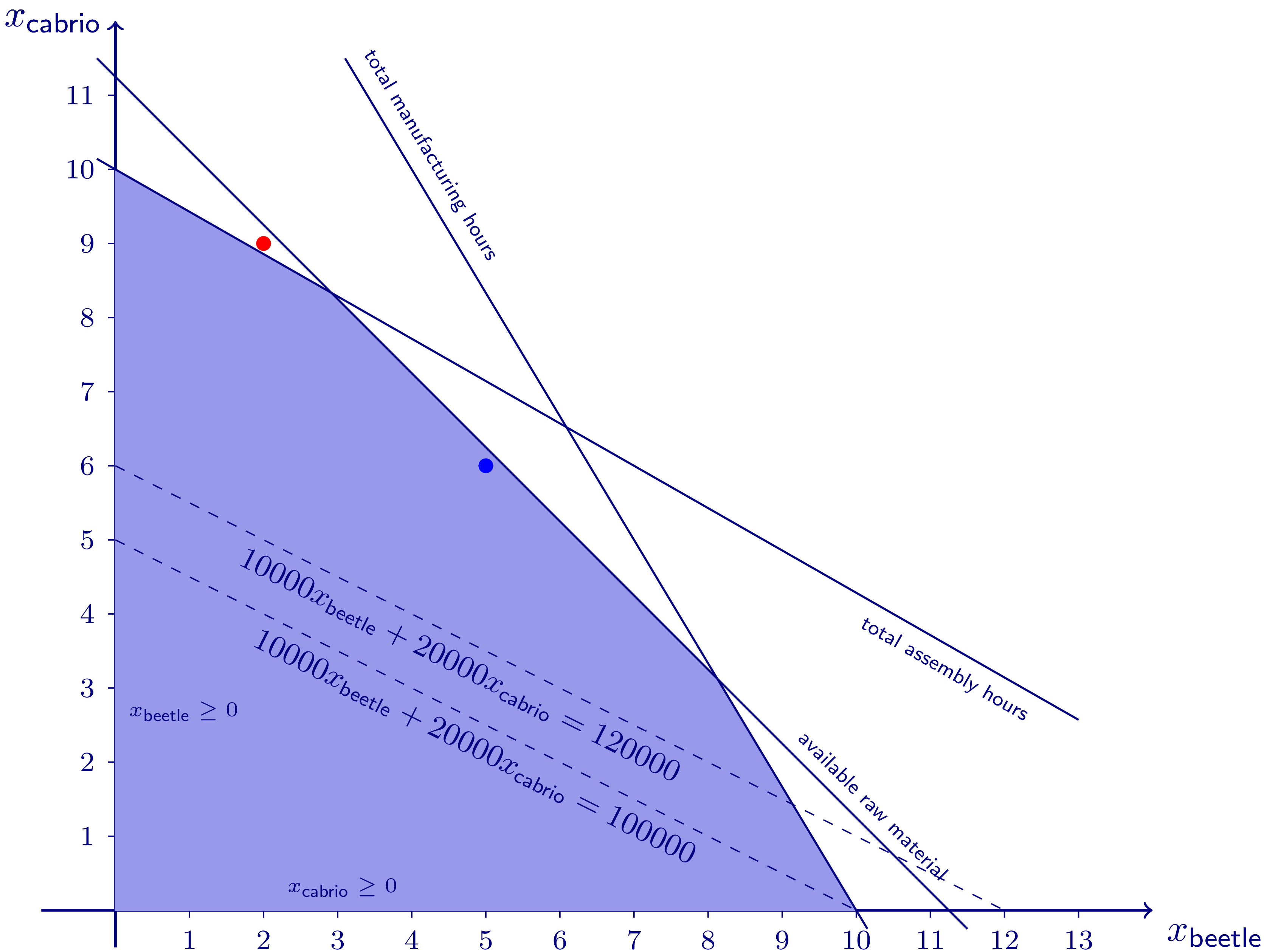


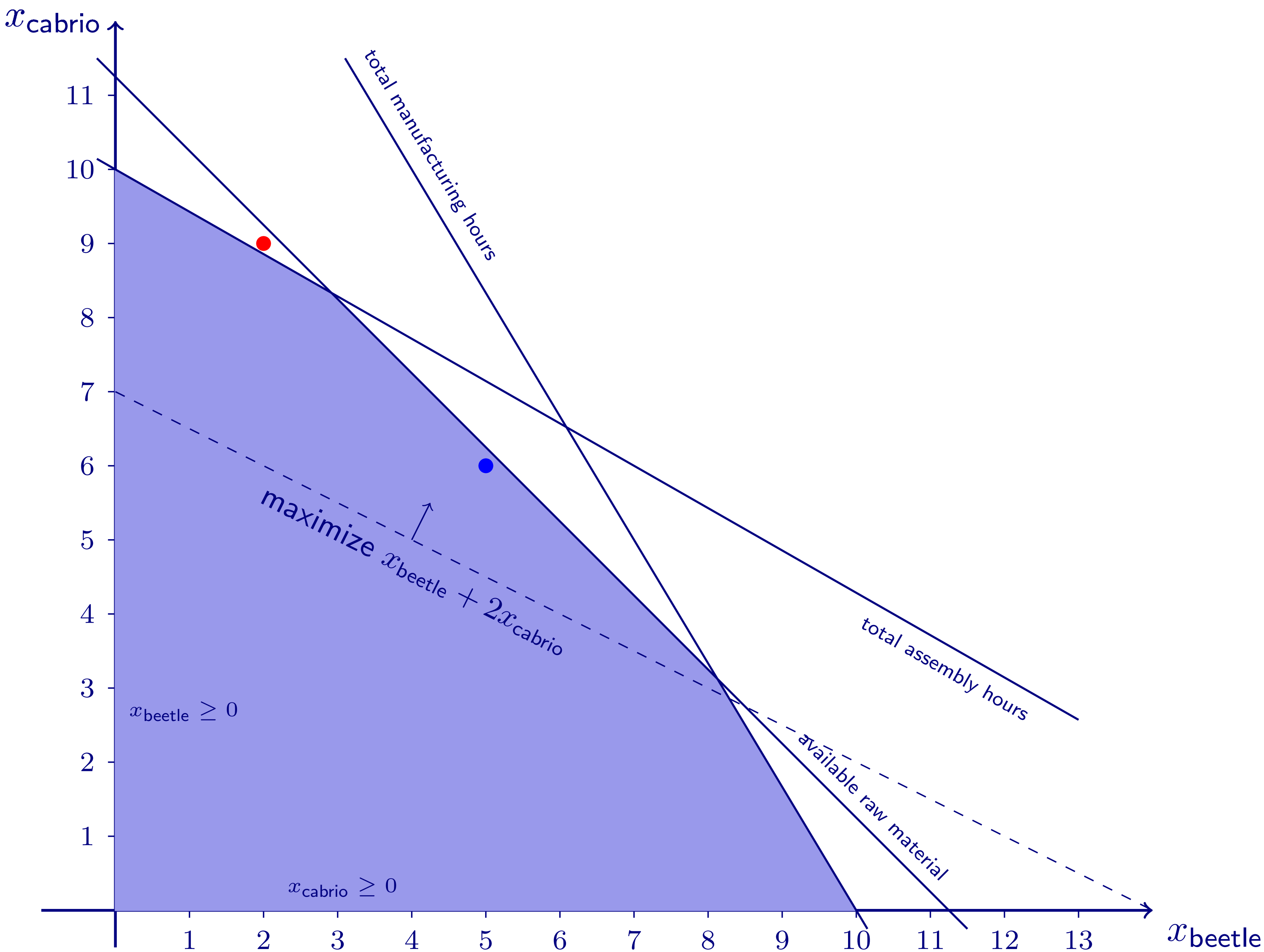
x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
total manufacturing hours
feasible region
total assembly hours
available raw material
$x_{\text{beetle}} \geq 0$
$x_{\text{cabrio}} \geq 0$
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}



x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}
total manufacturing hours
total assembly hours
available raw material
$10000x_{\text{beetle}} + 20000x_{\text{cabrio}} = 100000$
$x_{\text{beetle}} \geq 0$
$x_{\text{cabrio}} \geq 0$

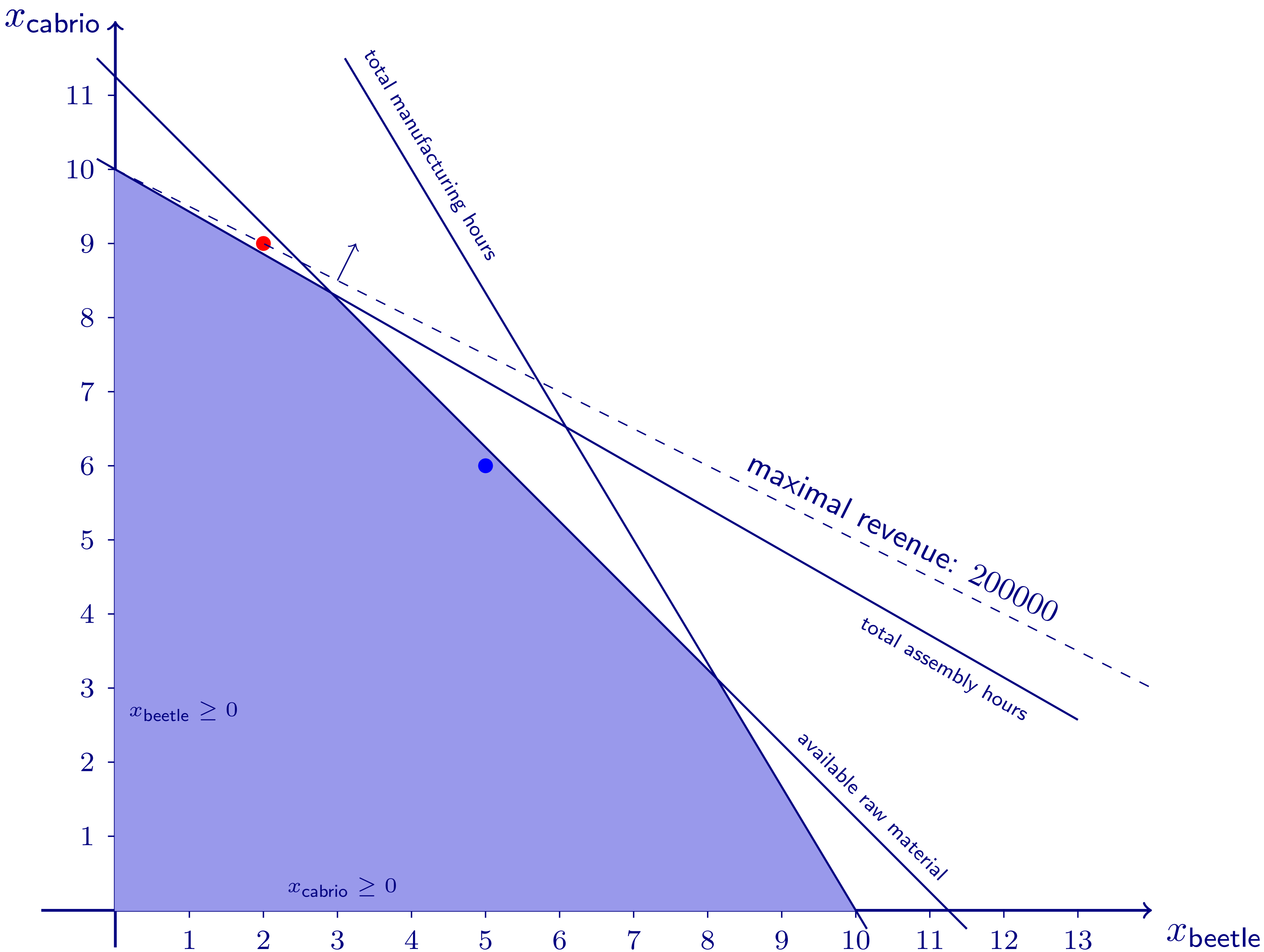
Production planning – geometric interpretation



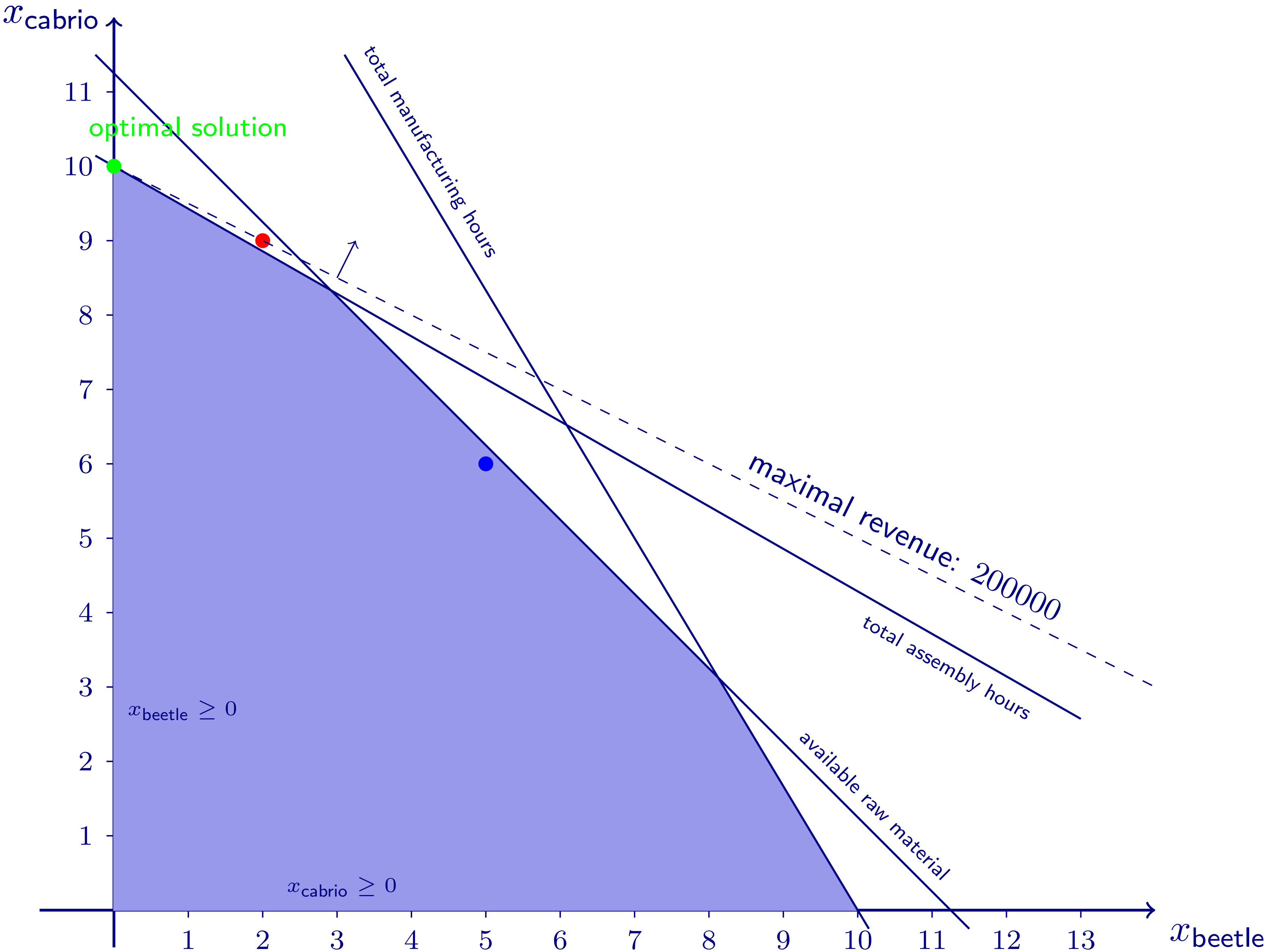


x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
total manufacturing hours
maximize $x_{\text{beetle}} + 2x_{\text{cabrio}}$
total assembly hours
available raw material
$x_{\text{beetle}} \geq 0$
$x_{\text{cabrio}} \geq 0$
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}

Production planning – geometric interpretation



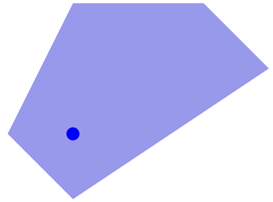
Production planning – geometric interpretation



- Feasible solution:

  All variable values satisfy all constraints

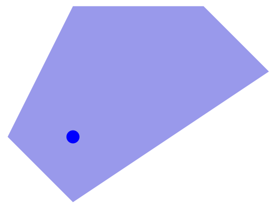
  ➡ Point in the feasible region



- ▷ Feasible solution:

  All variable values satisfy all constraints

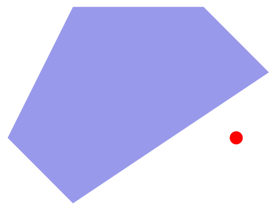
  ➡ Point in the feasible region



- ▷ Infeasible solution:

  The variable values violate at least one constraint

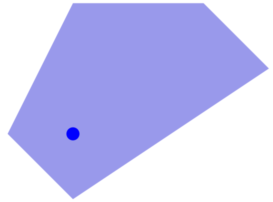
  ➡ Point outside the feasible region



- **Feasible solution:**

  All variable values satisfy all constraints

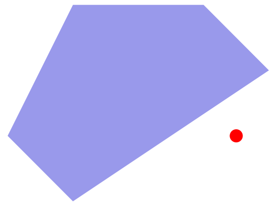
  ➡ Point in the feasible region



- **Infeasible solution:**

  The variable values violate at least one constraint

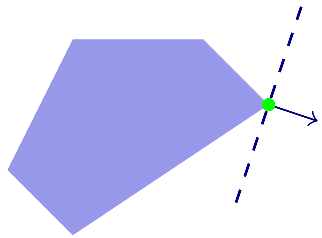
  ➡ Point outside the feasible region



- **Optimal solution:**

  Feasible solution such that no other feasible solution has a better objective value

  ➡ Point in the feasible region, on a line h (more generally: a hyperplane) such that the feasible region is completely on one side of h

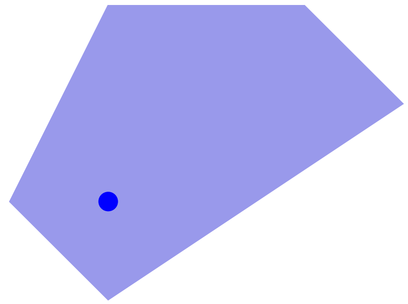


- A mathematical program is...

- A mathematical program is...

...feasible

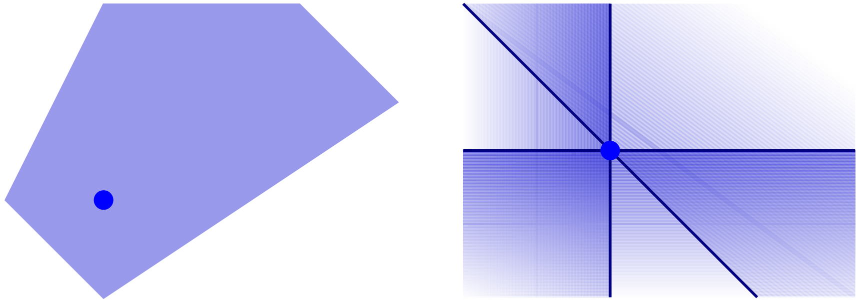
if there is at least one feasible solution



- A mathematical program is...

  ...feasible

  if there is at least one feasible solution

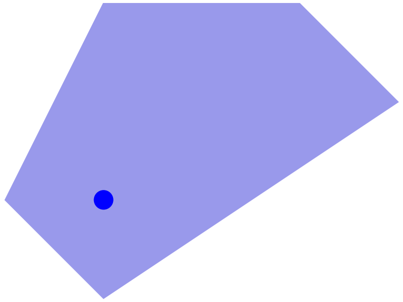


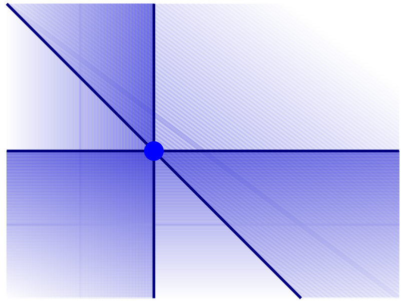
Feasibility regions

- A mathematical program is...

...feasible

if there is at least one feasible solution

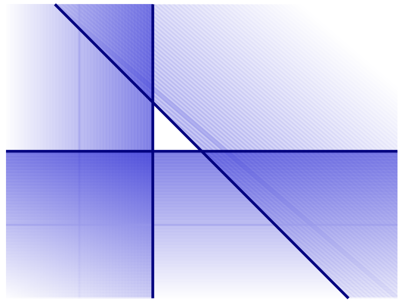




...infeasible

if there is no feasible solution

➡ no optimum

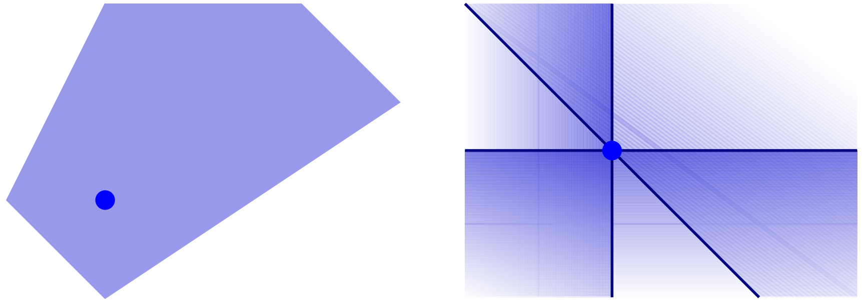


Feasibility regions

▷ A mathematical program is...

...feasible

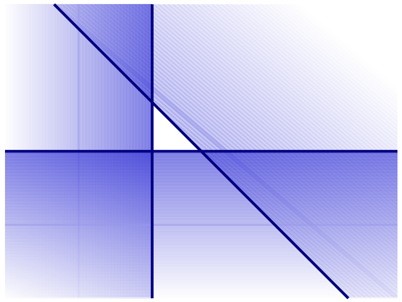
if there is at least one feasible solution



...infeasible

if there is no feasible solution

➡ no optimum



...unbounded

if there are feasible solutions with objective function value arbitrarily large (for maximizing), or small (for minimizing) respectively

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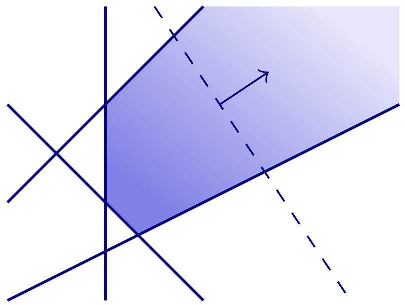
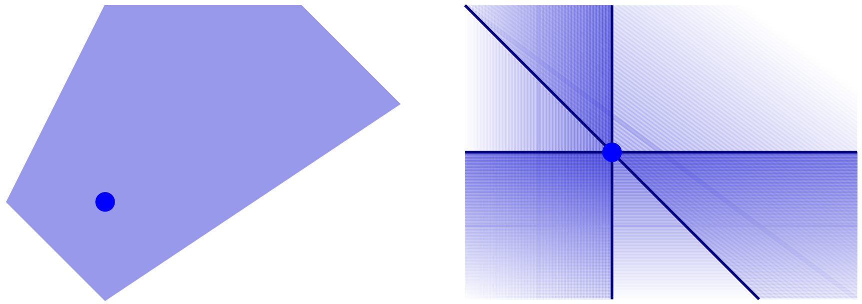


- A mathematical program is...

...feasible

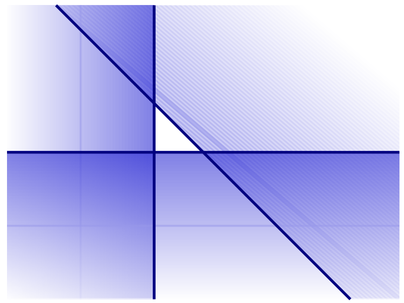
if there is at least one feasible solution



...infeasible

if there is no feasible solution

➡ no optimum



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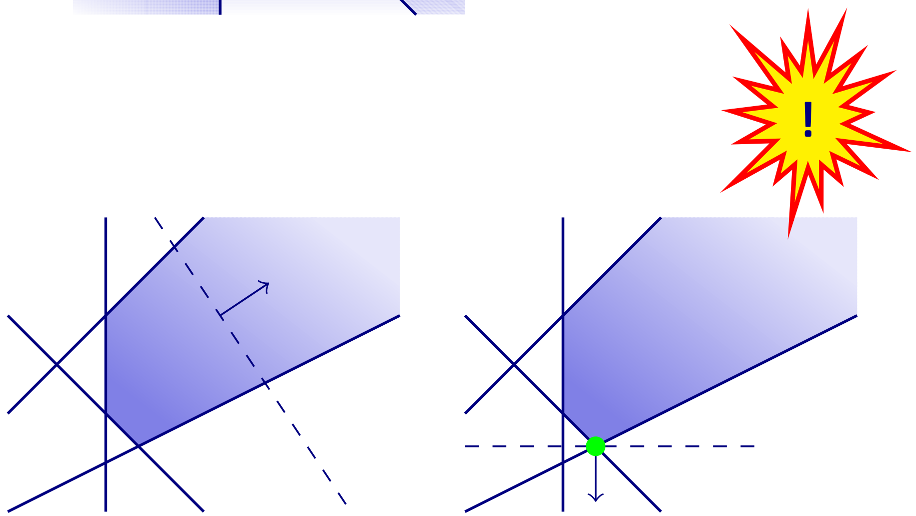

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    - one variable $\leq$ value (upper bound)

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  - ➡ Bounds on variables
    - one variable $\leq$ value (upper bound)
    - one variable $\geq$ value (lower bound)

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maximize/minimize $$\sum_{j=1}^{n} c_j x_j$$

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subject to	$\sum_{j=1}^{n} a_{ij} x_j \ \leq \ b_i$	for all $i = 1, \ldots, m$	**C**

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$\ell_j \leq x_j \leq u_j$ for all $j = 1, \ldots, n$

maximize/minimize	$\sum_{j=1}^{n} c_j x_j$	**Objective function**
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➡ n variables, m constraints

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- ➡ $(c_1, \ldots, c_n)$ is the objective function vector

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- n variables, m constraints
- $(c_1, \ldots, c_n)$ is the objective function vector
- $(b_1, \ldots, b_m)$ is the right-hand side vector

maximize/minimize	$\sum_{j=1}^{n} c_j x_j$	**Objective function**
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- $\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$ is the constraints matrix
- Some of the lower bounds ℓ_j could be $-\infty$, some of the upper bounds u_j could be $+\infty$

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling; Mathematical Background: Branch & Bound
- Branch & Bound, Cutting Planes; More Examples; Combinatorial Optimization
- Combinatorial Optimization: Examples, Graphs, Algorithms
- Complexity Theory
- Nonlinear Optimization
- Scheduling
- Lot Sizing
- Multicriteria Optimization
- Oral exam