Mathematical Tools for Engineering and Management

Lecture 4

09 Nov 2011

Overview

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis
- (Mixed) Integer Programming, Mathematical Background: Cuts, Branch & Bound
- Combinatorial Optimization
- Mathematical Background: Graphs, Algorithms
- Complexity Theory
- Nonlinear Optimization
- Scheduling
- Lot Sizing
- Multicriteria Optimization

- Exam

Sensitivity analysis

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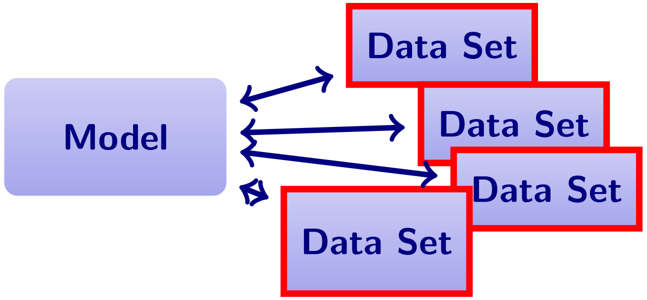

Model

Data Set

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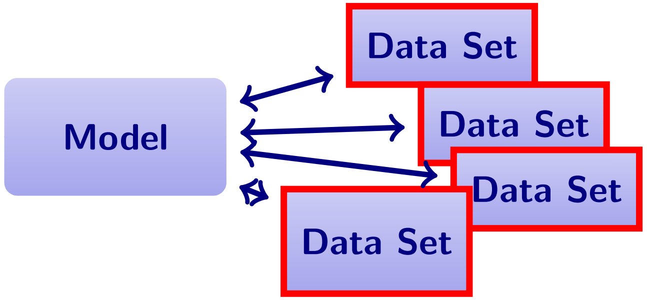
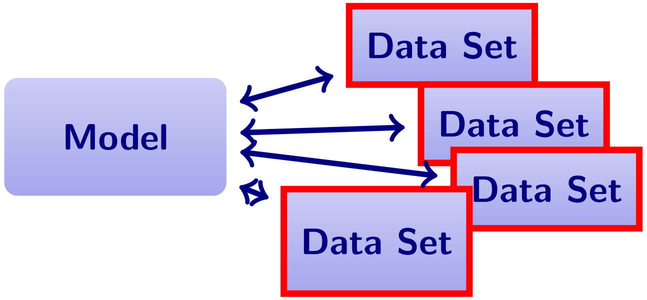


- Changes in the objective function

- ▷ Sensitivity Analysis: Analyse the influence of changes in the input data on the optimal solution.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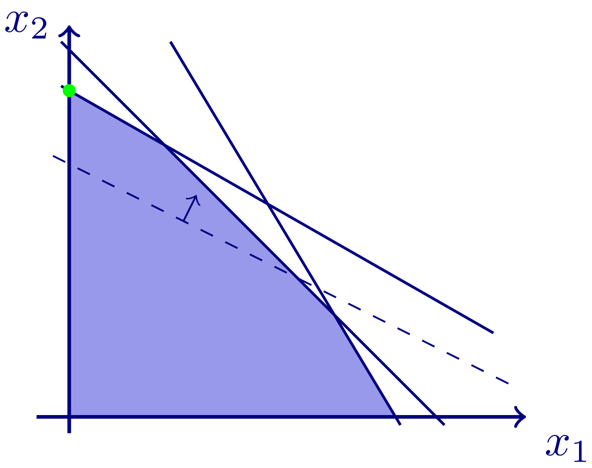
- ▷ Changes in the objective function

- ➡ Revenue per Beetle/per Cabrio:



$10000/$20000

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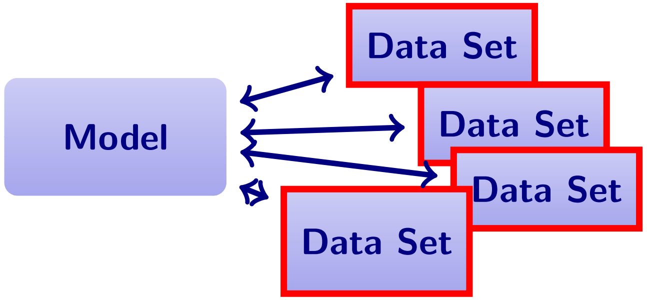

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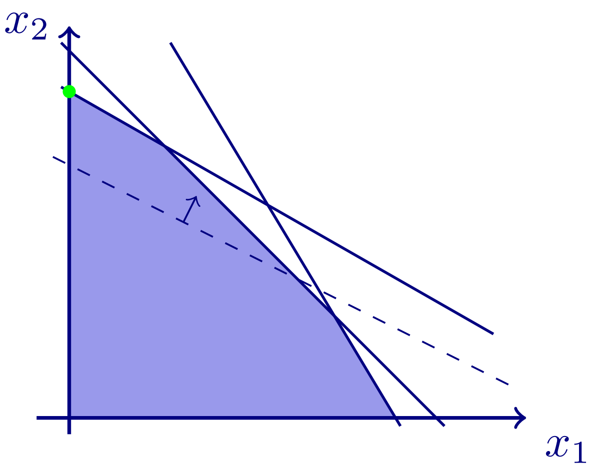
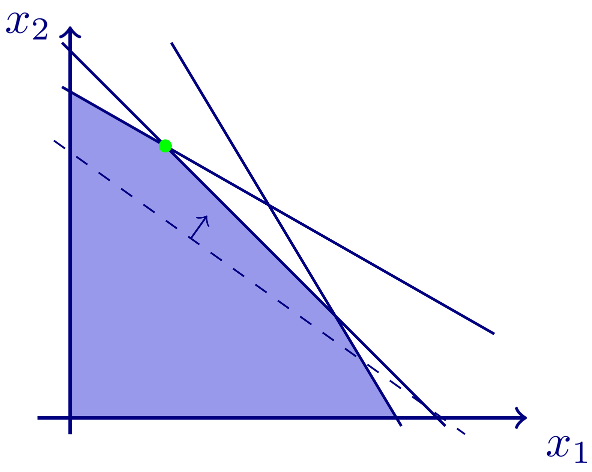


\$10000/\$20000



\$10000/\$14000

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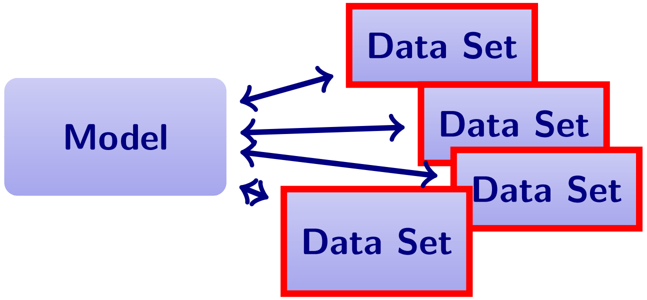

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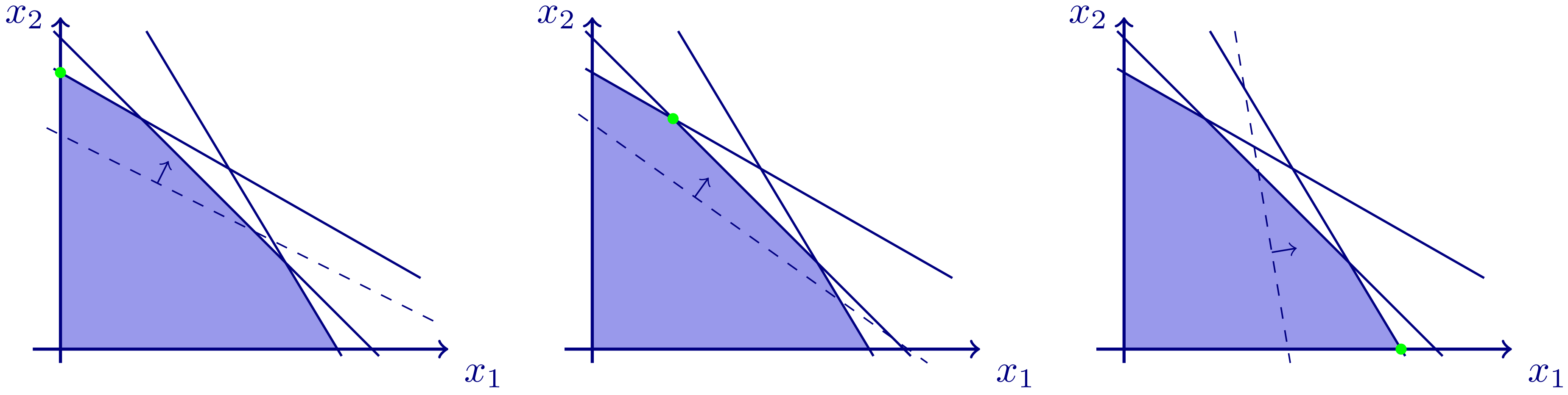

\$10000/\$20000 \$10000/\$14000 \$24000/\$4000

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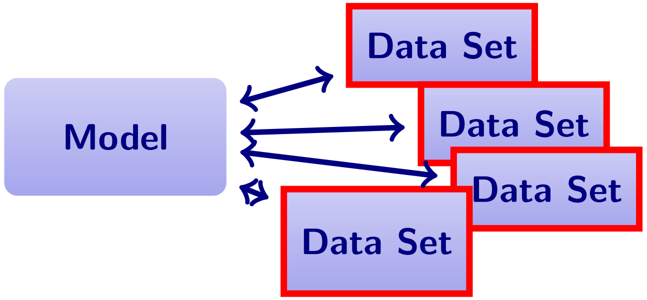

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- Changes in the objective function
- Changes in the right-hand side vector

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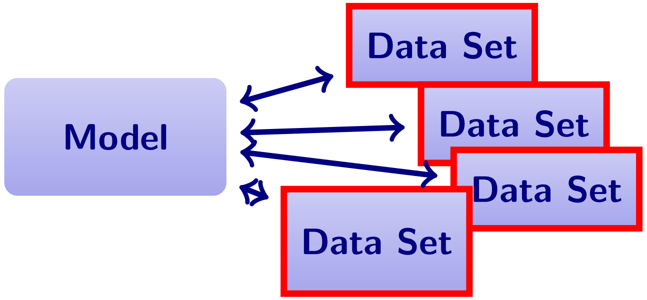

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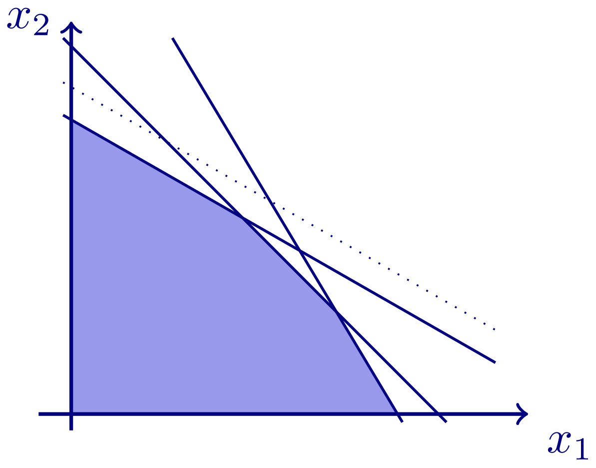

assembly capacity: 63h

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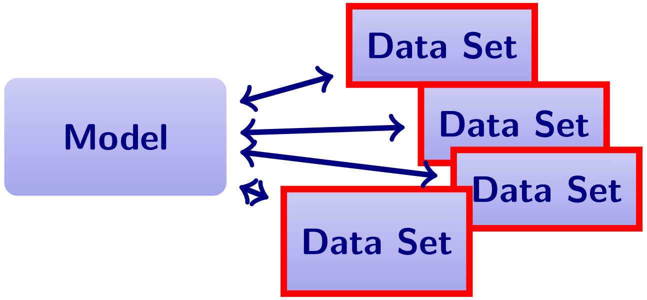

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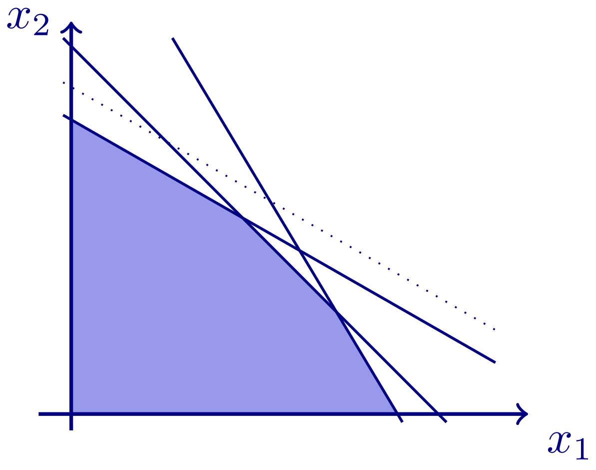

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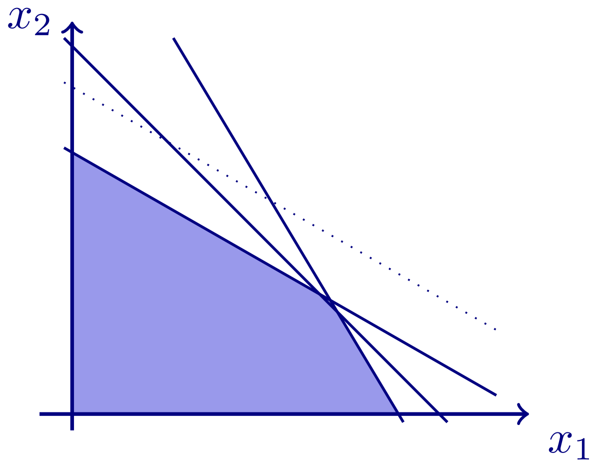

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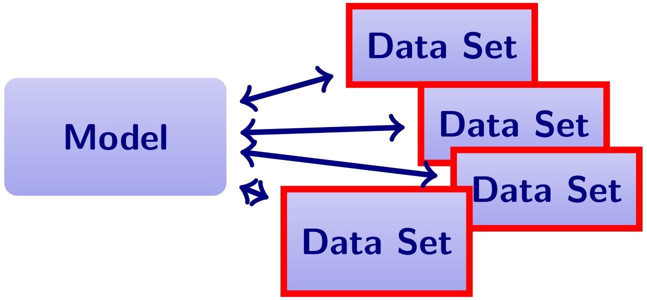

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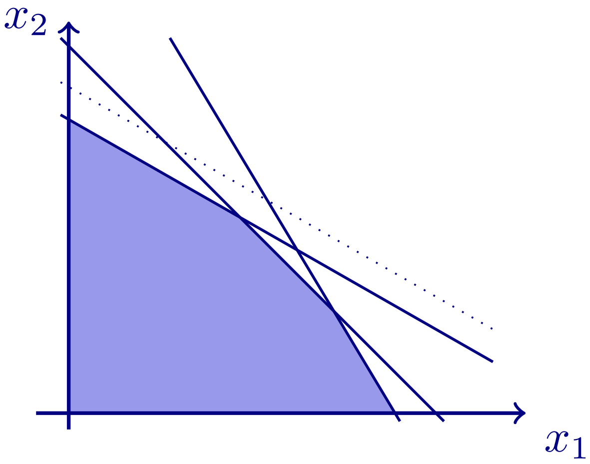

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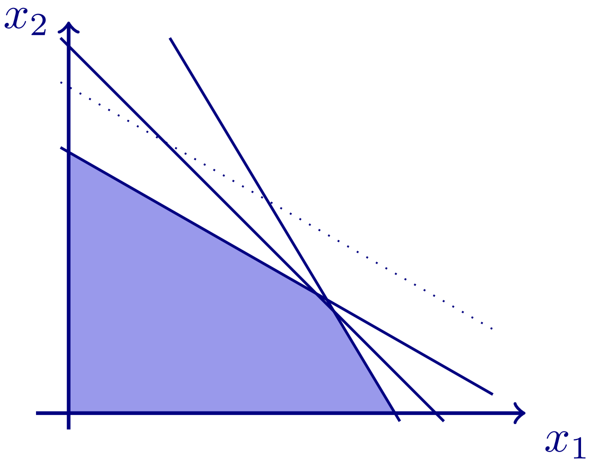

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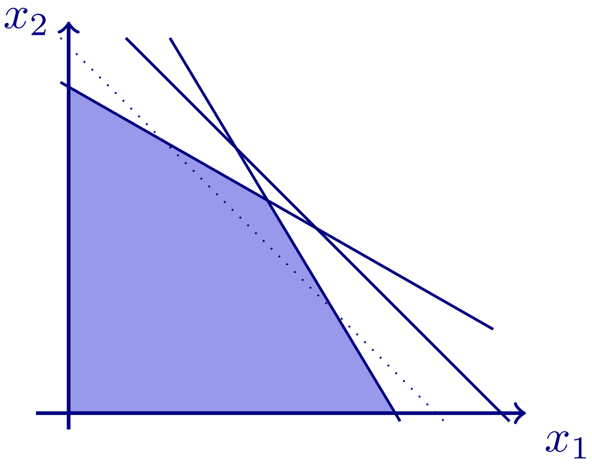

avail. raw material: 5300kg

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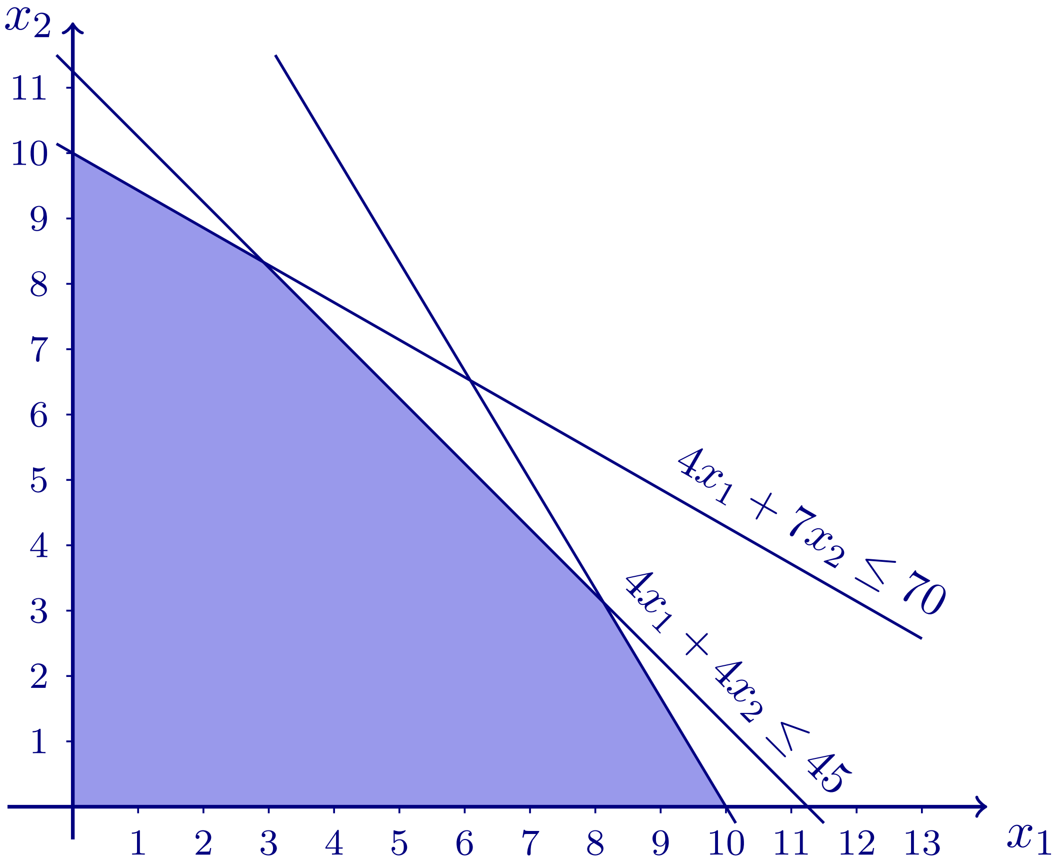
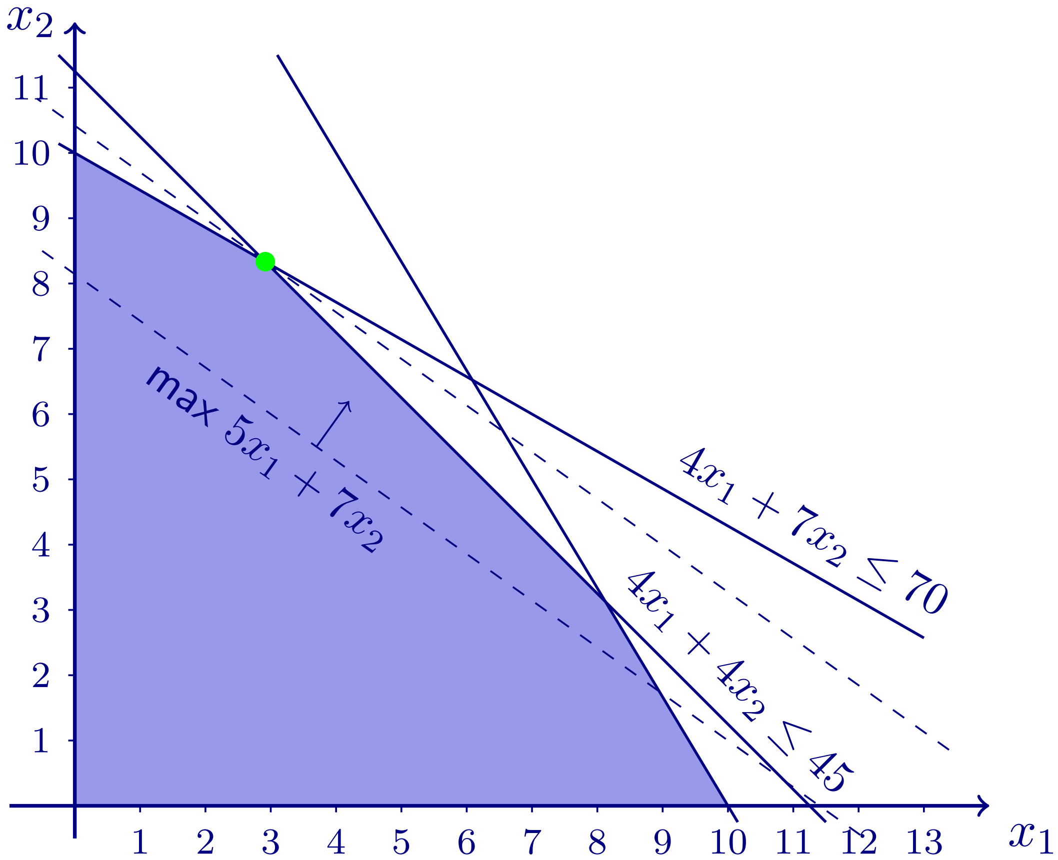
  - Sensitivity range information ➡ in which ranges are shadow prices valid?

Shadow prices



Shadow prices



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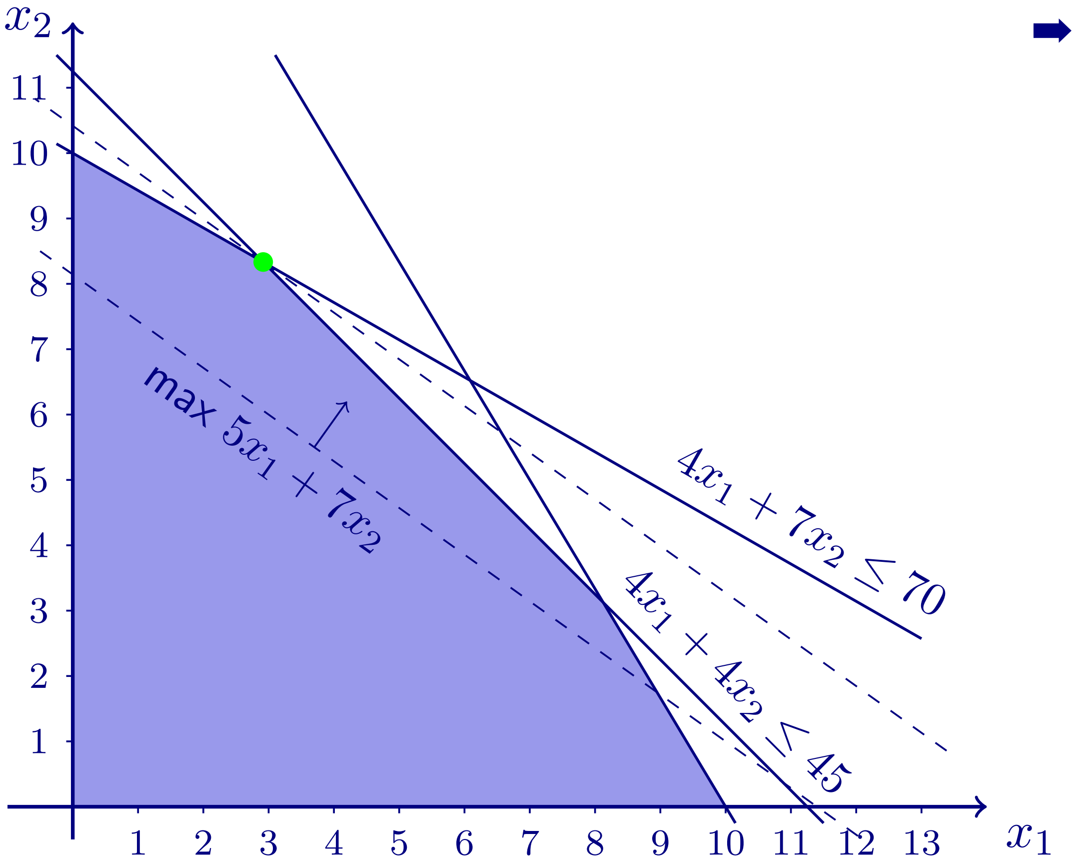

➡ compute optimal solution:

$$\begin{aligned} 4x_1 &+ 4x_2 &= 45 \\ 4x_1 &+ 7x_2 &= 70 \end{aligned}$$

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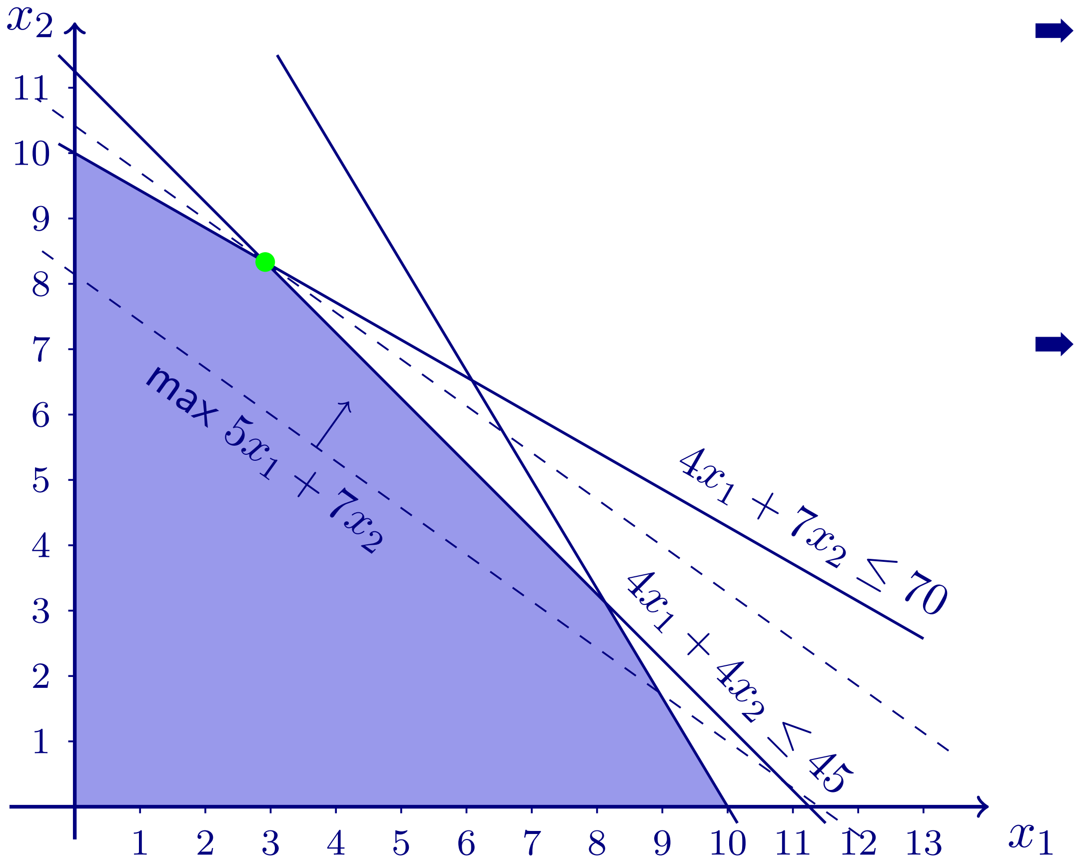

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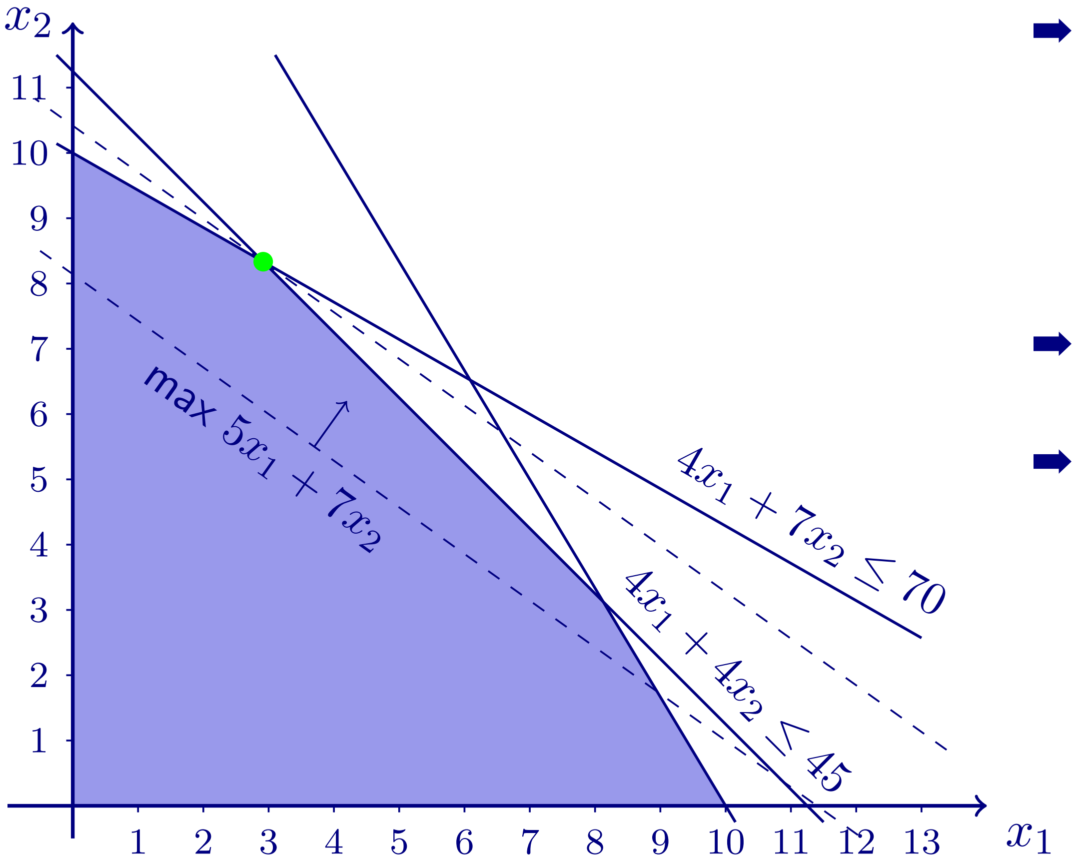

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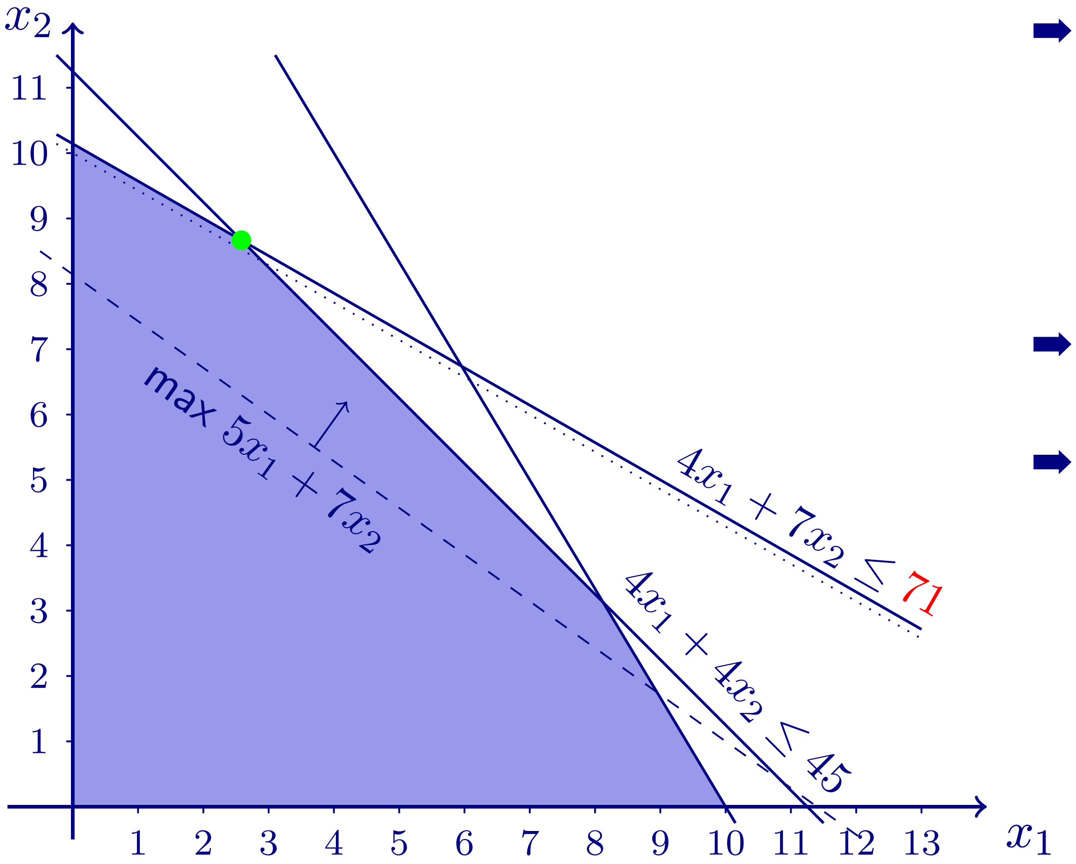

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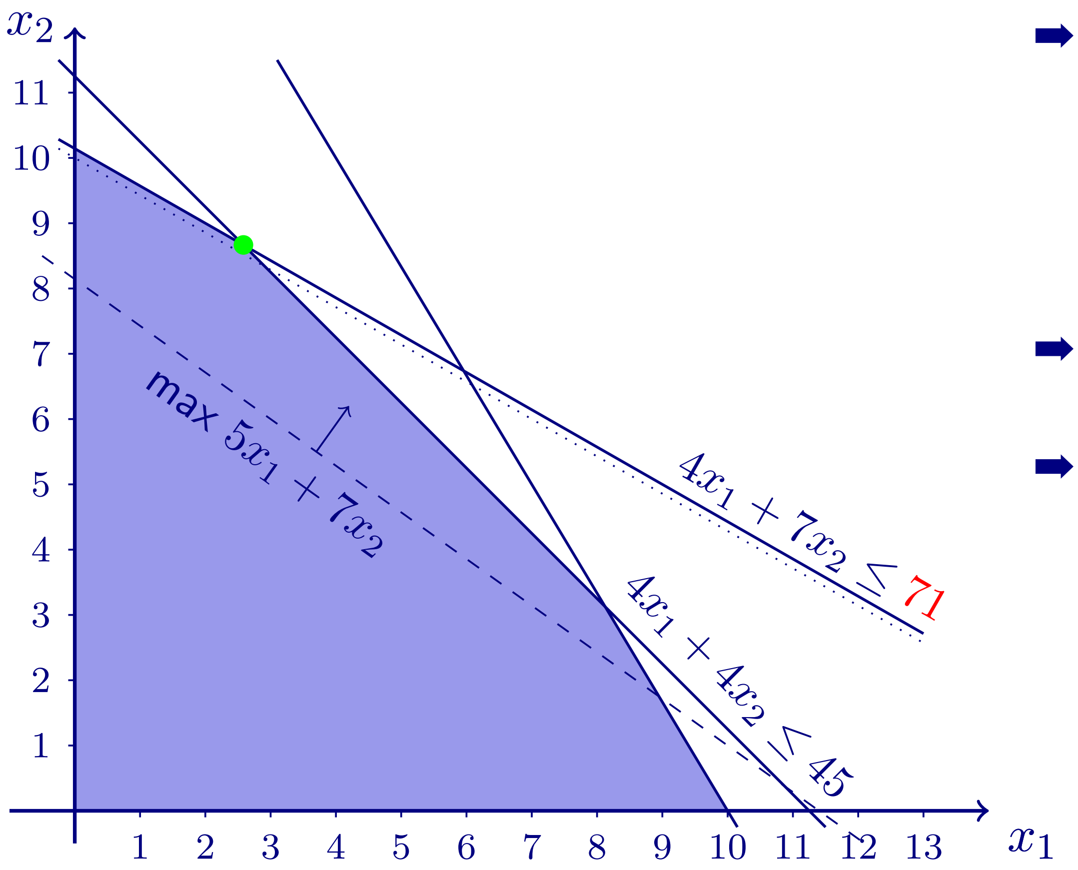

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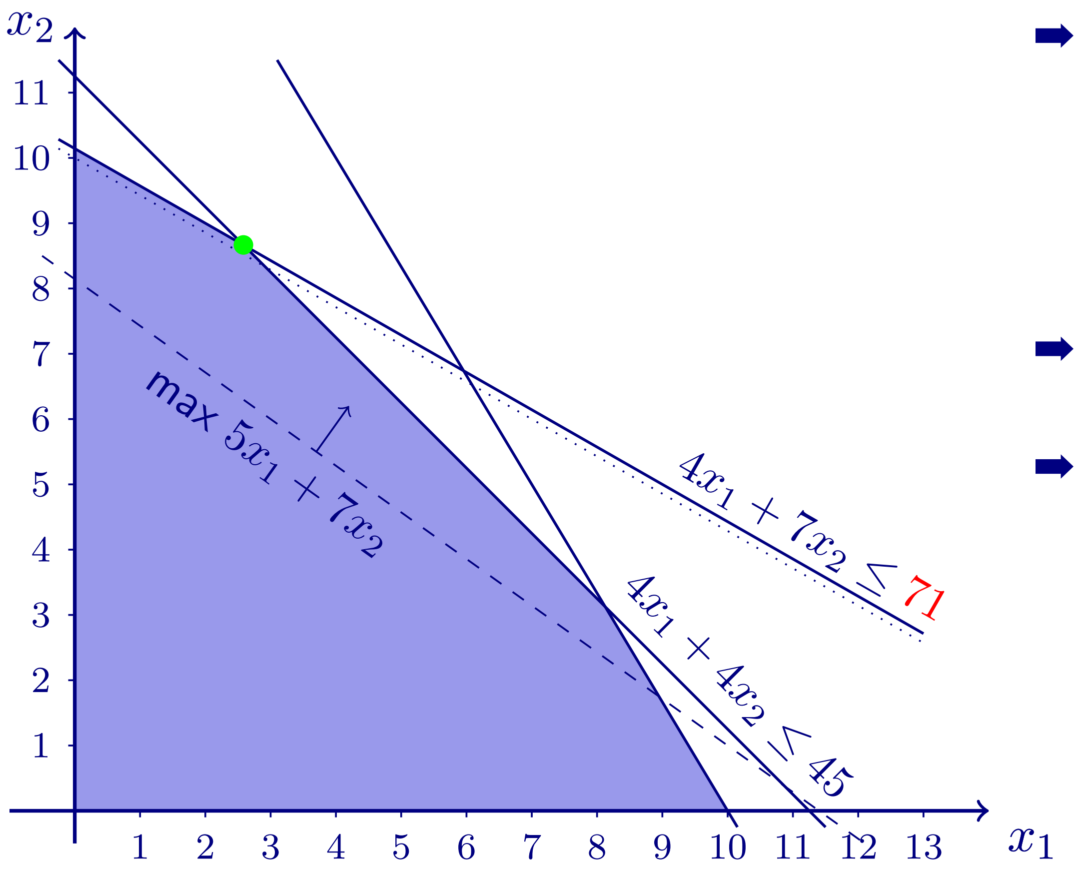

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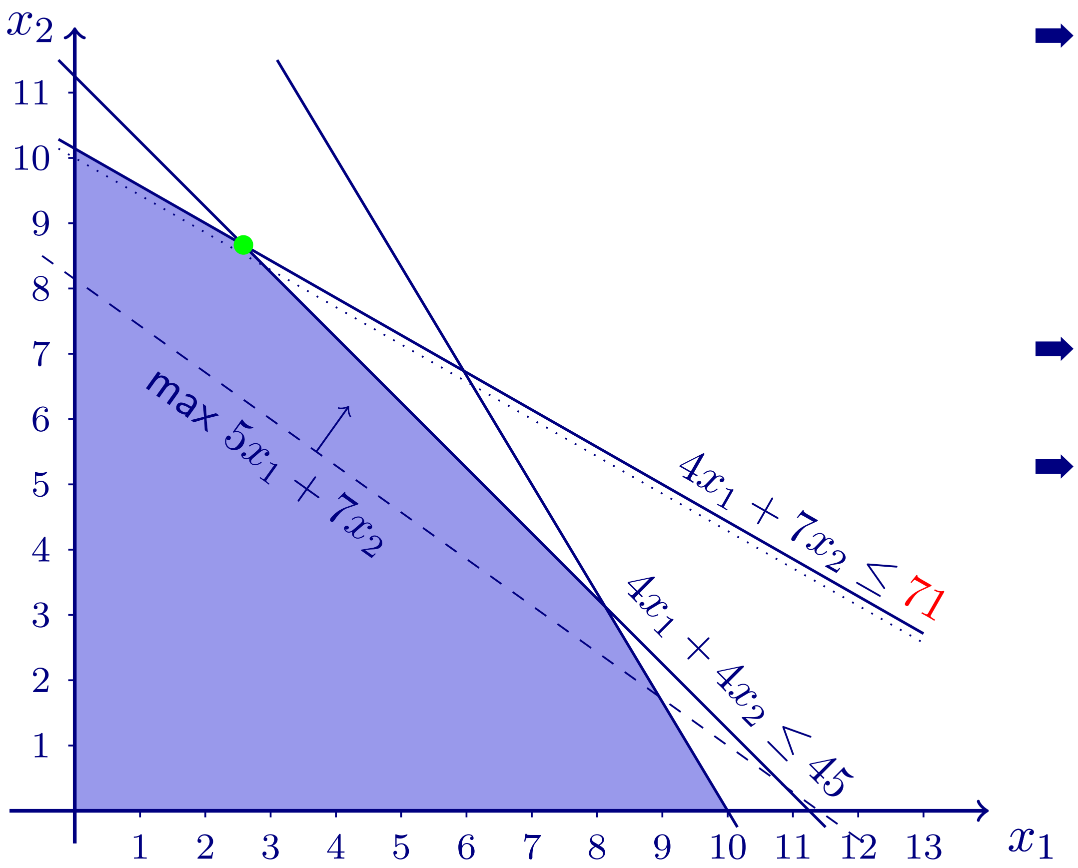

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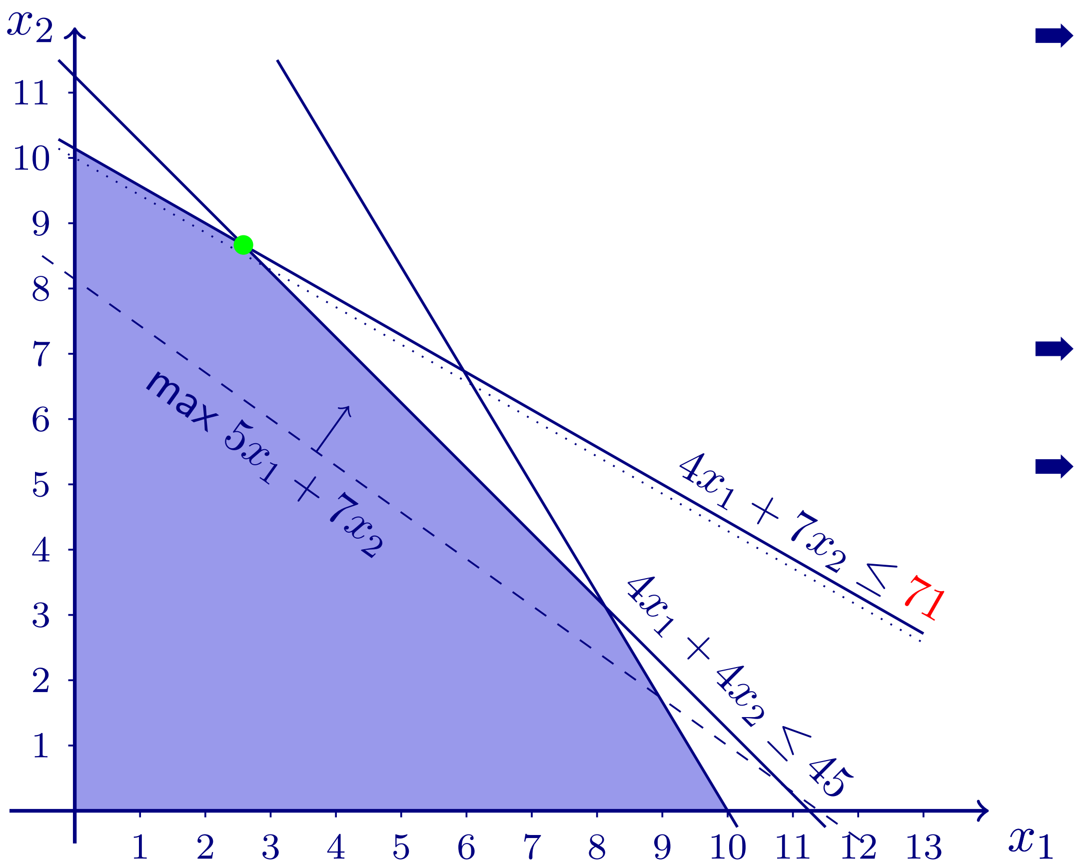

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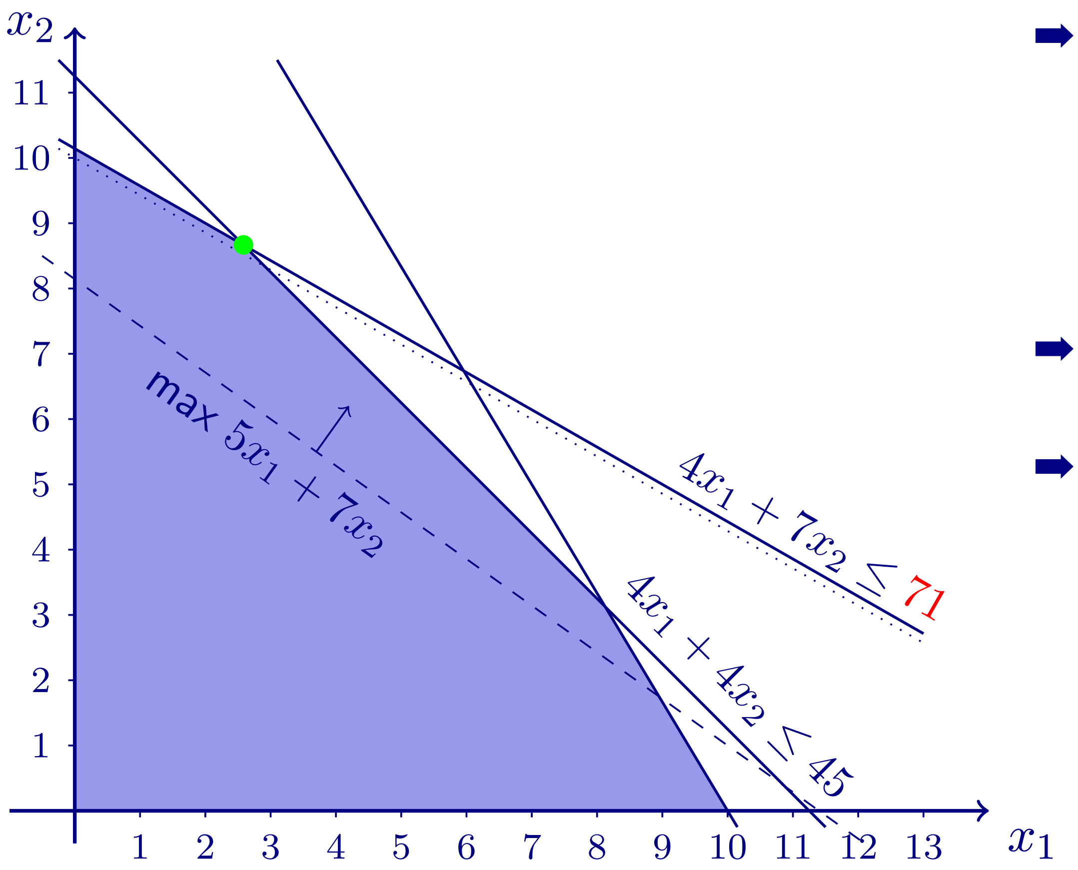
Shadow prices



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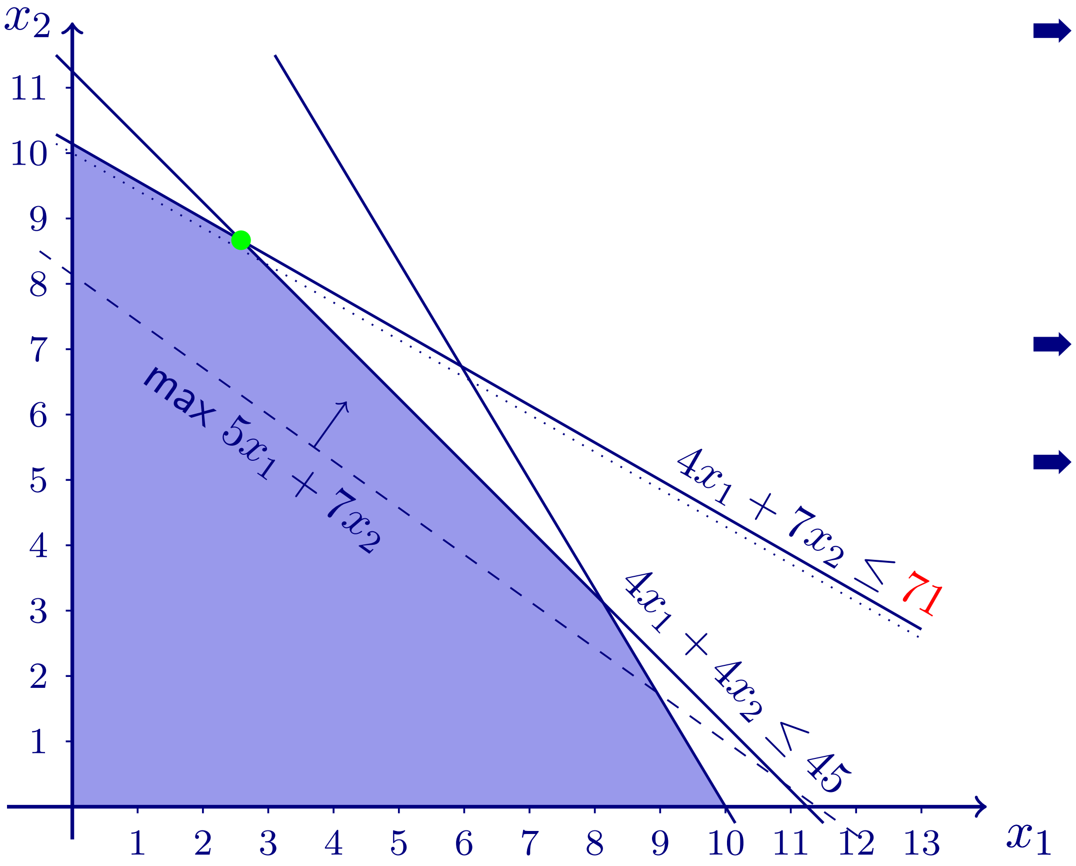

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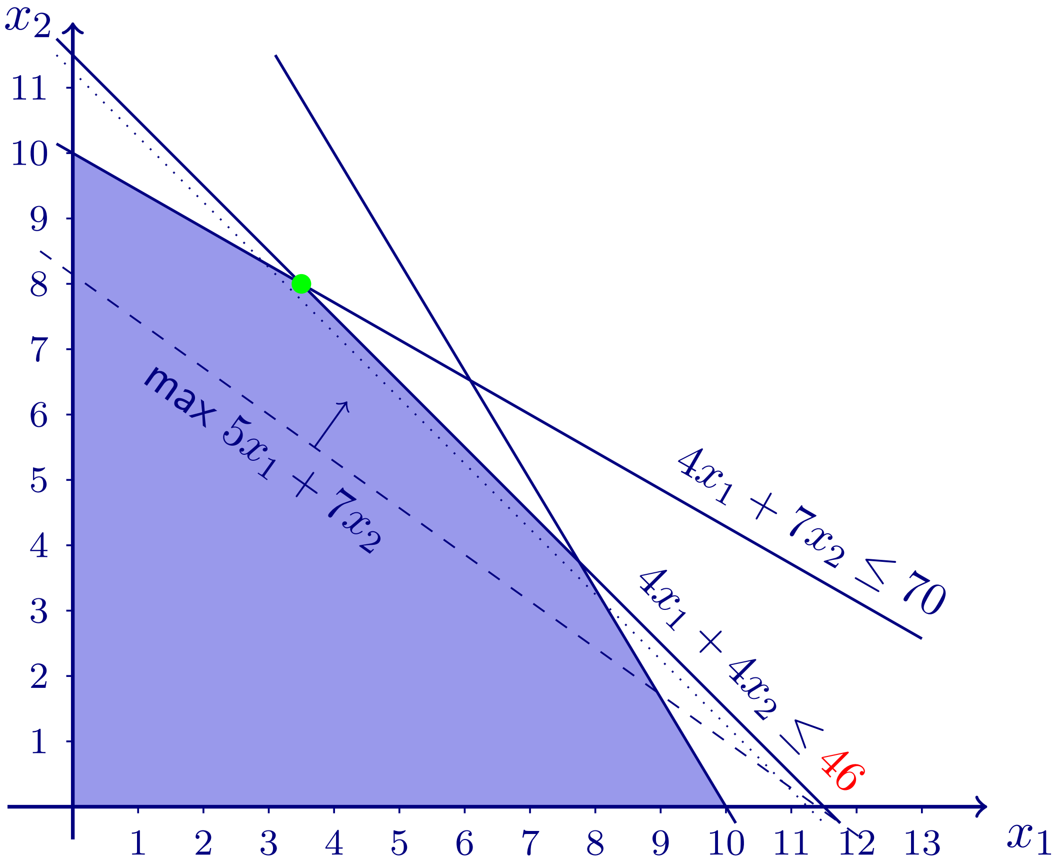
The shadow price of a constraint is the rate of change in the objective function per unit increase of the constraint's right-hand side

More shadow prices



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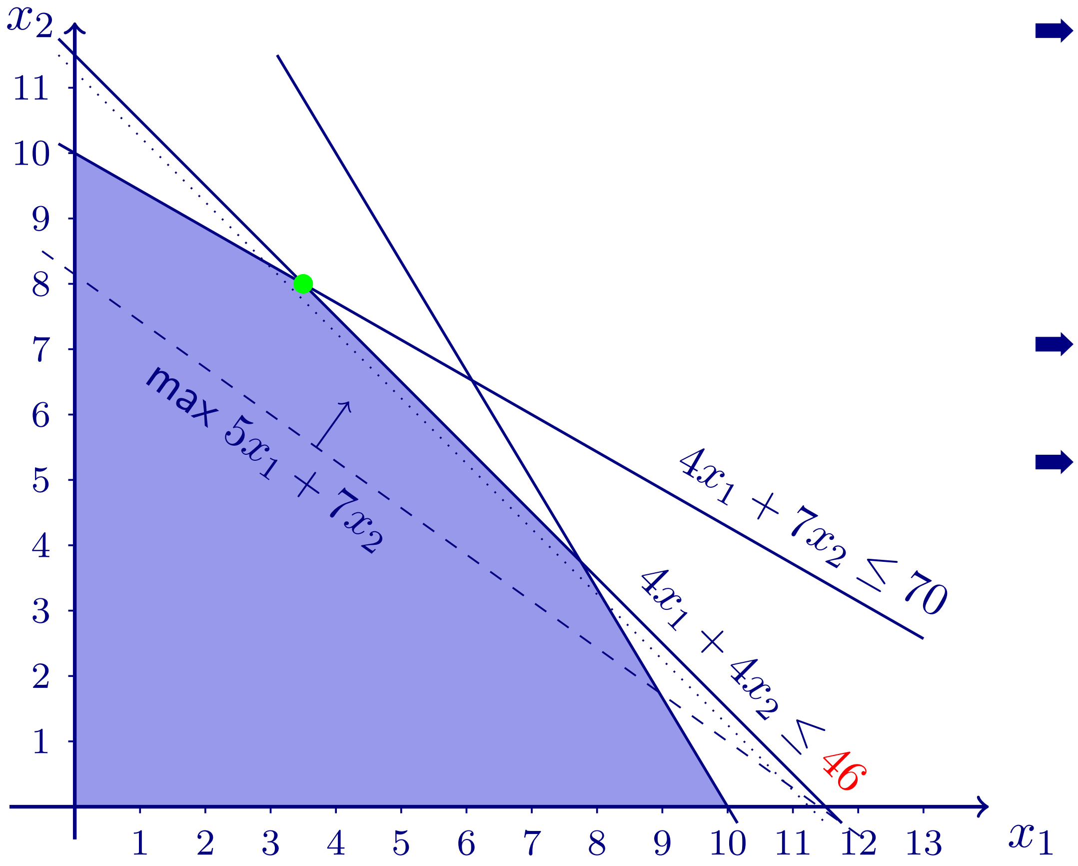

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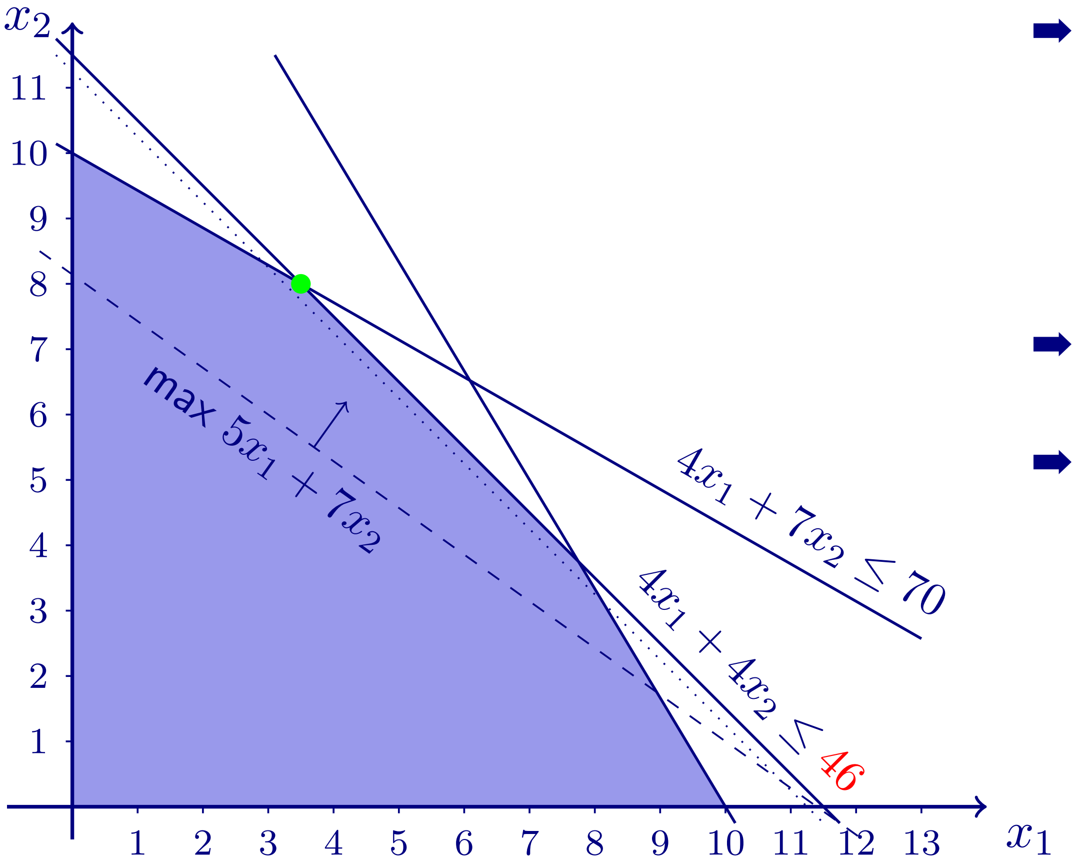

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$$
\begin{array}{ccccc}
4x_1 & + & 4x_2 & = & \cancel{45}\ 46 \\
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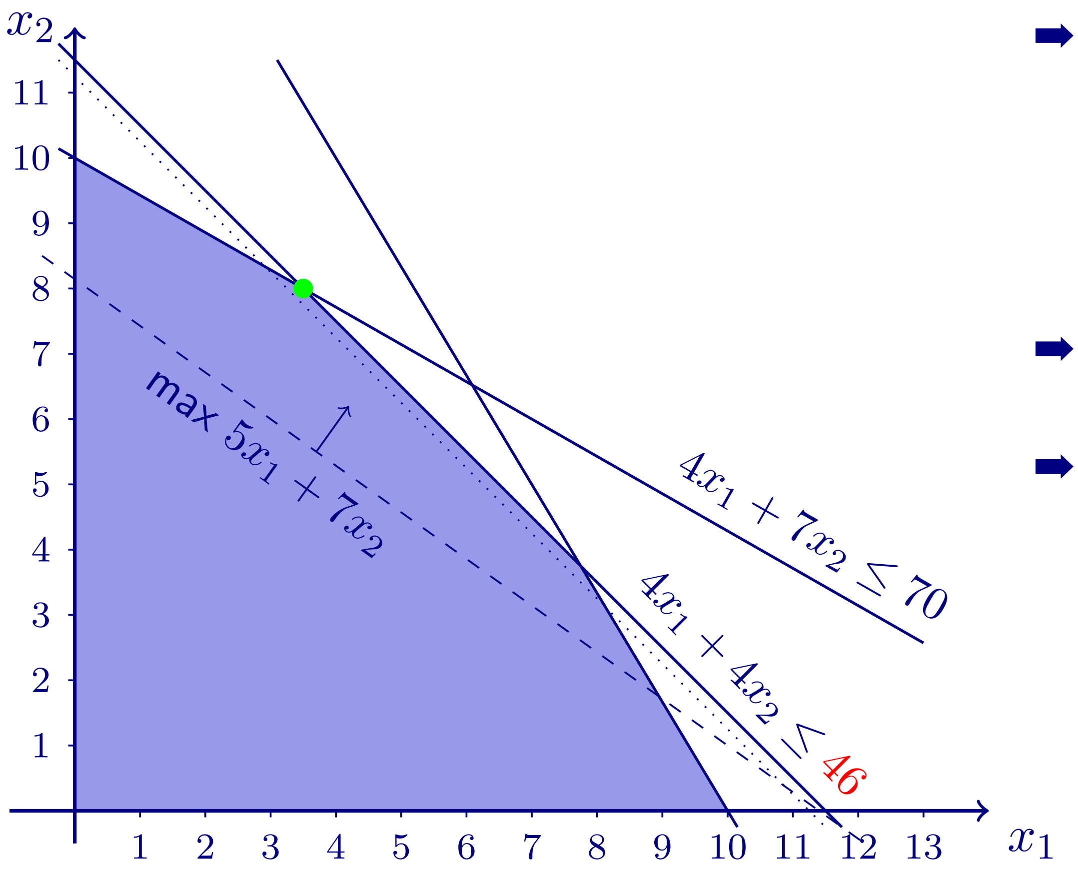

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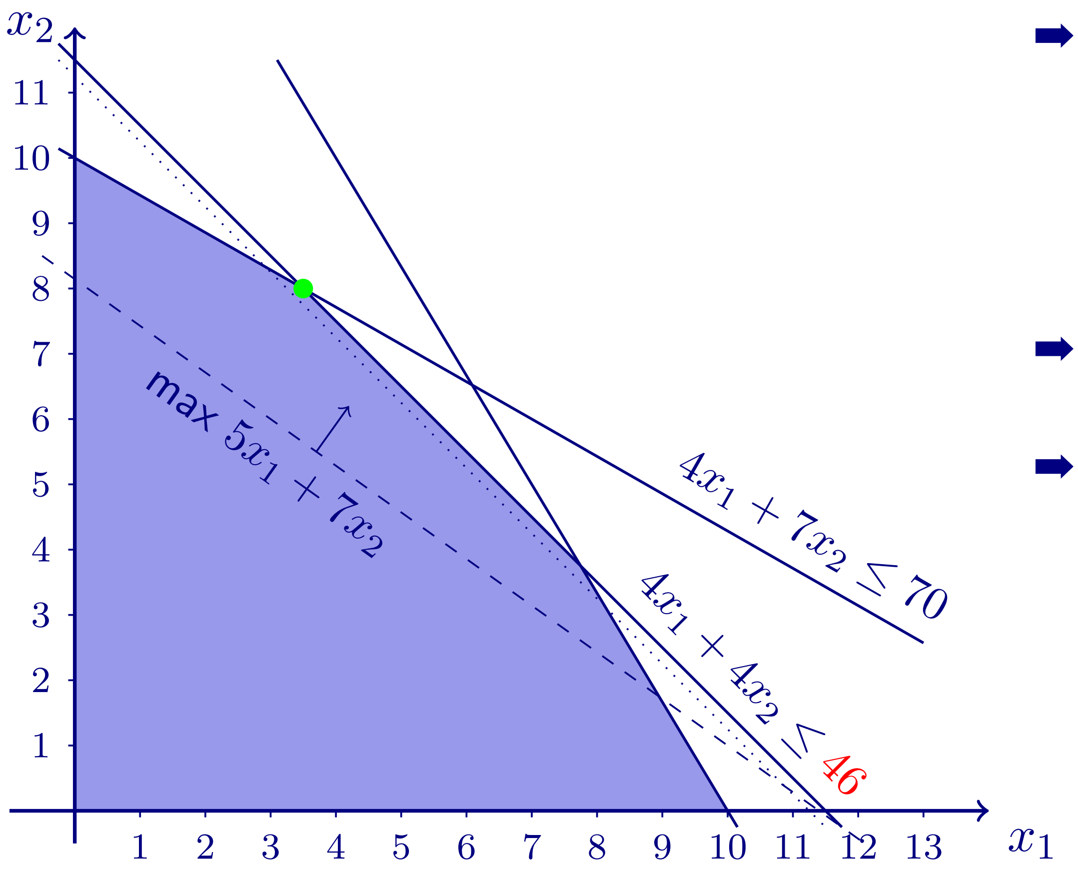

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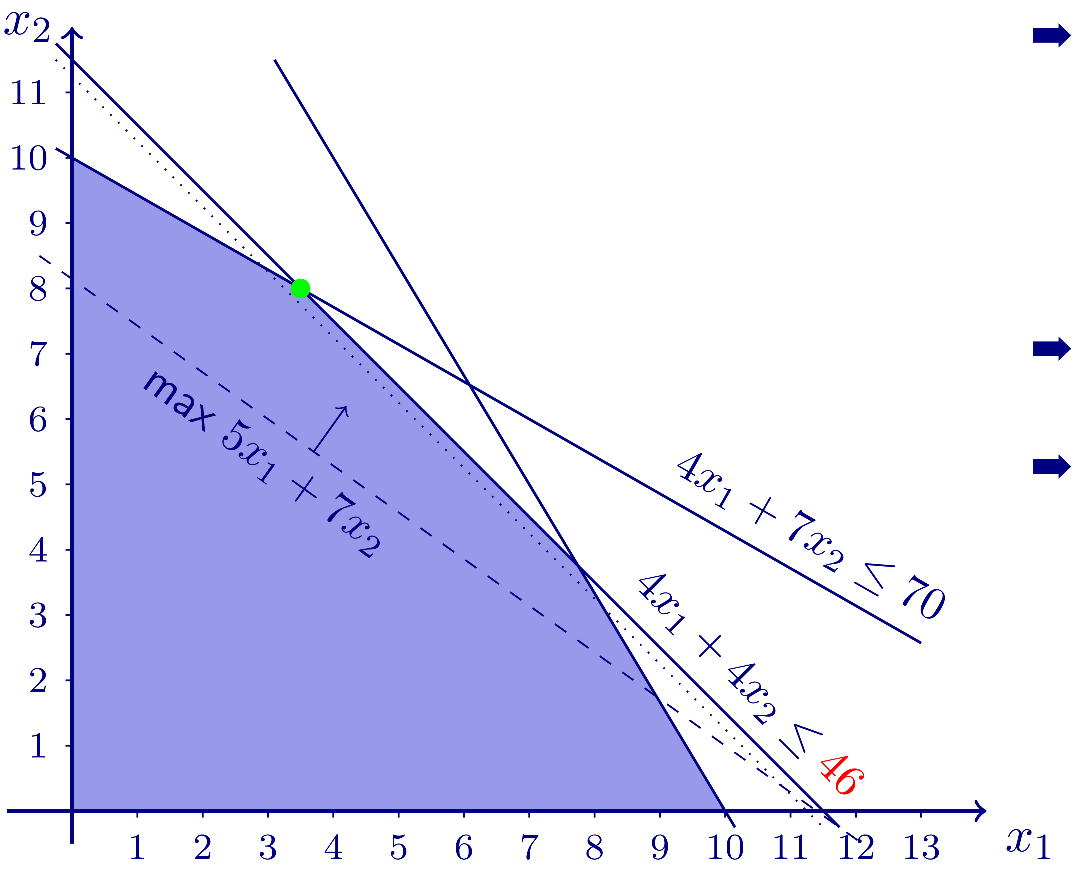

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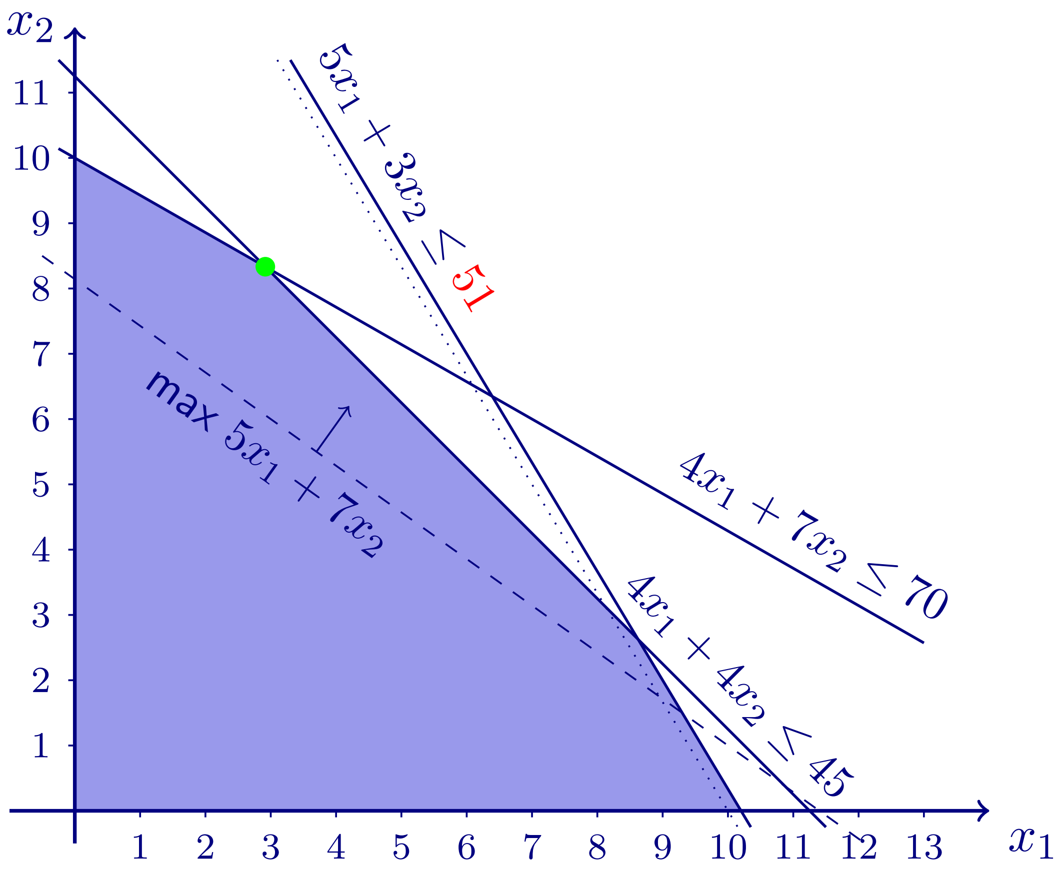
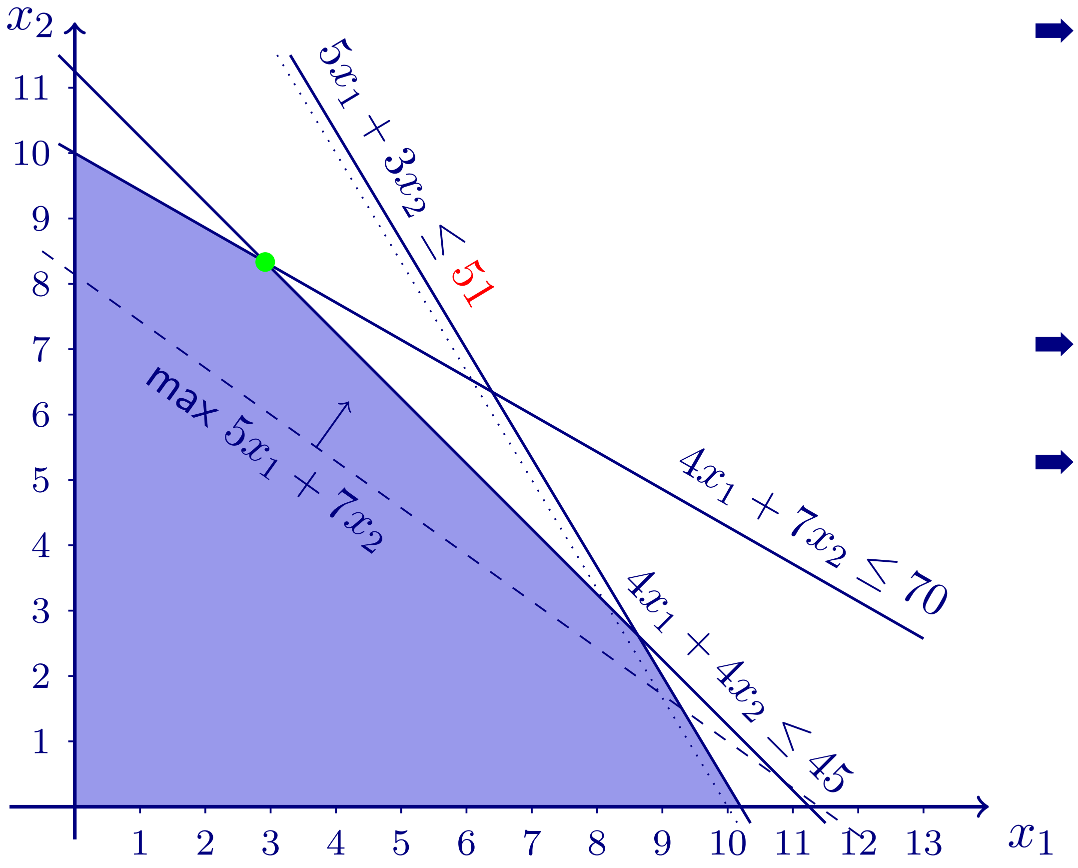
- ➡ Shadow price: $\frac{15}{12} = 1.25$

More shadow prices



More shadow prices



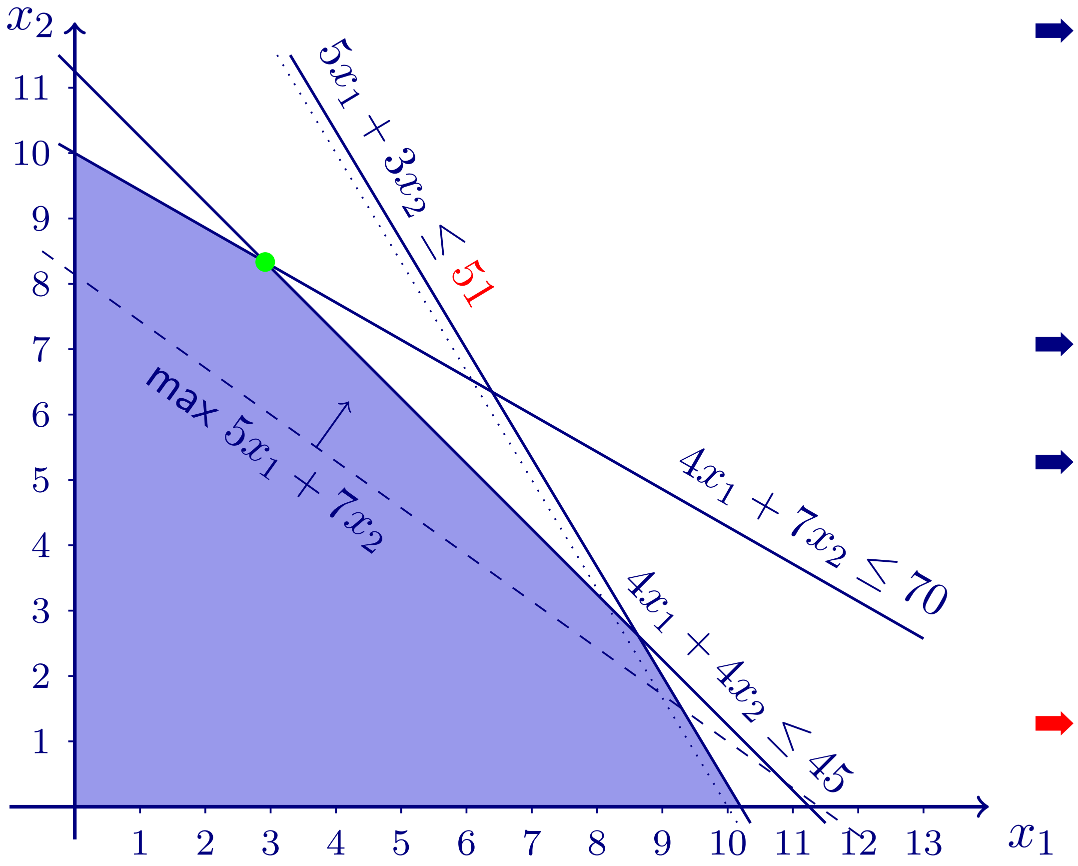
- ➡ compute optimal solution:

$$\begin{array}{ccccc} 4x_1 & + & 4x_2 & = & 45 \\ 4x_1 & + & 7x_2 & = & 70 \end{array}$$

- ➡ $(x_1, x_2) = \left(\frac{35}{12}, \frac{25}{3}\right)$
- ➡ compute optimal value:

$$5 \cdot \frac{35}{12} + 7 \cdot \frac{25}{3} = \frac{875}{12} \approx 72,91667$$

More shadow prices



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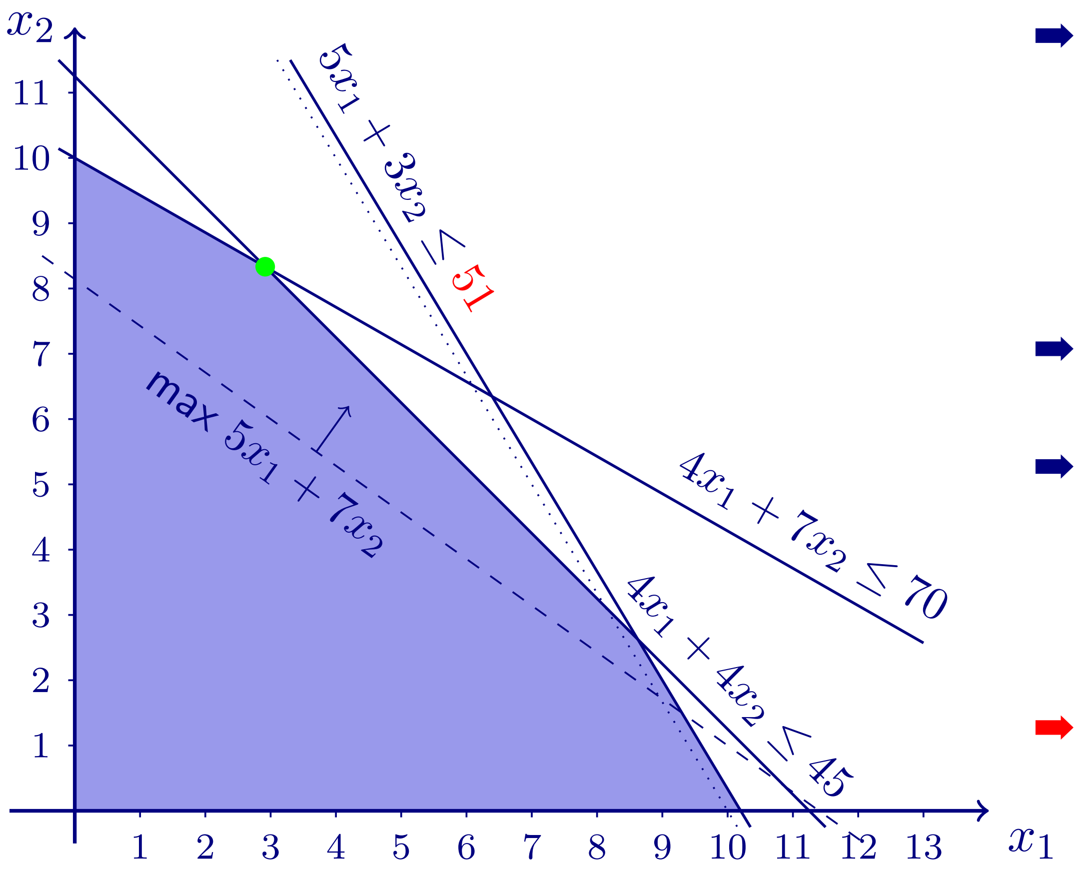
- ➡ Optimal solution doesn't change!

More shadow prices



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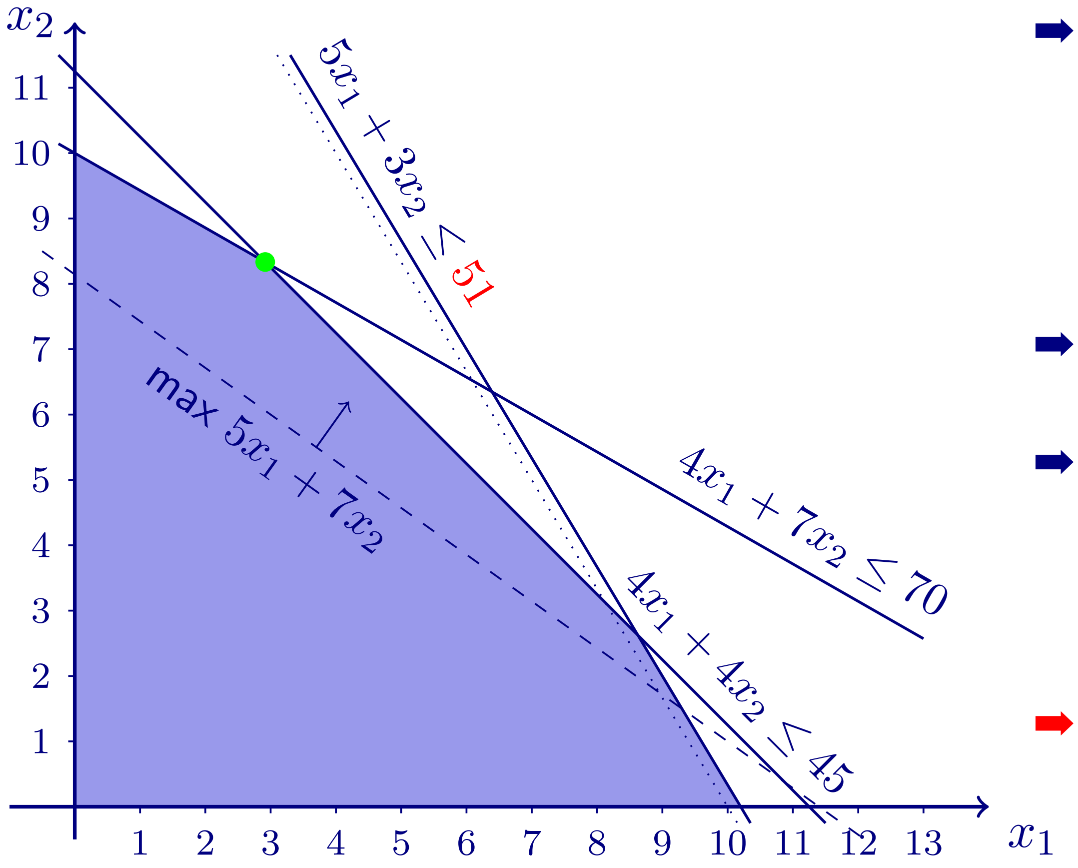
  - ➡ Shadow price: 0

More shadow prices



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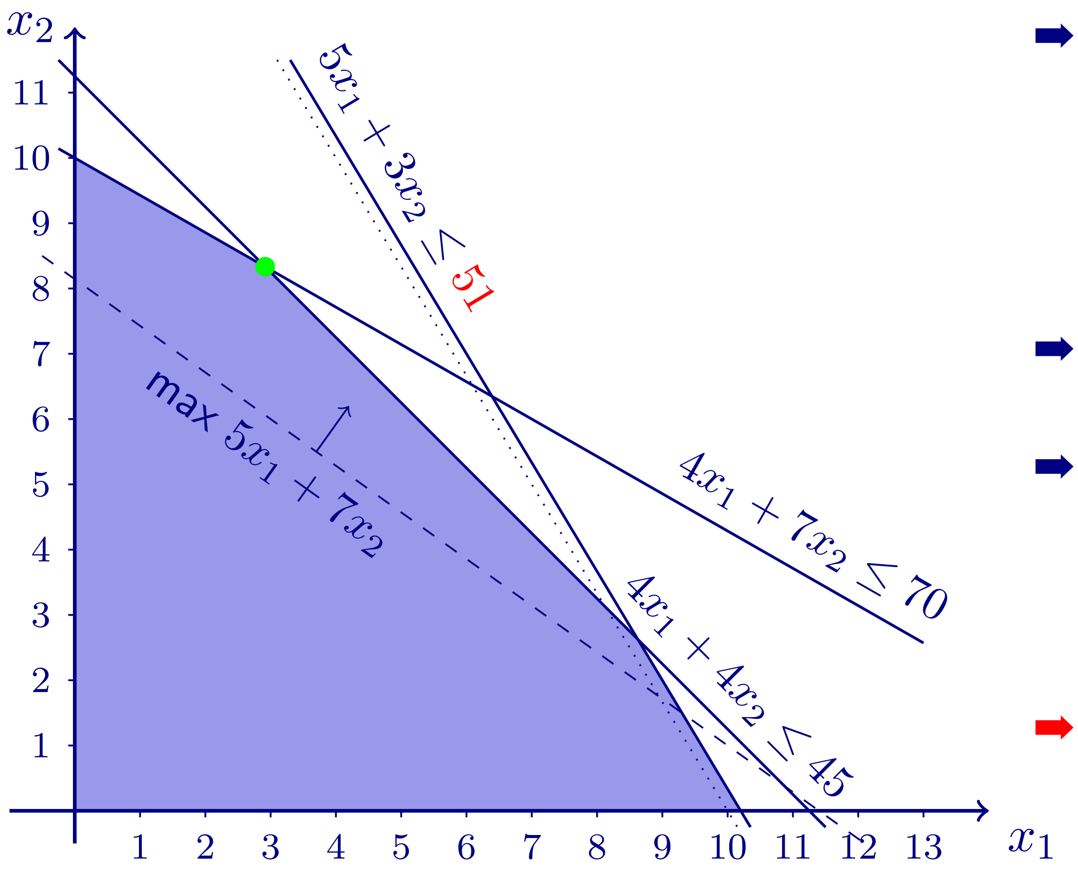
▷ Shadow prices of non-binding constraints are always 0

More shadow prices



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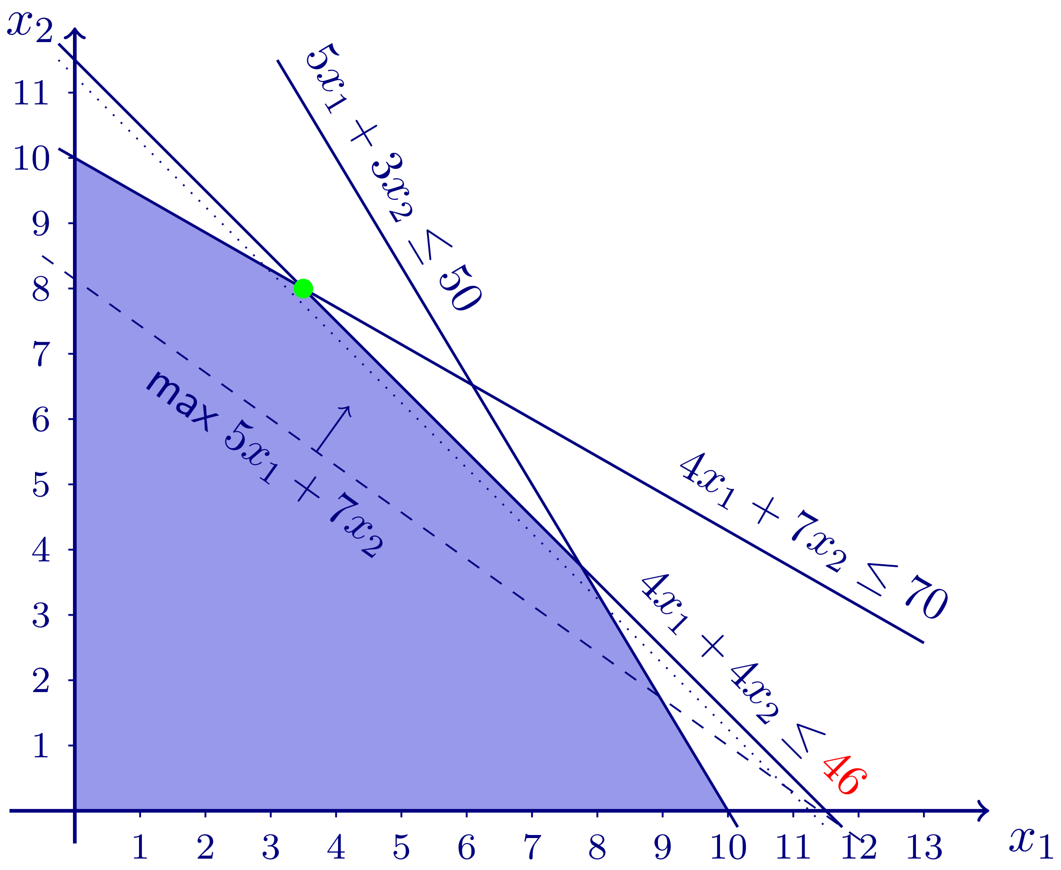
▷ Shadow prices of non-binding constraints are always 0

▷ Shadow prices of binding constraints are usually non-zero (but not always!)

Valid ranges of shadow prices



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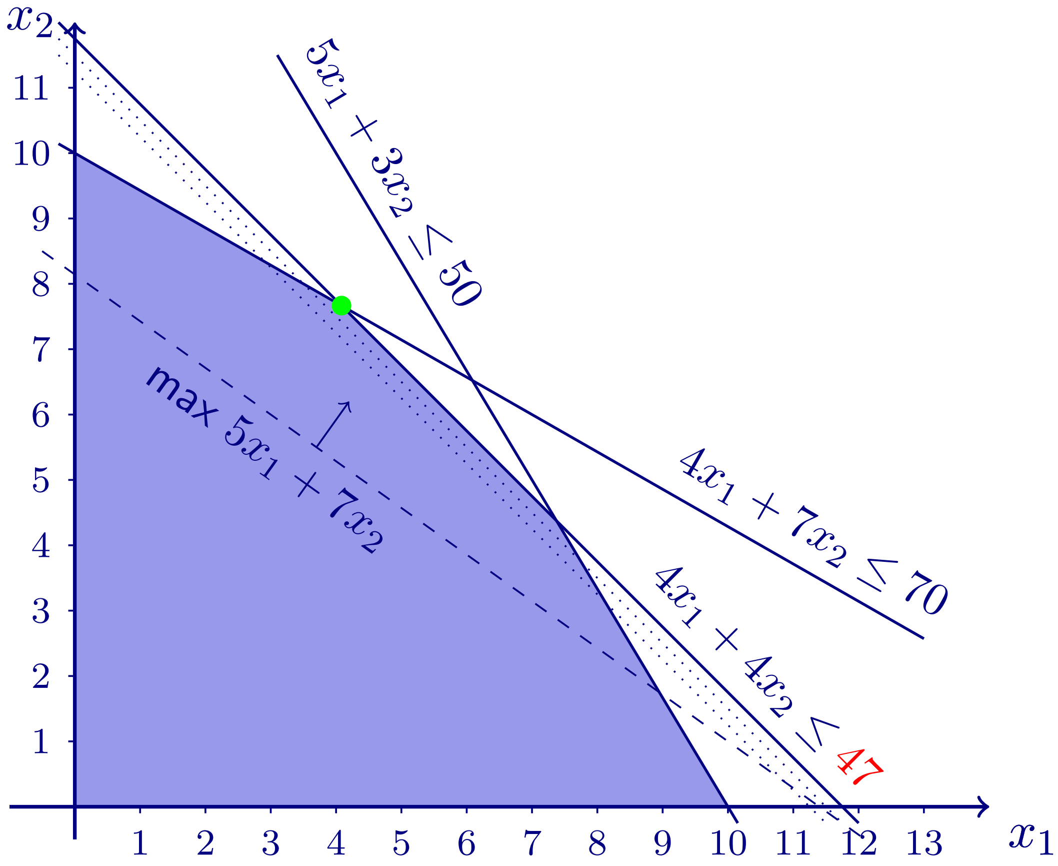
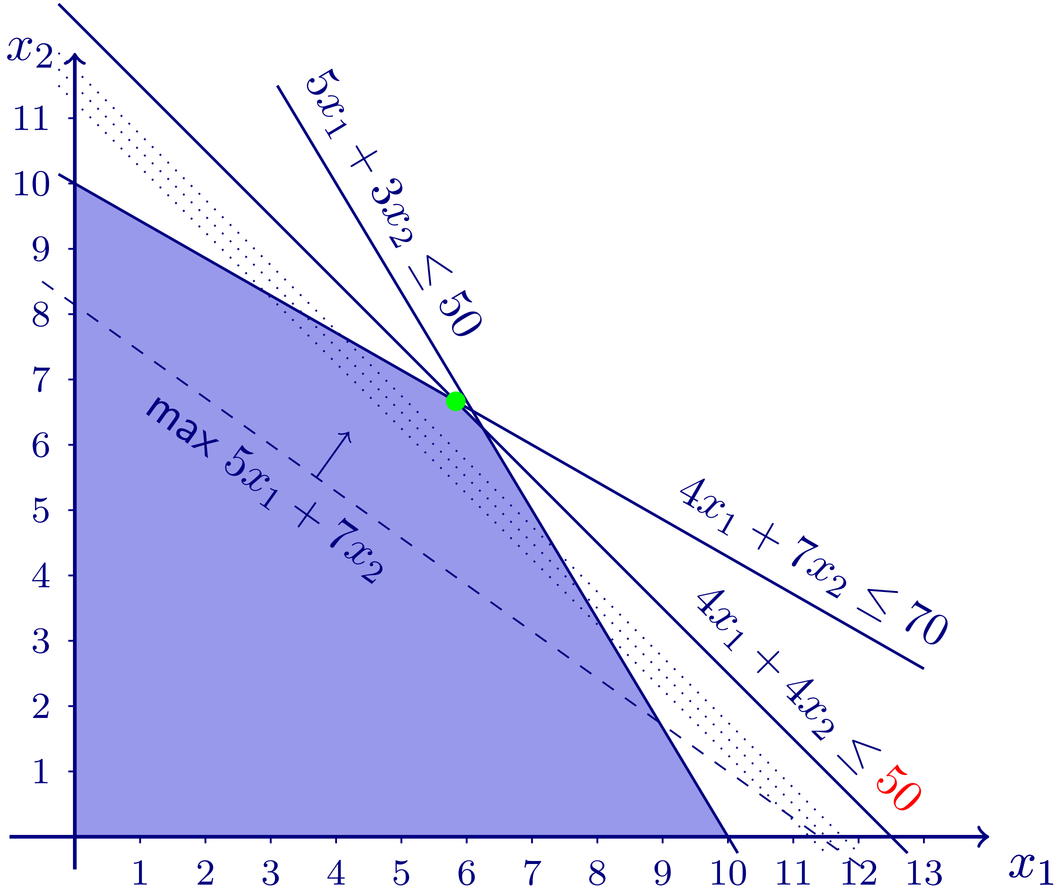


Valid ranges of shadow prices



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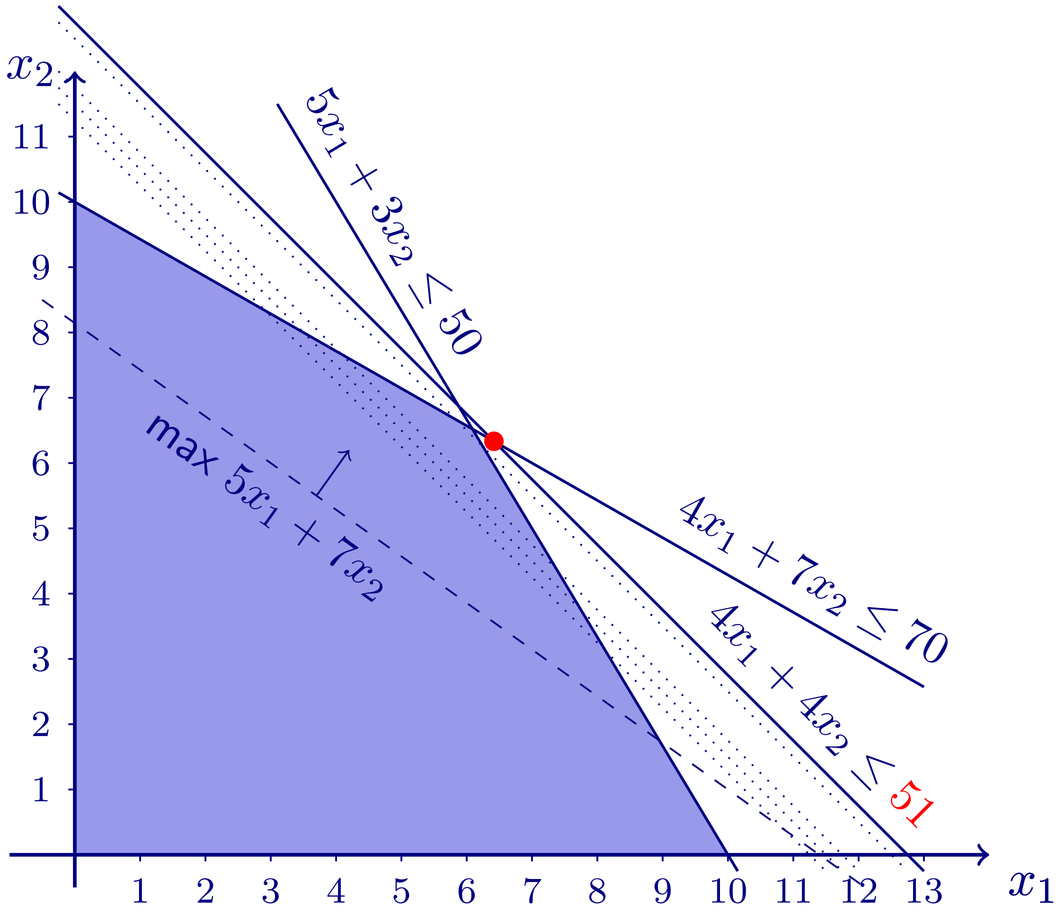
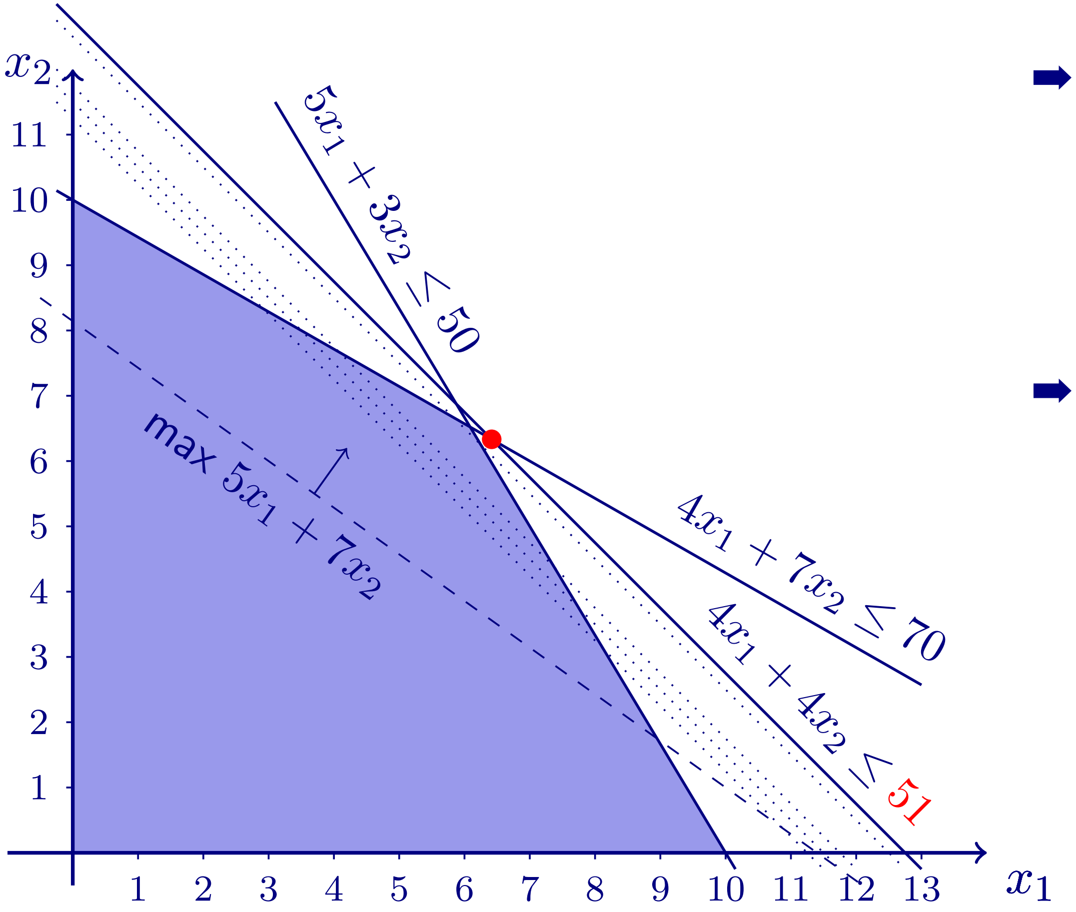


Valid ranges of shadow prices

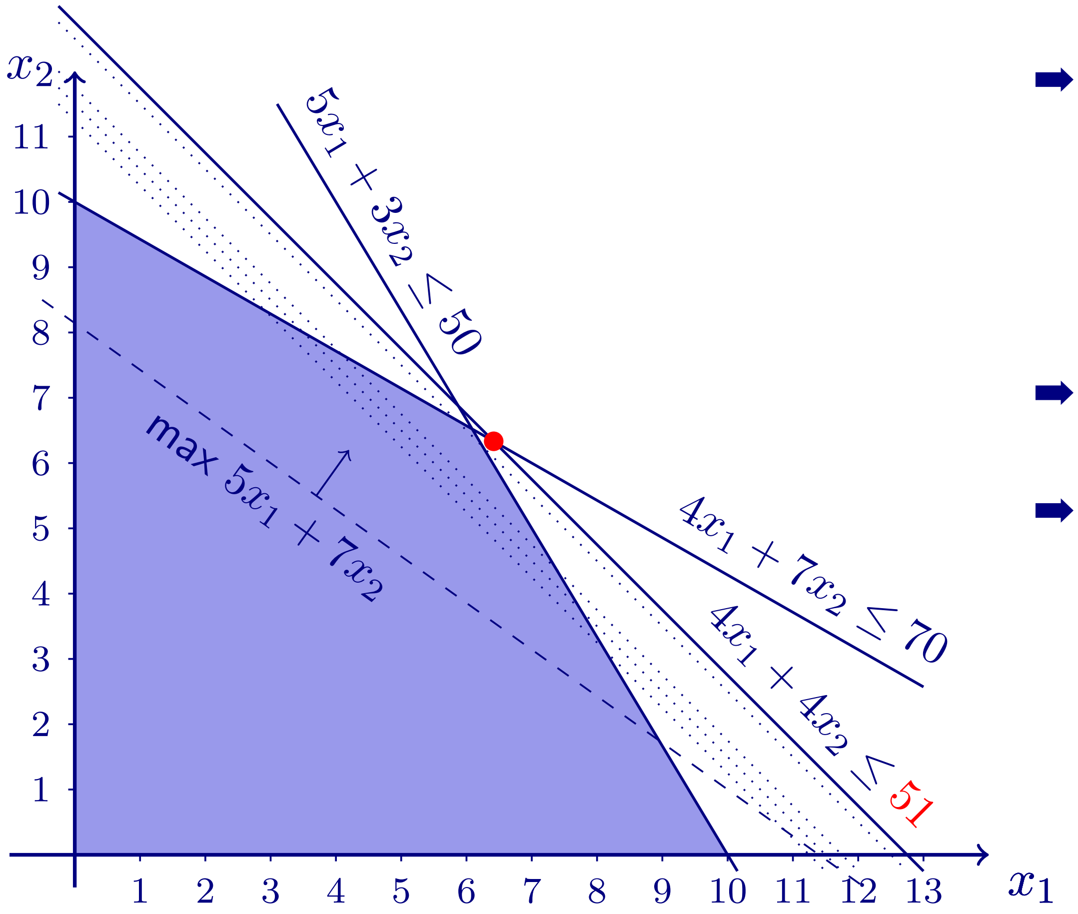


- ➡ compute intersection point:

$$\begin{array}{rcrcl} 4x_1 & + & 4x_2 & = & 51 \\ 4x_1 & + & 7x_2 & = & 70 \end{array}$$

- ➡ $(x_1, x_2) = \left(\frac{77}{12}, \frac{19}{3}\right)$

Valid ranges of shadow prices



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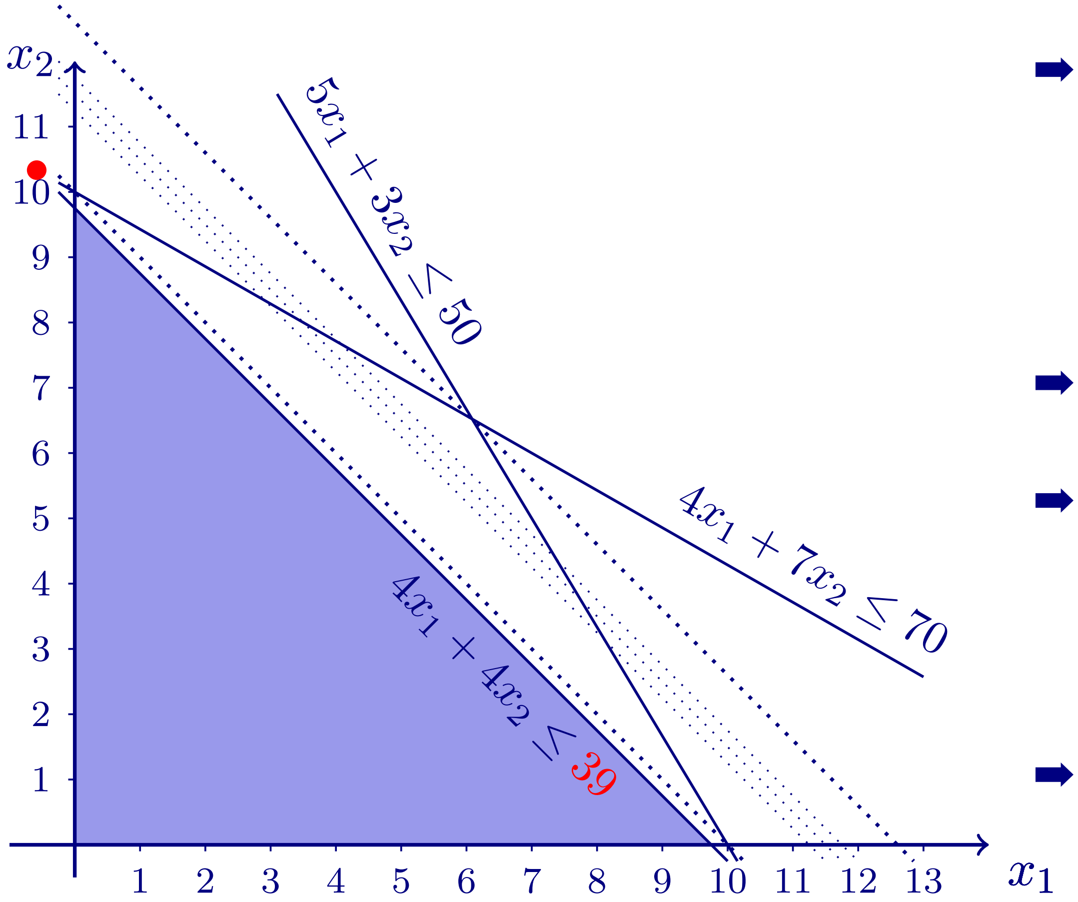
- ➡ violates other constraint:

$$5 \cdot \frac{77}{12} + 3 \cdot \frac{19}{3} = \frac{613}{12} \approx 51.08333 > 50$$

Valid ranges of shadow prices



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- ➡ similarly for decrease of RHS

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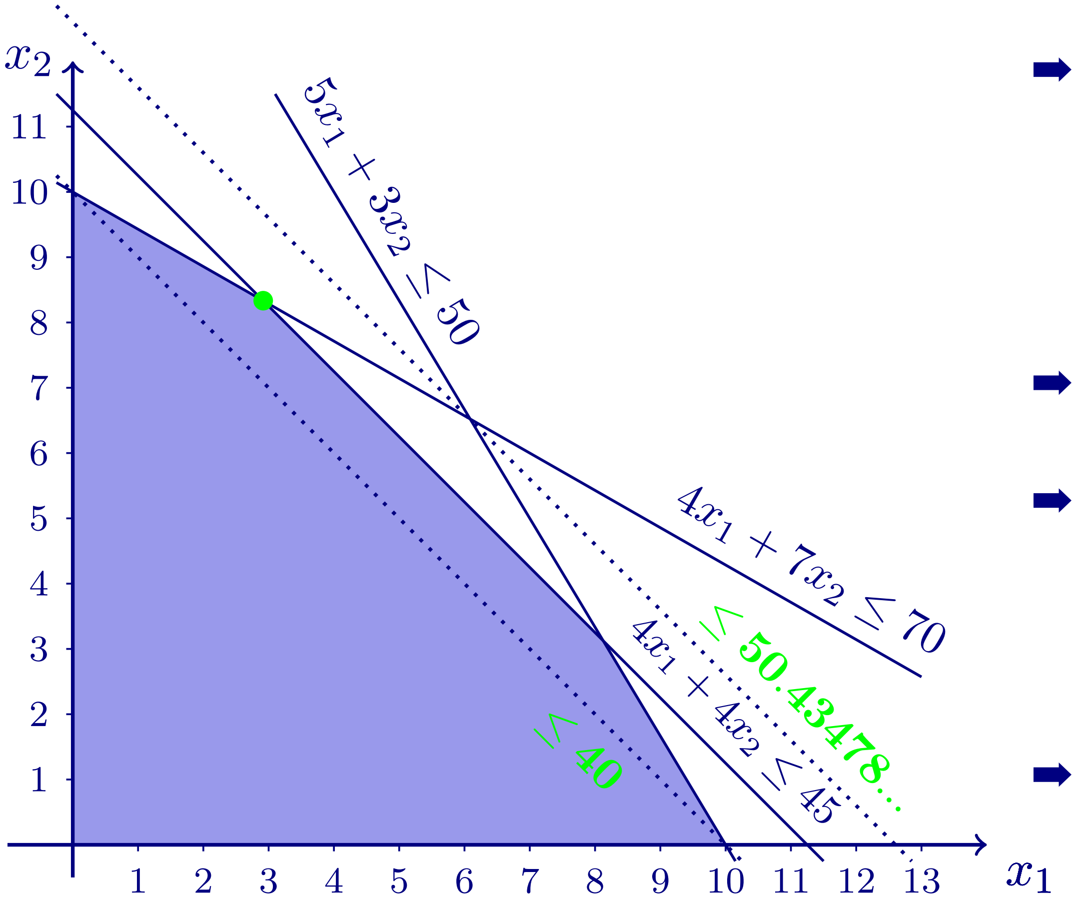

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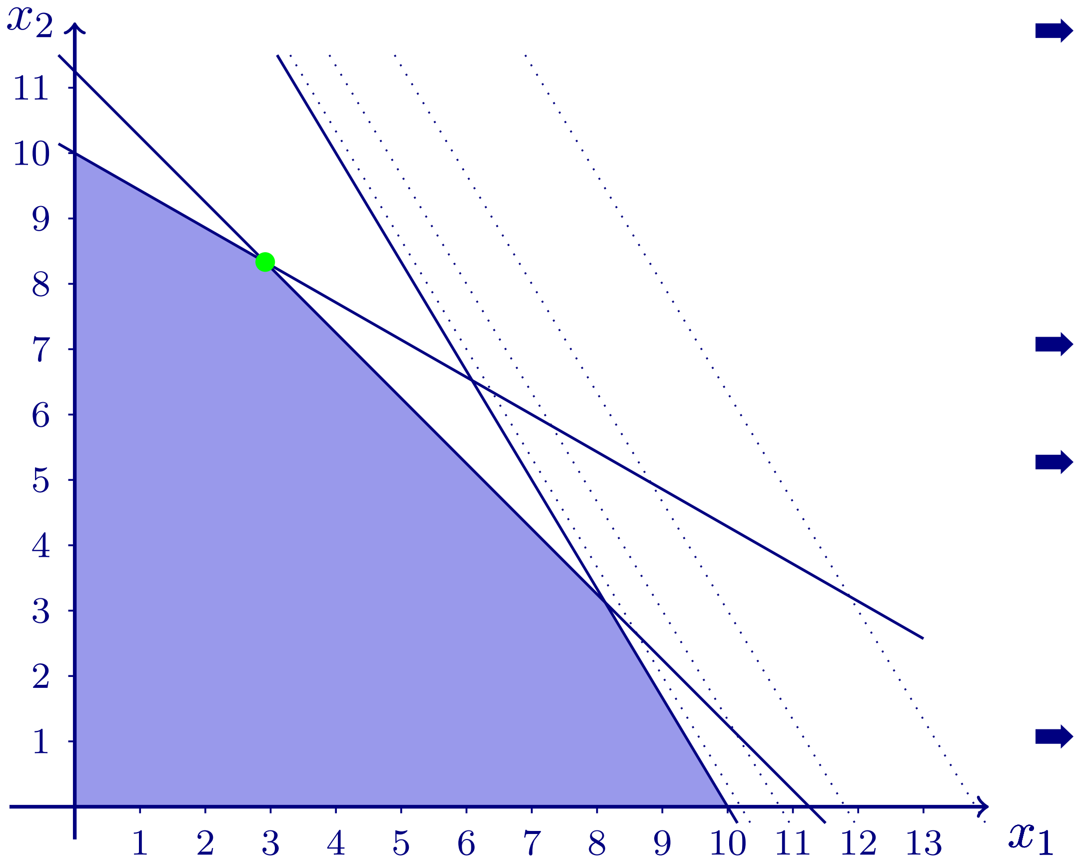
▷ Shadow price for constraint is only valid if RHS is in a certain range

Valid ranges of shadow prices



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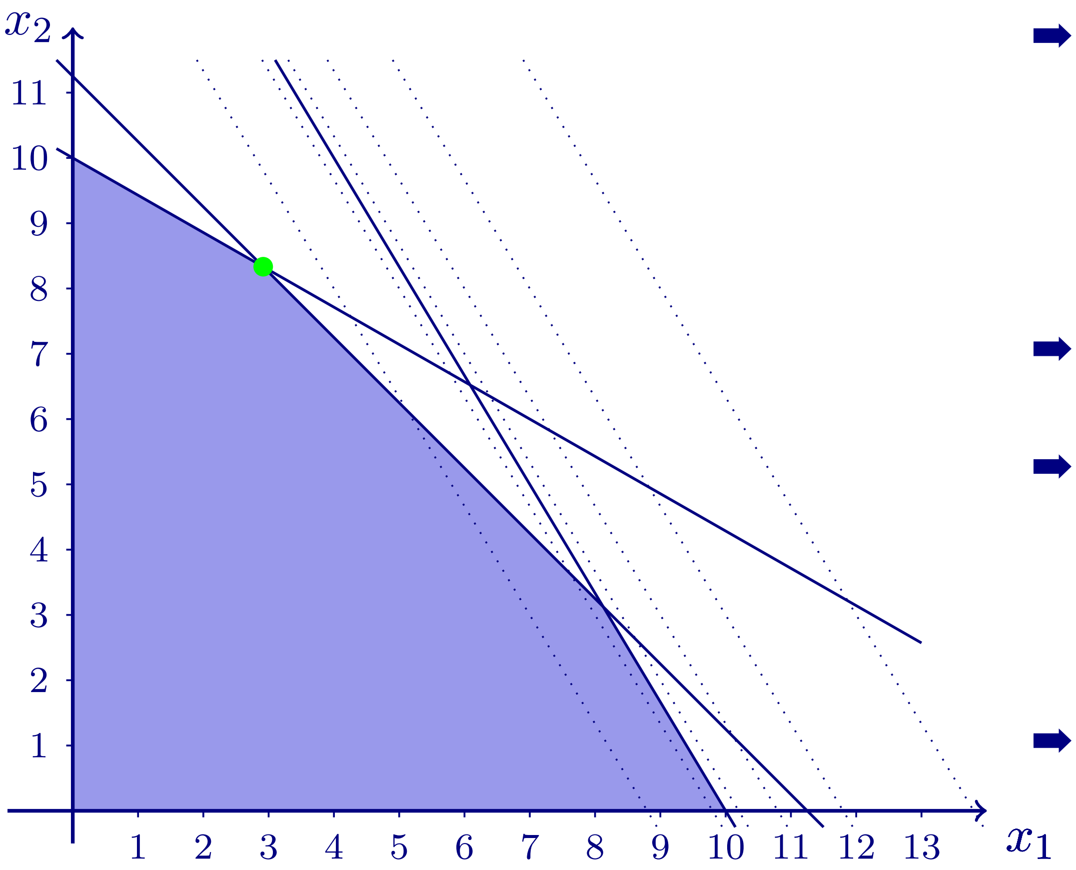

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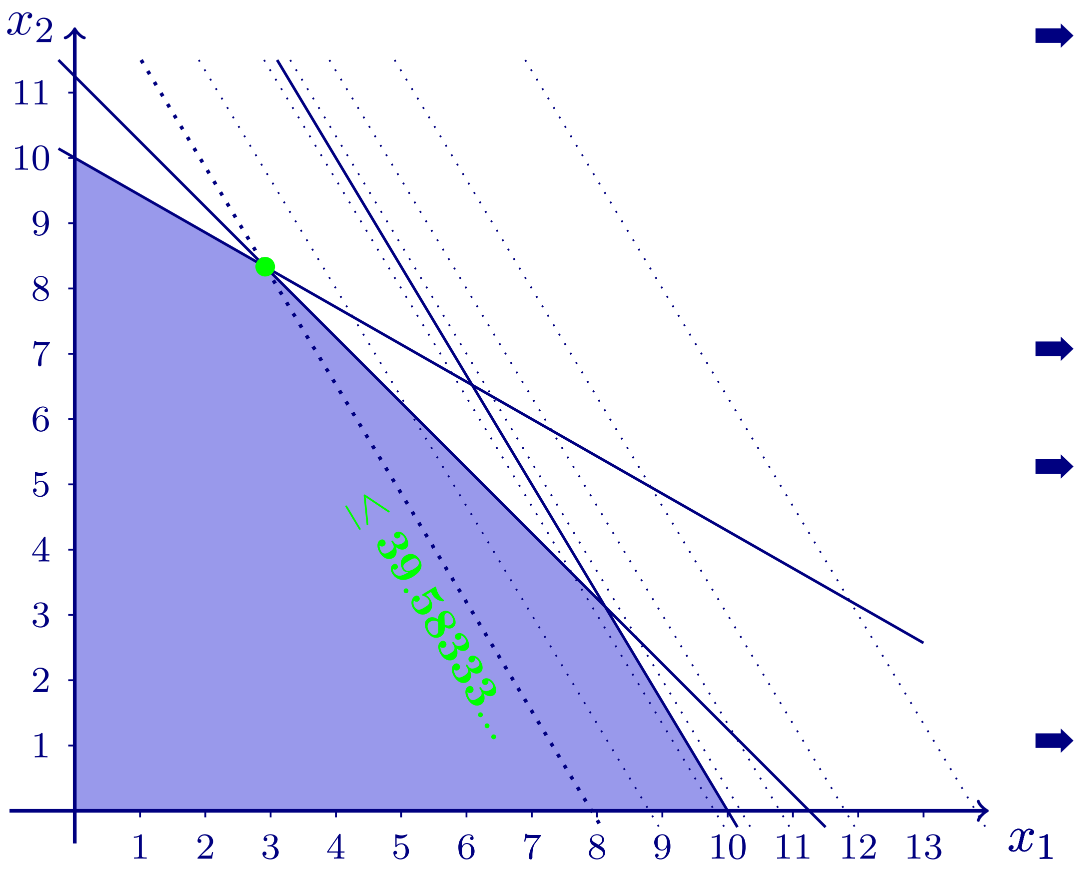

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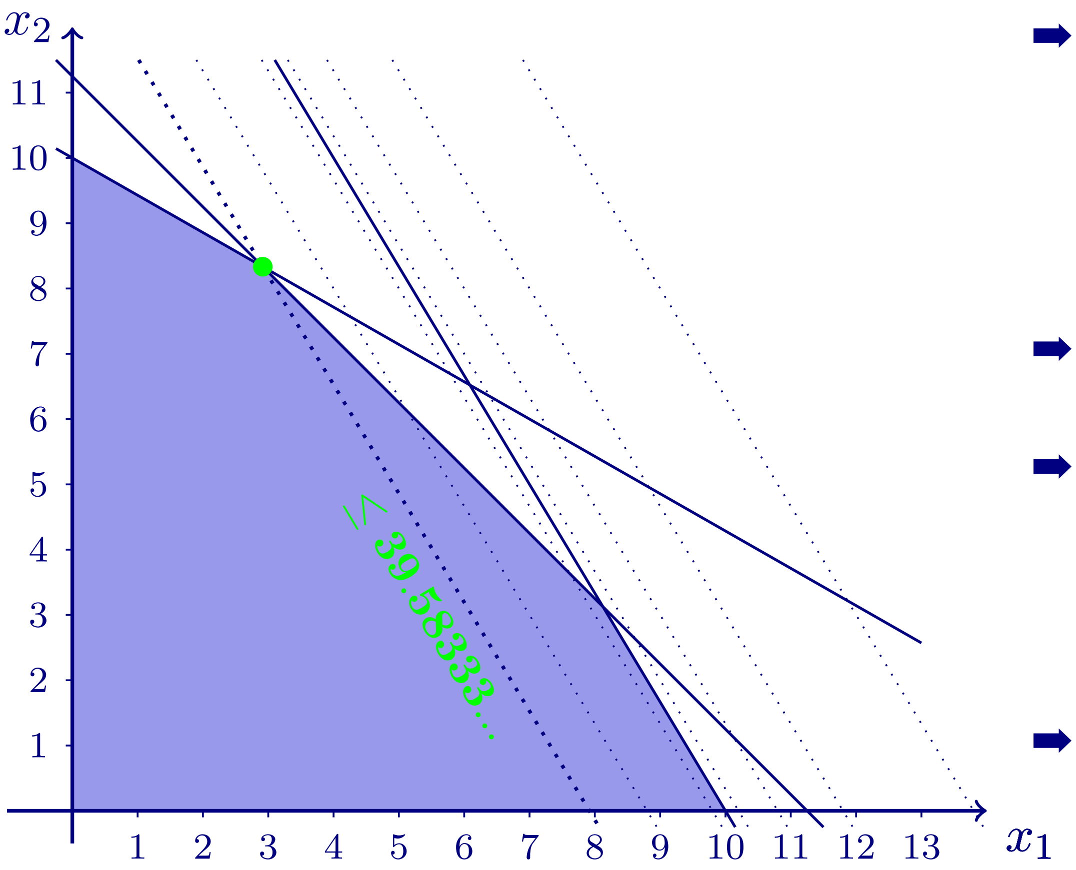

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➡ similarly for decrease of RHS

- ▷ Shadow price for constraint is only valid if RHS is in a certain range
- ▷ Range for non-binding constraints is only bounded from below and extends to $+\infty$

Changing objective – production planning revisited

▷ Production Planning in Automobile Industry

Product	**Beetle**	**Cabrio**
Revenue	$10000	$20000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 4500kg

➡ Question: How many cars of each type should be produced to maximize the profit?

Changing objective – production planning revisited

- ▷ Production Planning in Automobile Industry

Product	**Beetle**	**Cabrio**
Revenue	$10000	$14000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

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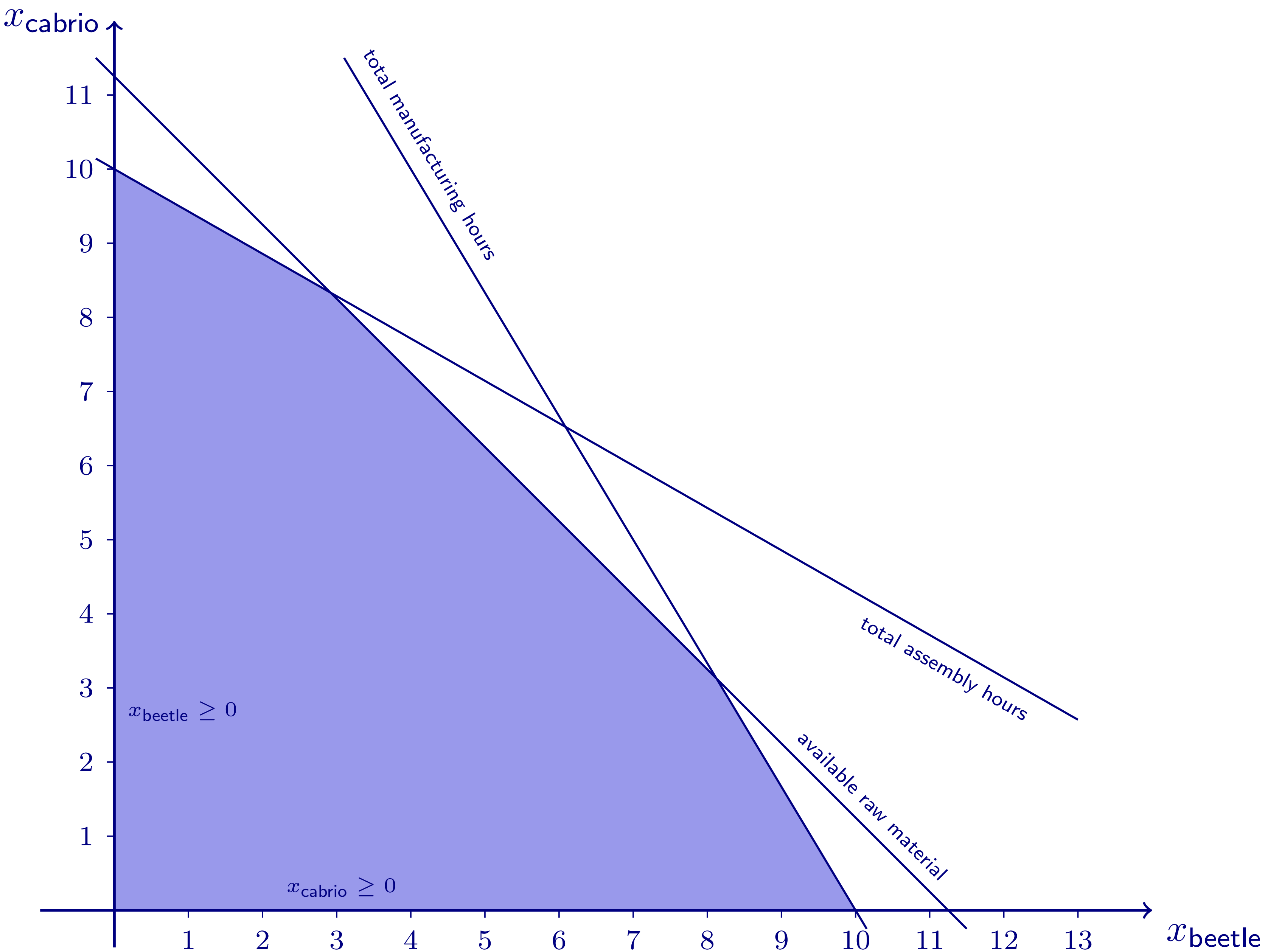
Solving the changed model



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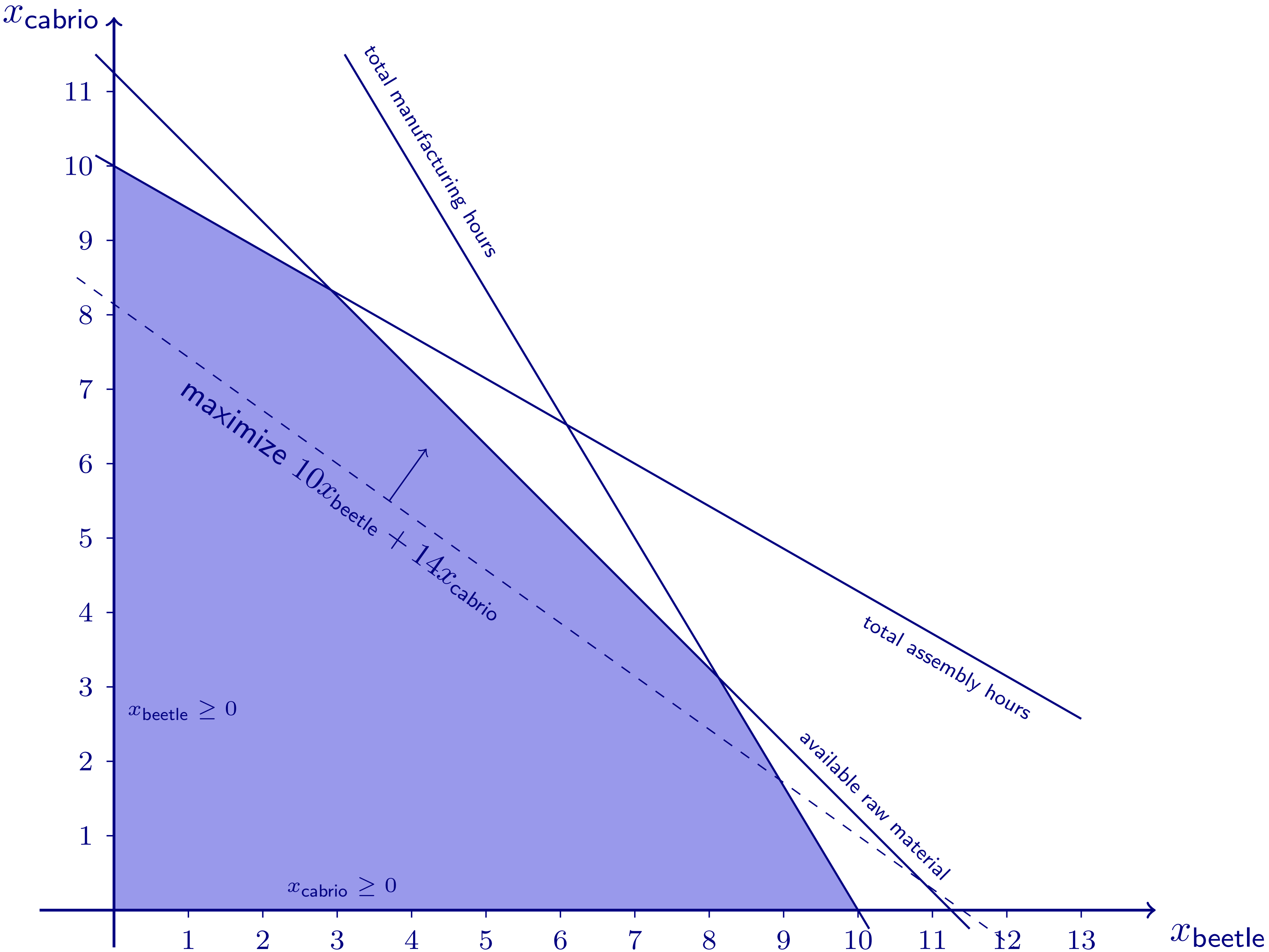

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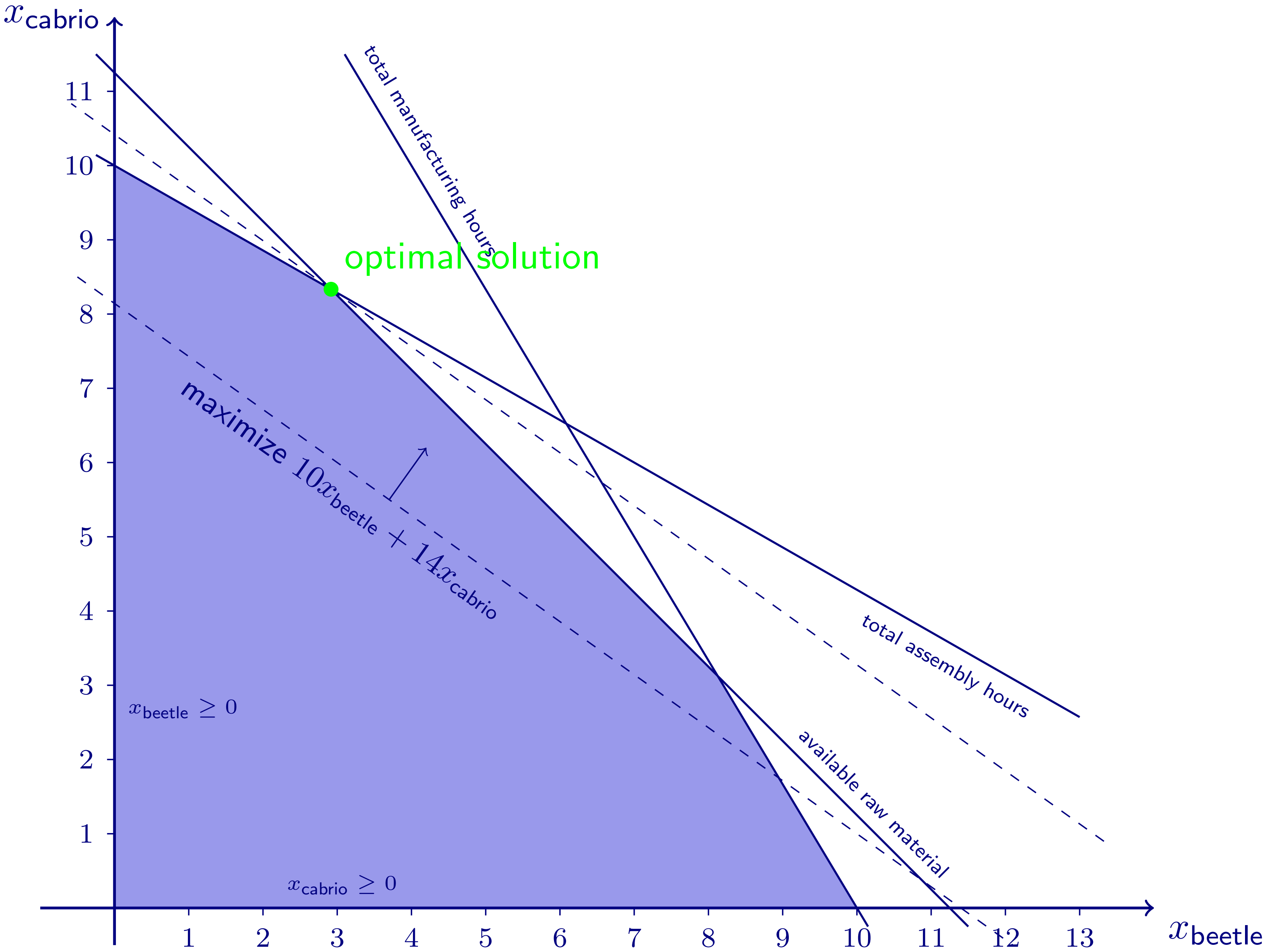

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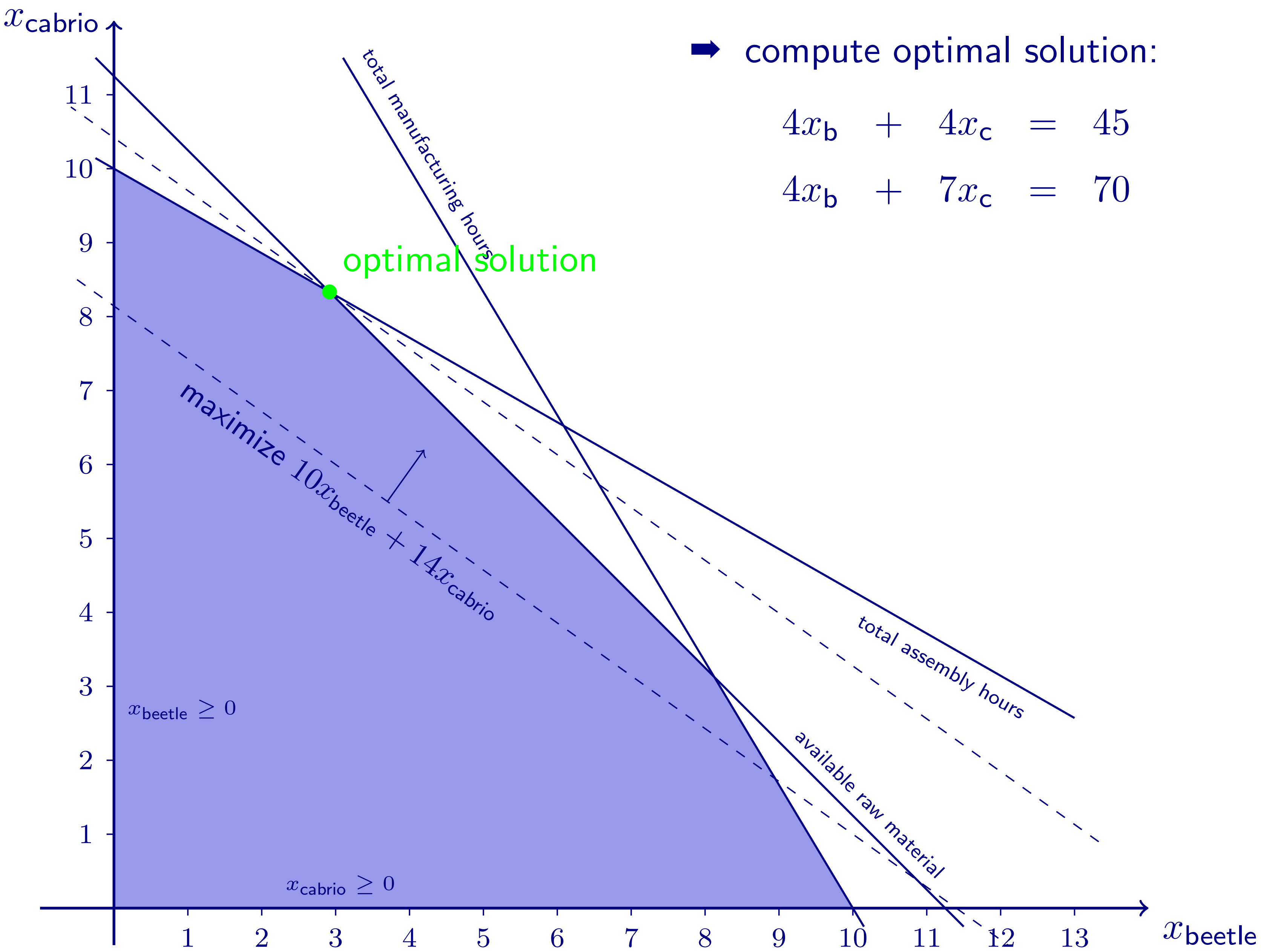

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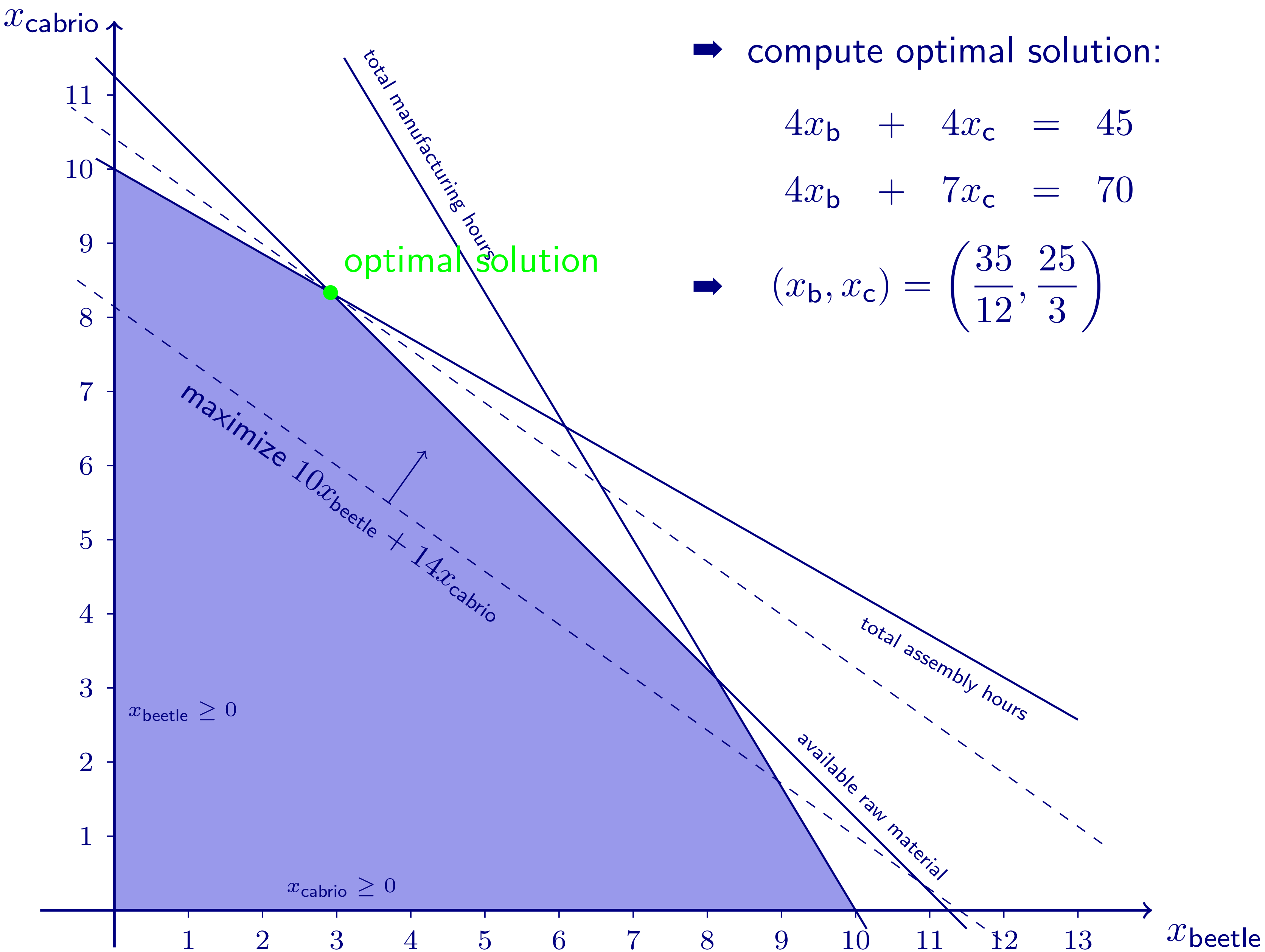

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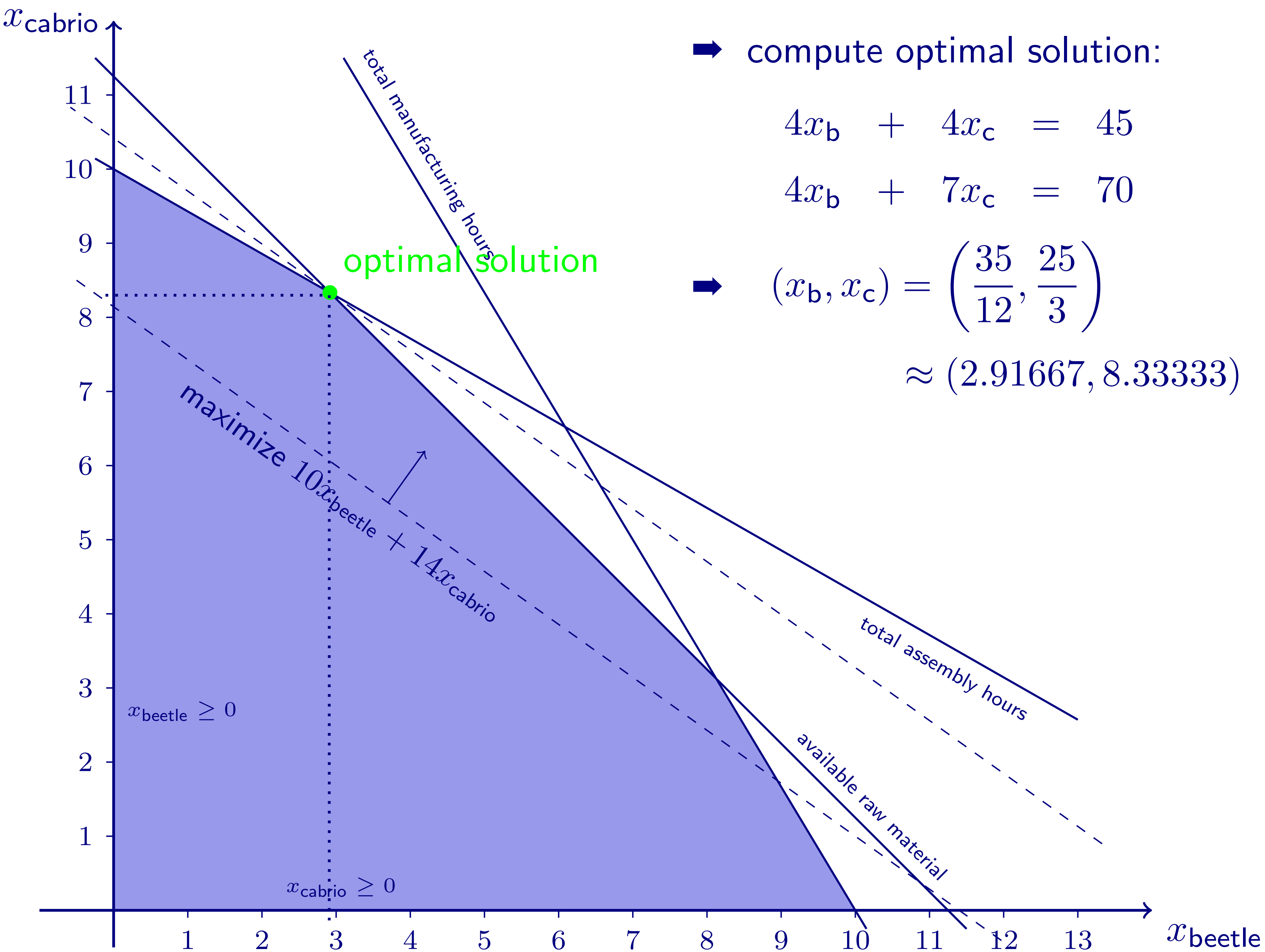

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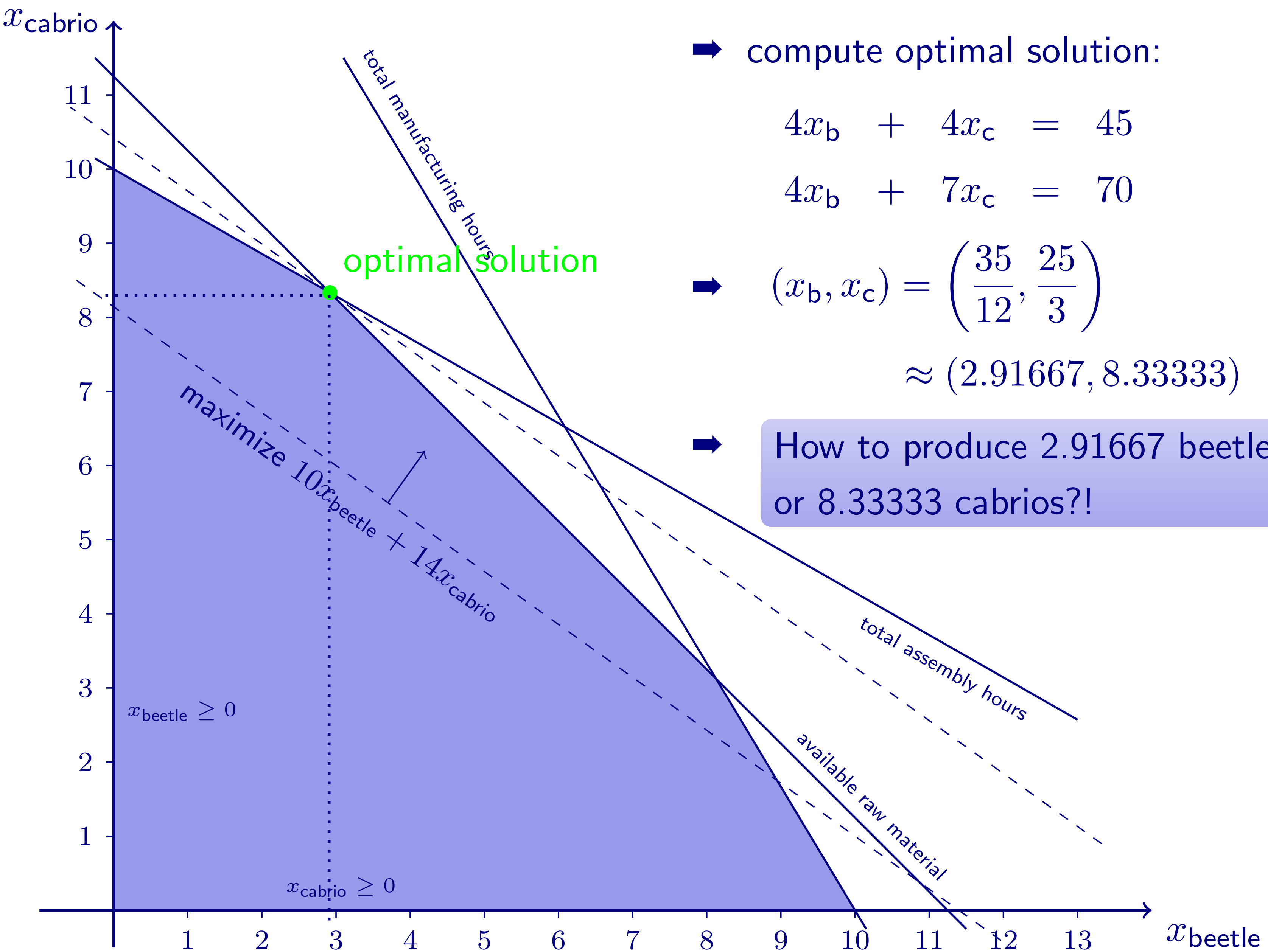
$\approx (2.91667, 8.33333)$

Solving the changed model



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$$\approx (2.91667, 8.33333)$$

- ➡ How to produce 2.91667 beetles, or 8.33333 cabrios?!

Overview

- ▷ Models, Data and Algorithms
- ▷ Linear Optimization
- ▷ Mathematical Background: Polyhedra, Simplex-Algorithm
- ▷ Sensitivity Analysis
- ▷ (Mixed) Integer Programming, Mathematical Background: Cuts, Branch & Bound
- ▷ Combinatorial Optimization
- ▷ Mathematical Background: Graphs, Algorithms
- ▷ Complexity Theory
- ▷ Nonlinear Optimization
- ▷ Scheduling
- ▷ Lot Sizing
- ▷ Multicriteria Optimization

- ▷ Exam