Mathematical Tools for Engineering and Management

Lecture 5

16 Nov 2011

Overview

- ▷ Models, Data and Algorithms
- ▷ Linear Optimization
- ▷ Mathematical Background: Polyhedra, Simplex-Algorithm
- ▷ Sensitivity Analysis; (Mixed) Integer Programming
- ▷ MIP Modelling
- ▷ Branch & Bound, Cutting Planes; More Examples; Combinatorial Optimization
- ▷ Combinatorial Optimization: Examples, Graphs, Algorithms
- ▷ Complexity Theory
- ▷ Nonlinear Optimization
- ▷ Scheduling
- ▷ Lot Sizing
- ▷ Multicriteria Optimization
- ▷ Oral exam

▷ Production Planning in Automobile Industry



Product	**Beetle**	**Cabrio**
Revenue	$10000	$20000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 4500kg

➡ Question: How many cars of each type should be produced to maximize the profit?

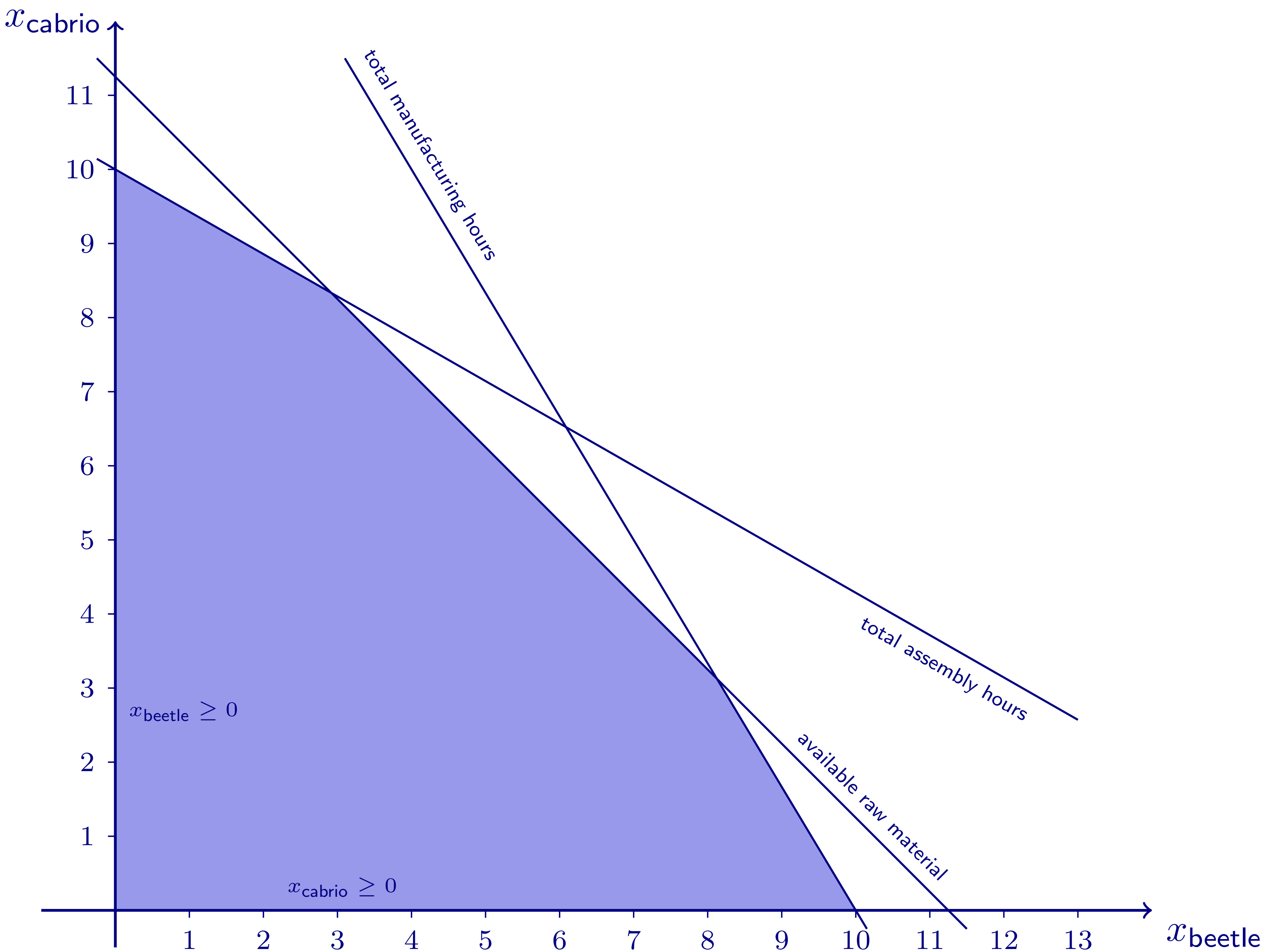
▷ Production Planning in Automobile Industry

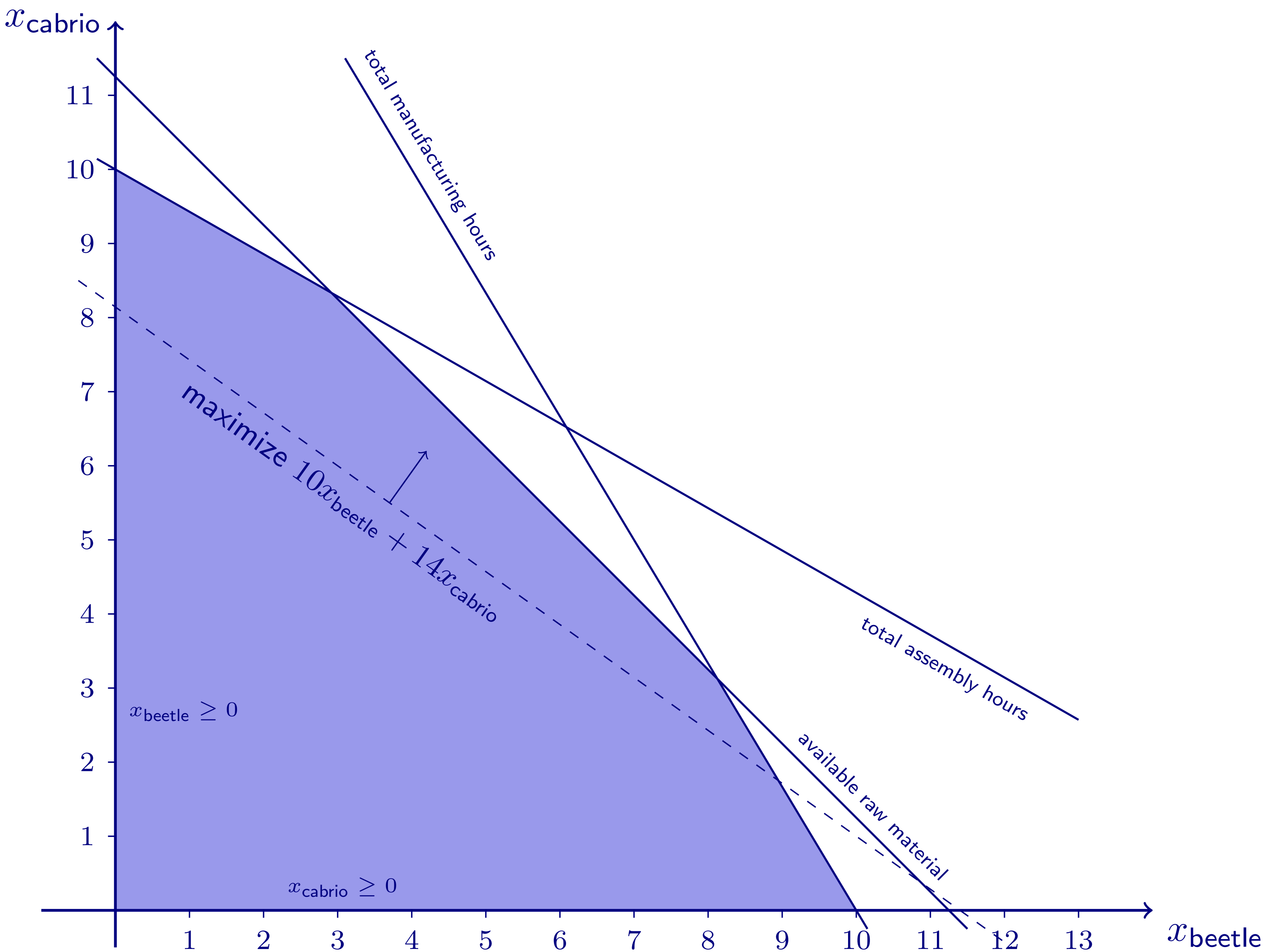
Product	**Beetle**	**Cabrio**
Revenue	\$10000	\$14000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 4500kg

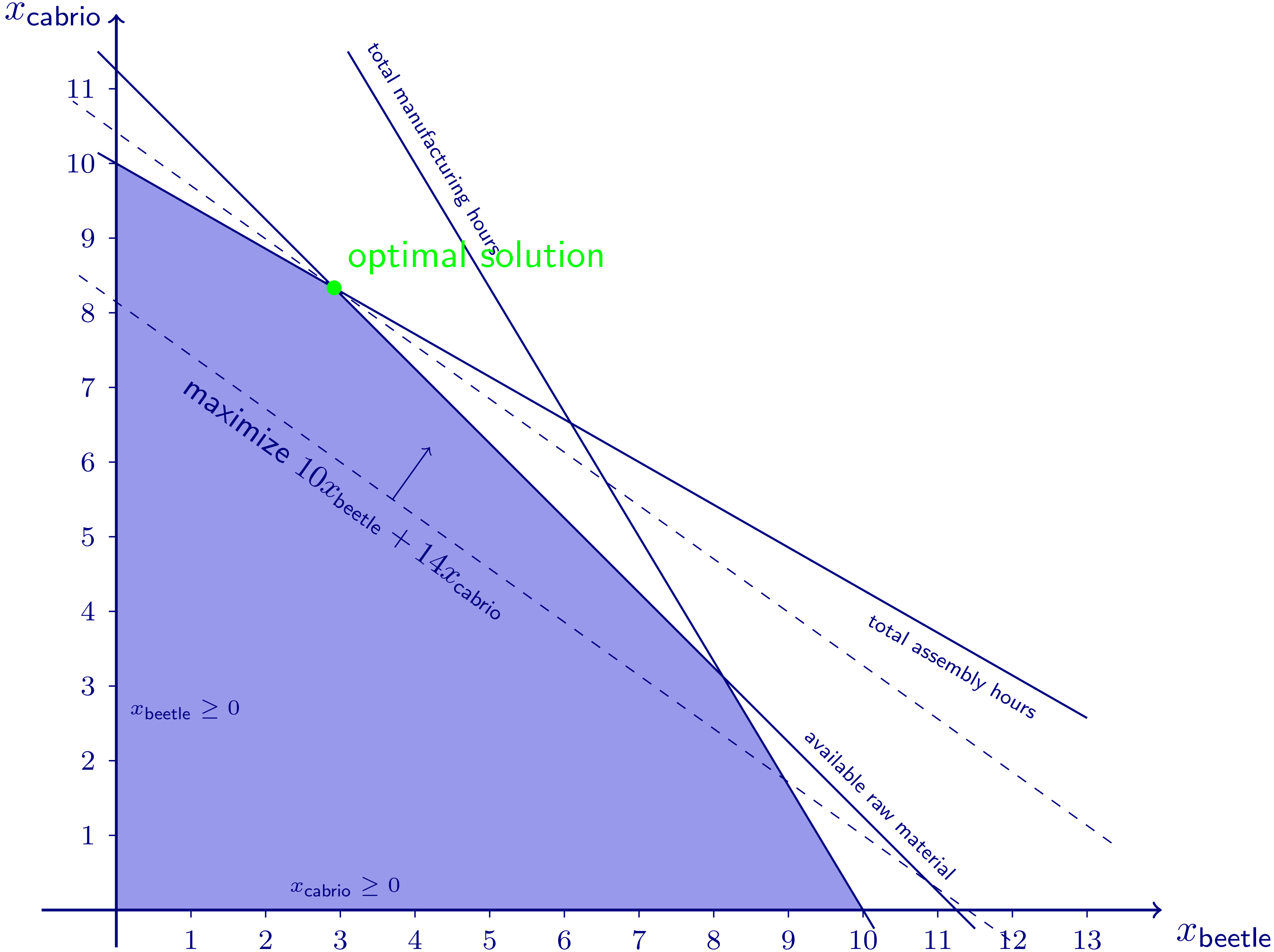
➡ Question: How many cars of each type should be produced to maximize the profit?

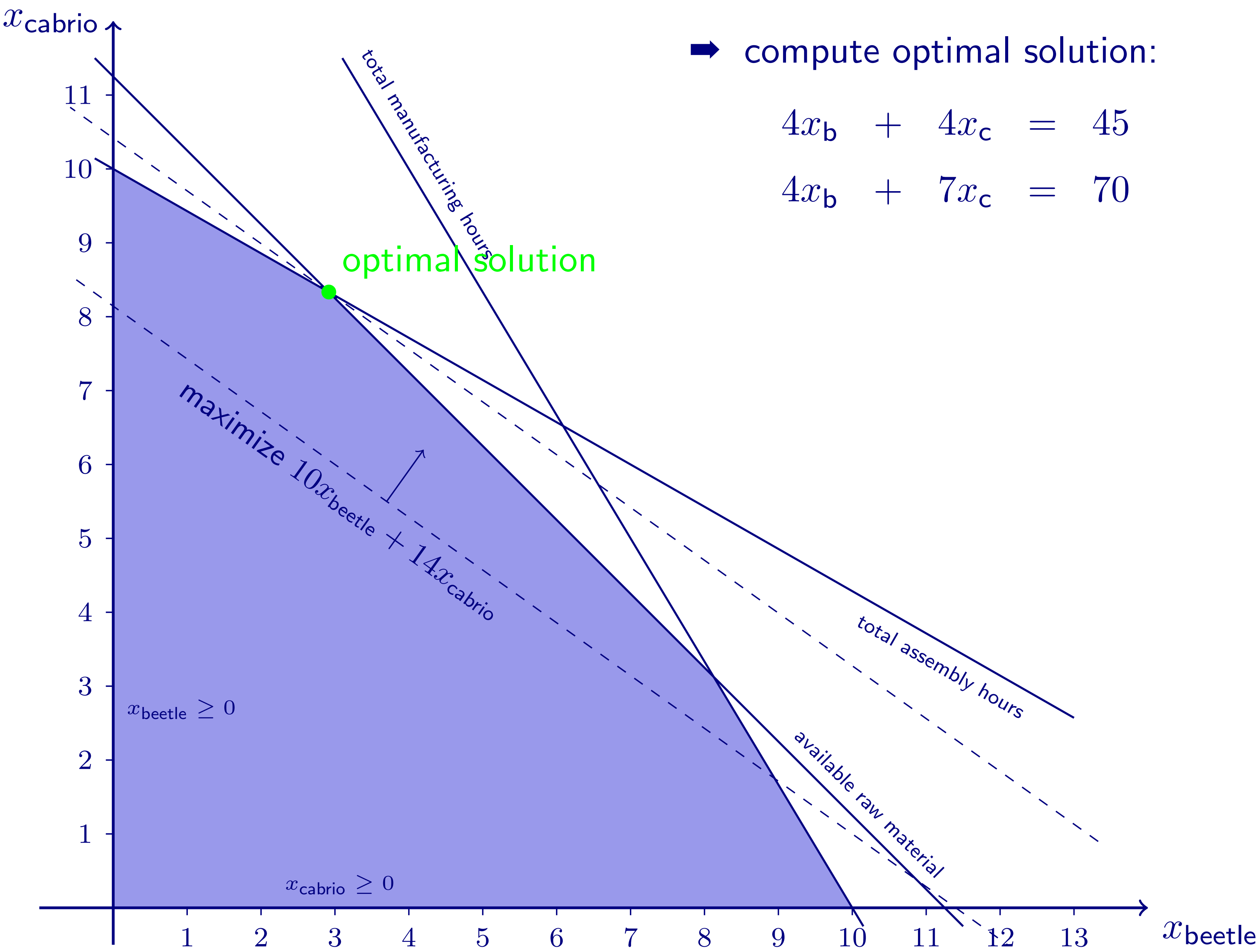

x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
total manufacturing hours
total assembly hours
available raw material
$x_{\text{beetle}} \geq 0$
$x_{\text{cabrio}} \geq 0$
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}



x_{cabrio}
11
10
9
8
7
6
5
4
3
2
1
total manufacturing hours
maximize $10x_{\text{beetle}} + 14x_{\text{cabrio}}$
total assembly hours
available raw material
$x_{\text{beetle}} \geq 0$
$x_{\text{cabrio}} \geq 0$
1 2 3 4 5 6 7 8 9 10 11 12 13
x_{beetle}

Solving the changed model

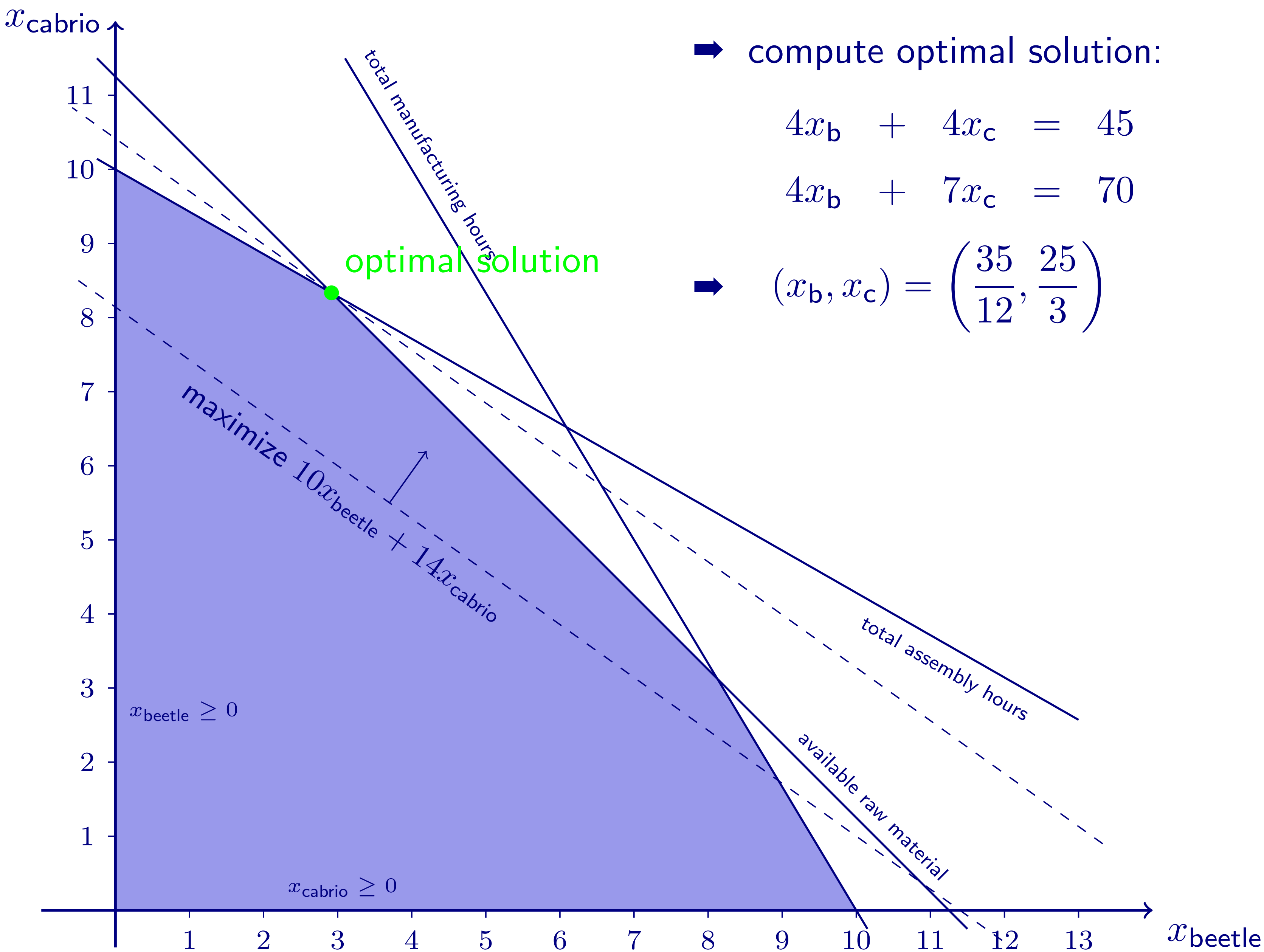




➡ compute optimal solution:

$$
\begin{aligned}
4x_b &+ 4x_c &= 45 \\
4x_b &+ 7x_c &= 70
\end{aligned}
$$

Solving the changed model

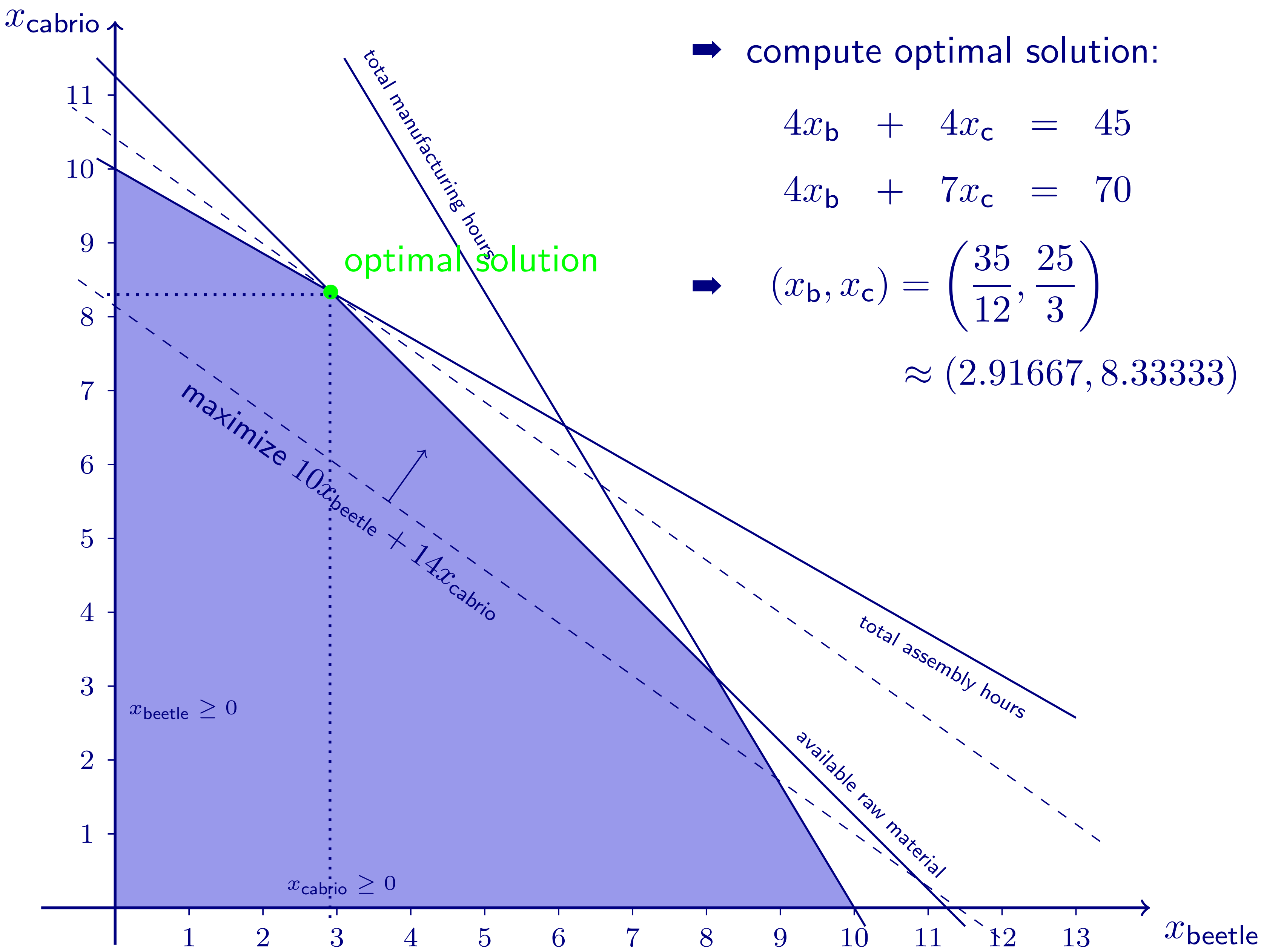


➡ compute optimal solution:

$$
\begin{aligned}
4x_{\mathsf{b}} &+ 4x_{\mathsf{c}} &= 45 \\
4x_{\mathsf{b}} &+ 7x_{\mathsf{c}} &= 70
\end{aligned}
$$

➡ $(x_{\mathsf{b}}, x_{\mathsf{c}}) = \left(\frac{35}{12}, \frac{25}{3}\right)$

Solving the changed model

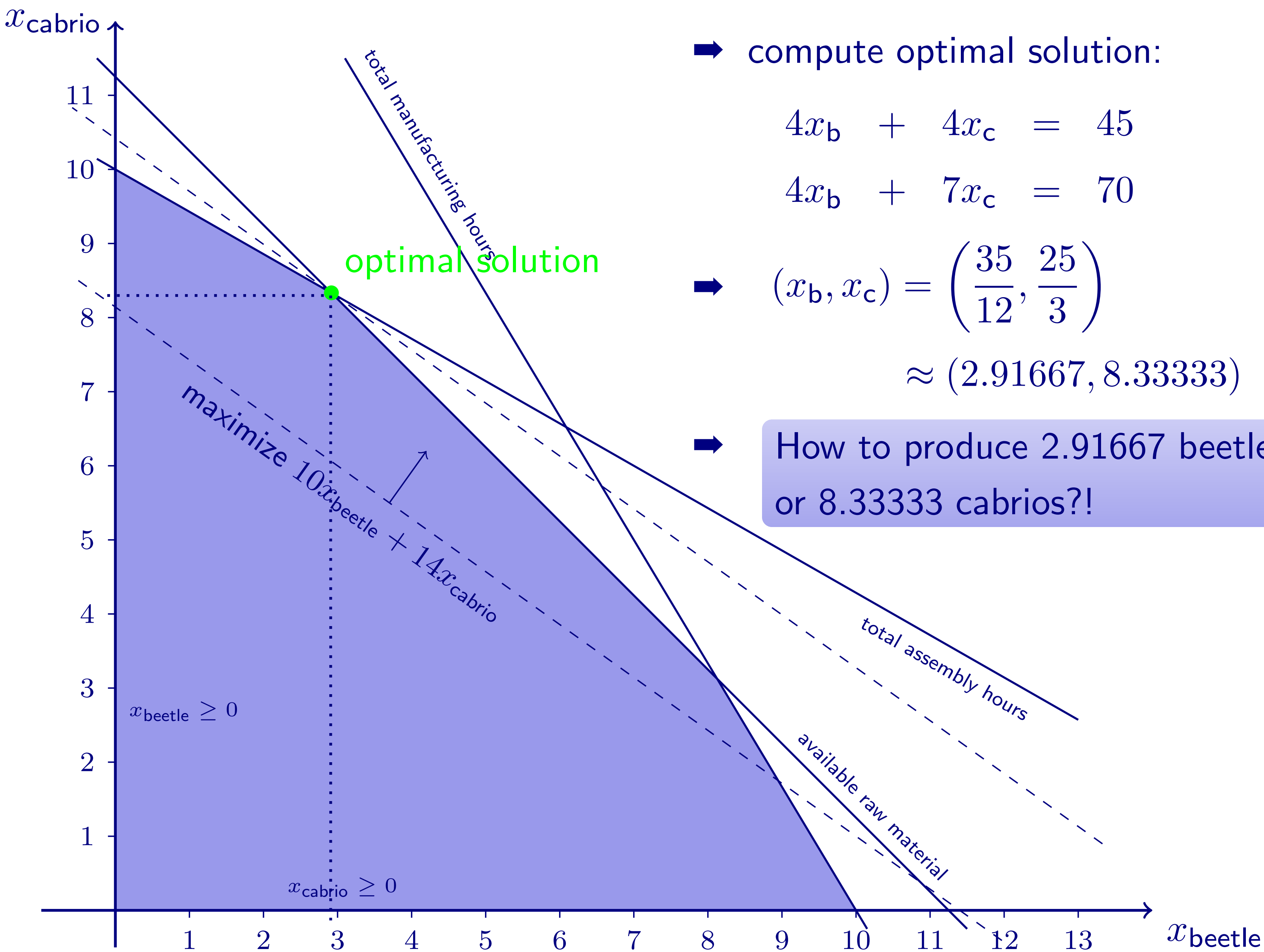


➡ compute optimal solution:

$$\begin{aligned} 4x_b &+ 4x_c &= 45 \\ 4x_b &+ 7x_c &= 70 \end{aligned}$$

➡ $$(x_b, x_c) = \left(\frac{35}{12}, \frac{25}{3}\right)$$

$$\approx (2.91667, 8.33333)$$



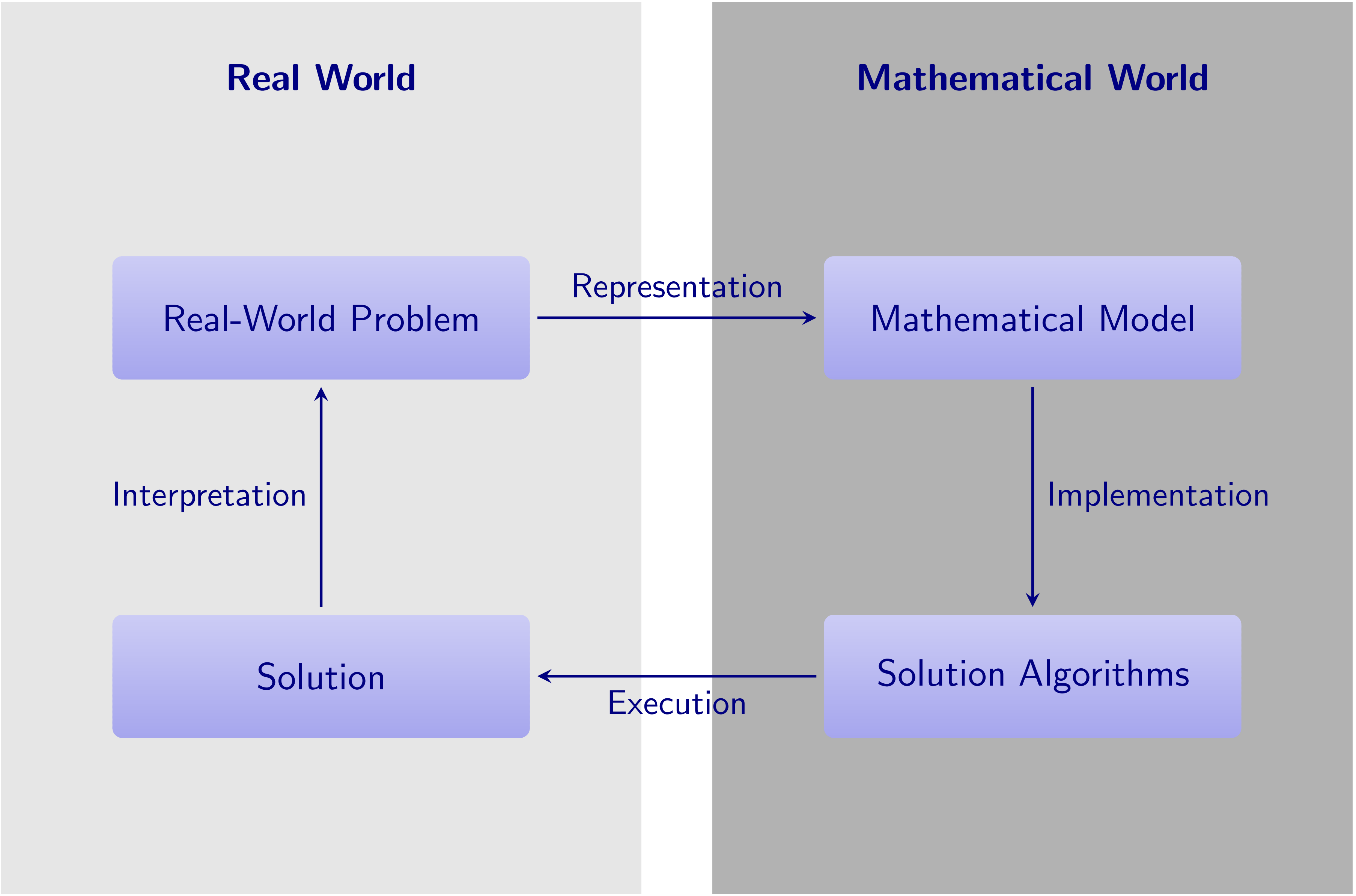
➡ compute optimal solution:

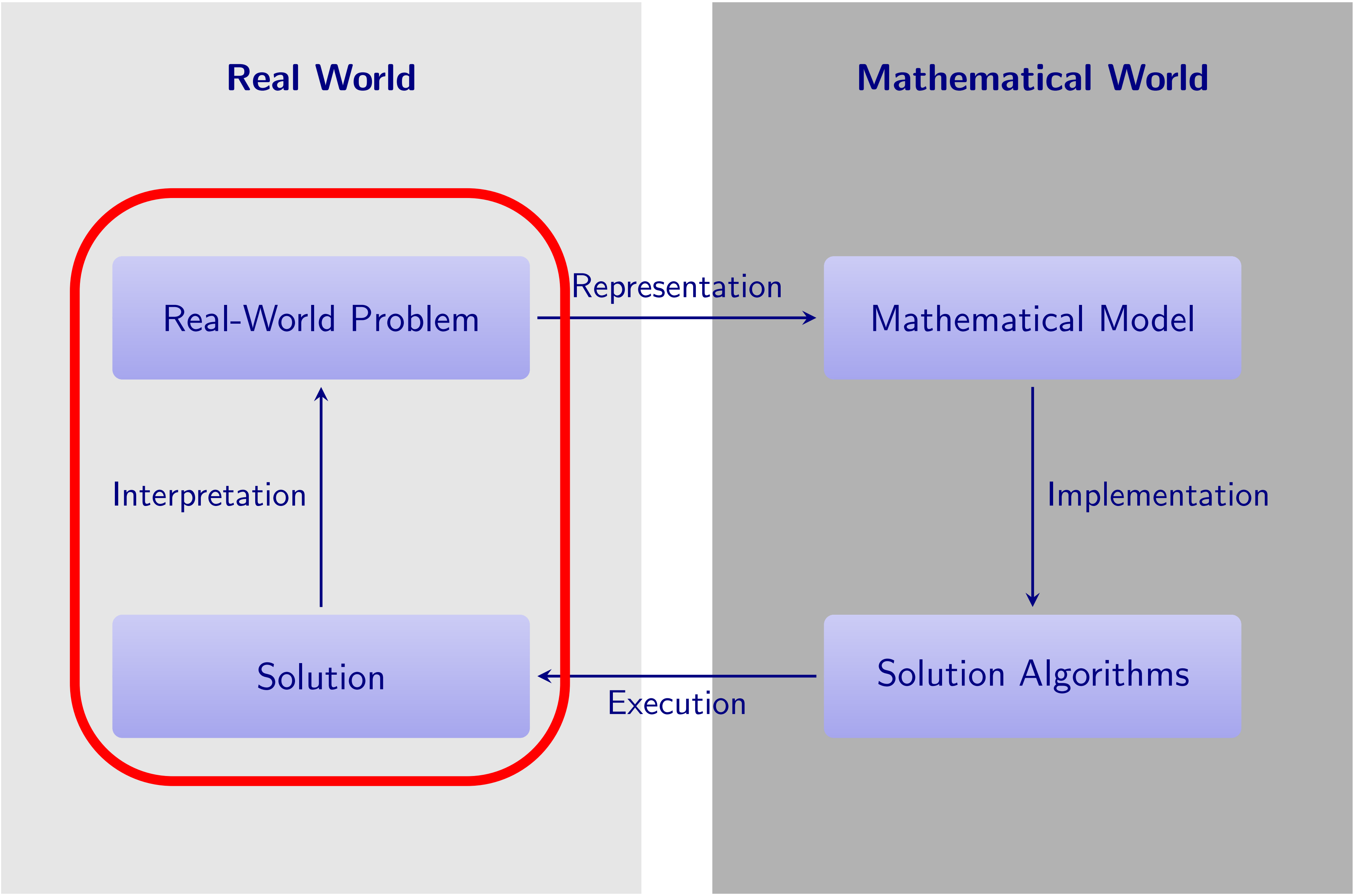
$$\begin{aligned} 4x_{\text{b}} &+ 4x_{\text{c}} &= 45 \\ 4x_{\text{b}} &+ 7x_{\text{c}} &= 70 \end{aligned}$$

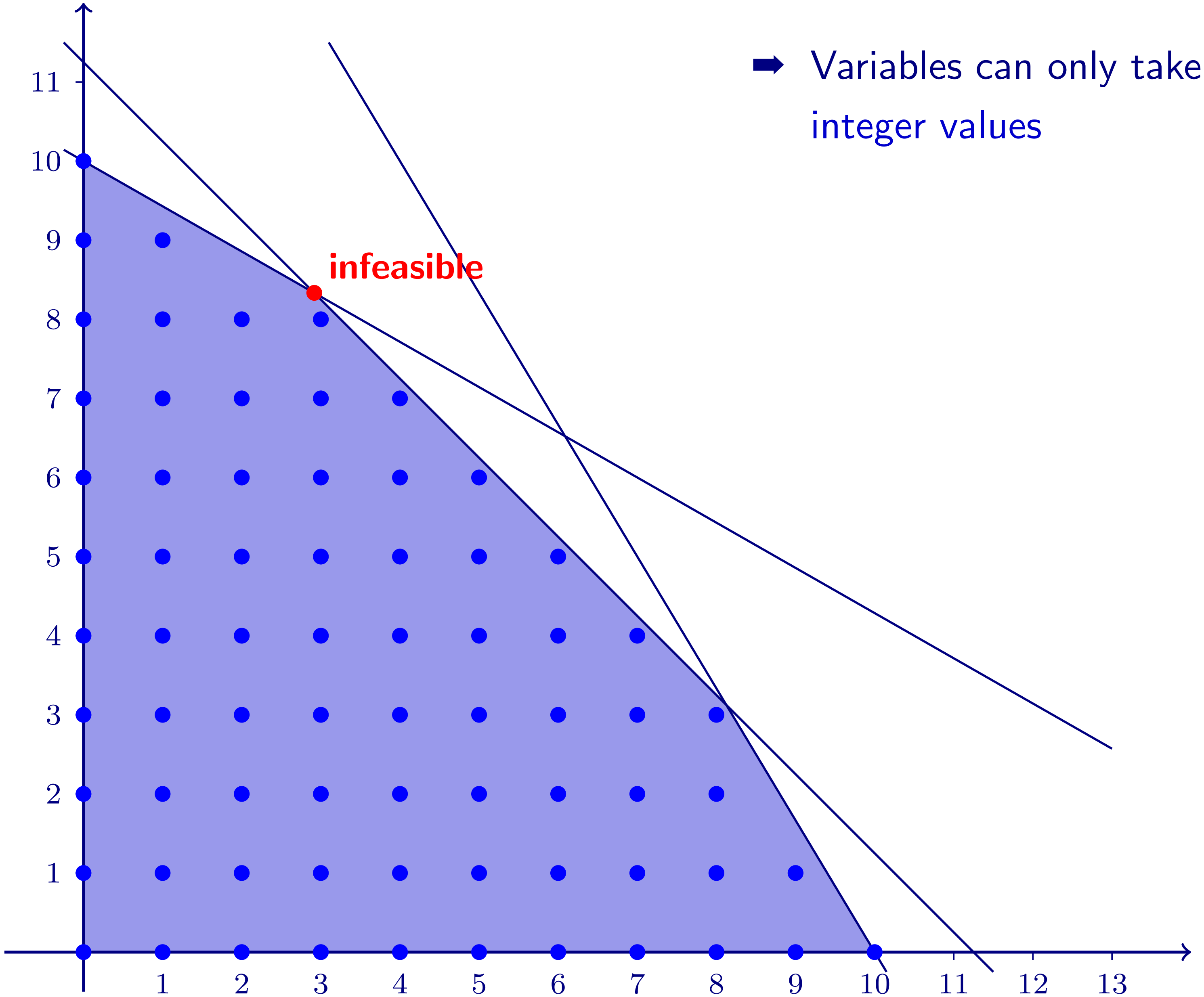
➡ $$(x_{\text{b}}, x_{\text{c}}) = \left(\frac{35}{12}, \frac{25}{3}\right)$$

$$\approx (2.91667, 8.33333)$$

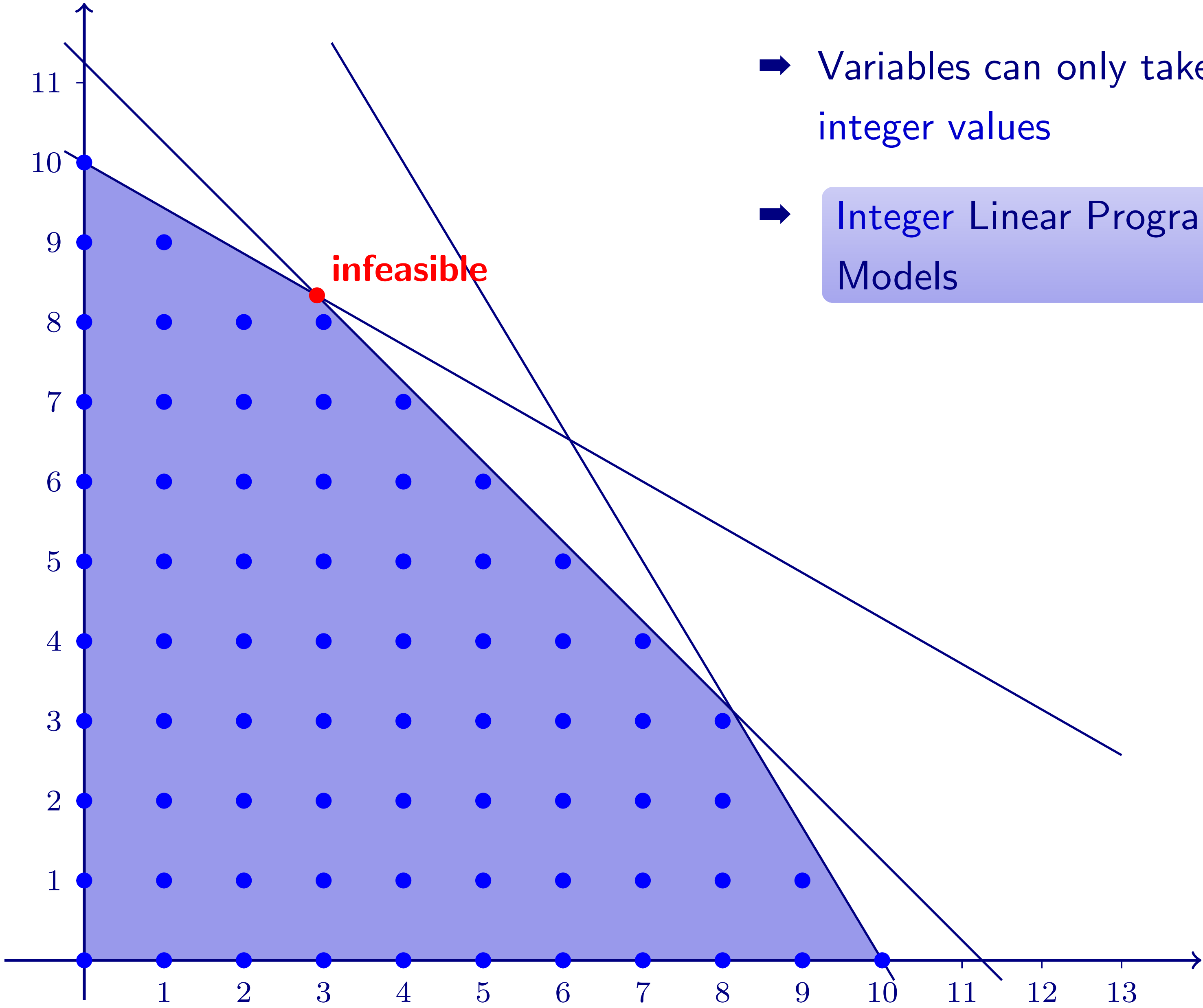
➡ How to produce 2.91667 beetles, or 8.33333 cabrios?!


Real World
Mathematical World
Real-World Problem
Representation
Mathematical Model
Interpretation
Implementation
Solution
Execution
Solution Algorithms

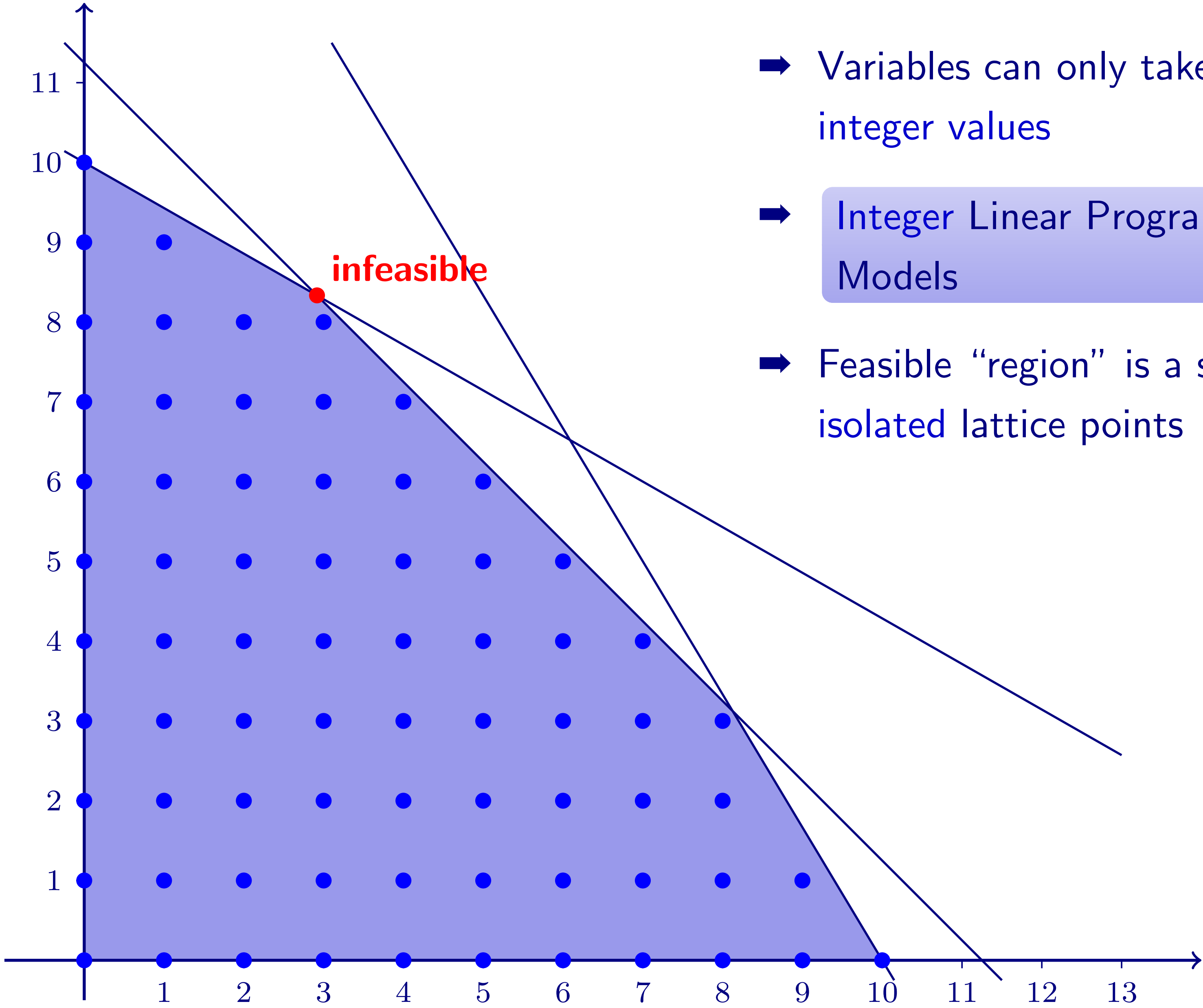

Real World
Mathematical World
Real-World Problem
Representation
Mathematical Model
Interpretation
Implementation
Solution
Execution
Solution Algorithms



➡ Variables can only take integer values



- ➡ Variables can only take integer values
- ➡ Integer Linear Programming Models



- ➡ Variables can only take integer values
- ➡ Integer Linear Programming Models
- ➡ Feasible "region" is a set of isolated lattice points

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = 1, \ldots, n$ **V**

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = 1, \ldots, n$ **V**

x_j integer for all $j = 1, \ldots, n$

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \le b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \le x_j \le u_j$ for all $j = 1, \ldots, n$ **V**

➡ Integer Program: x_j integer for all $j = 1, \ldots, n$

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = 1, \ldots, n$ **V**

➡ Integer Program: x_j integer for all $j = 1, \ldots, n$

➡ Mixed Integer Program: x_j integer for all $j = 1, \ldots, \ell$ $(\ell < n)$

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = 1, \ldots, n$ **V**

➡ Integer Program: x_j integer for all $j = 1, \ldots, n$

➡ Mixed Integer Program: x_j integer for all $j = 1, \ldots, \ell$ $(\ell < n)$

➡ The LP obtained by skipping all of the integrality constraints is called the Linear Programming Relaxation of the (M)IP

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = \ell + 1, \ldots, n$ **V**

x_j integer for all $j = 1, \ldots, \ell$

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = \ell + 1, \ldots, n$ **V**

$0 \leq x_j \leq 1$ for all $j = 1, \ldots, \ell$ $(\ell < n)$

x_j integer for all $j = 1, \ldots, \ell$

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = \ell + 1, \ldots, n$ **V**

$0 \leq x_j \leq 1$ for all $j = 1, \ldots, \ell$ $(\ell < n)$

x_j integer for all $j = 1, \ldots, \ell$

$\Updownarrow$

$x_j \in \{0, 1\}$ for all $j = 1, \ldots, \ell$ $(\ell < n)$

Special case: Binary MIP models

maximize/minimize	$\sum_{j=1}^{n} c_j x_j$		**Objective function**
subject to	$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$	for all $i = 1, \ldots, m$	**C**
	$\ell_j \leq x_j \leq u_j$	for all $j = \ell + 1, \ldots, n$	**V**

$$0 \leq x_j \leq 1 \quad \text{for all } j = 1, \ldots, \ell \qquad (\ell < n)$$

$$x_j \text{ integer} \quad \text{for all } j = 1, \ldots, \ell$$

$$\Updownarrow$$

$$x_j \in \{0, 1\} \quad \text{for all } j = 1, \ldots, \ell \qquad (\ell < n)$$

➡ Binary variables used to model yes/no decisions

▷ Original problem: How many cars should be produced to maximize the profit?

Product	**Beetle**	**Cabrio**
Revenue	\$10000	\$14000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 8000kg

▷ Original problem: How many cars should be produced to maximize the profit?

Product	**Beetle**	**Cabrio**
Revenue	\$10000	\$14000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 8000kg

▷ Additional Options: Investments in manufacturing and/or assembly units

▷ Original problem: How many cars should be produced to maximize the profit?

Product	Beetle	Cabrio
Revenue	$10000	$14000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 8000kg

▷ Additional Options: Investments in manufacturing and/or assembly units

	Investment	Extra Cap.
Manufacturing	$10000	19h
Assembly, Option 1	$10000	20h
Assembly, Option 2	$25000	32h
Assembly, Option 3	$40000	45h

▷ Original problem: How many cars should be produced to maximize the profit?

Product	Beetle	Cabrio
Revenue	$10000	$14000
Manufacturing	5h	3h
Assembly	4h	7h
Raw material	400kg	400kg

Plant capacity and available raw materials:

- Manufacturing capacity: 50h
- Assembly capacity: 70h
- Raw material: 8000kg

▷ Additional Options: Investments in manufacturing and/or assembly units

	Investment	Extra Cap.
Manufacturing	$10000	19h
Assembly, Option 1	$10000	20h
Assembly, Option 2	$25000	32h
Assembly, Option 3	$40000	45h

Budget for investment:

max. $50000

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

V

(non-negativity of variables) $x_c \geq 0$ for all $c \in C$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

V

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

V

(0/1 decision variables) $y_{\text{man1}}, y_{\text{ass1}}, y_{\text{ass2}}, y_{\text{ass3}} \in \{0, 1\}$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

V

(0/1 decision variables) $y_{\text{man1}}, y_{\text{ass1}}, y_{\text{ass2}}, y_{\text{ass3}} \in \{0, 1\}$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

V

(0/1 decision variables) $y_i \in \{0,1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- ▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- ▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

(available budget) $b_{\text{man1}} y_{\text{man1}} + b_{\text{ass1}} y_{\text{ass1}} + b_{\text{ass2}} y_{\text{ass2}} + b_{\text{ass3}} y_{\text{ass3}} \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$

▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

(available budget) $\sum_{i \in I} b_i \, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in each department) $t_{\text{beetle},d} x_{\text{beetle}} + t_{\text{cabrio},d} x_{\text{cabrio}} \leq T_d$ for all $d \in D$

(available budget) $\sum_{i \in I} b_i \, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in manufacturing) $t_{\text{beetle,man}} x_{\text{beetle}} + t_{\text{cabrio,man}} x_{\text{cabrio}} \leq T_{\text{man}}$

(available budget) $\sum_{i \in I} b_i \, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- ▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- ▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in manufacturing) $t_{\text{beetle,man}} x_{\text{beetle}} + t_{\text{cabrio,man}} x_{\text{cabrio}} \leq T_{\text{man}} + \tau_{\text{man1}}\, y_{\text{man1}}$

(available budget) $\sum_{i \in I} b_i\, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- ▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- ▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in manufacturing) $t_{\text{beetle,man}} x_{\text{beetle}} + t_{\text{cabrio,man}} x_{\text{cabrio}} - \tau_{\text{man1}}\, y_{\text{man1}} \leq T_{\text{man}}$

(available budget) $\sum_{i \in I} b_i\, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in manufacturing) $\sum_{c \in C} t_{c,\text{man}} x_c - \tau_{\text{man1}}\, y_{\text{man1}} \leq T_{\text{man}}$

(available budget) $\sum_{i \in I} b_i\, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- ▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- ▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in manufacturing) $\sum_{c \in C} t_{c,\text{man}} x_c - \tau_{\text{man1}}\, y_{\text{man1}} \leq T_{\text{man}}$

(time spent in assembly) $\sum_{c \in C} t_{c,\text{ass}}\, x_c \leq T_{\text{ass}} + \tau_{\text{ass1}} y_{\text{ass1}} + \tau_{\text{ass2}} y_{\text{ass2}} + \tau_{\text{ass3}} y_{\text{ass3}}$

(available budget) $\sum_{i \in I} b_i\, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0, 1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- ▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- ▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in manufacturing) $\sum_{c \in C} t_{c,\text{man}} x_c - \tau_{\text{man1}}\, y_{\text{man1}} \leq T_{\text{man}}$

(time spent in assembly) $\sum_{c \in C} t_{c,\text{ass}}\, x_c \leq T_{\text{ass}} + \sum_{i \in \{\text{ass1},\text{ass2},\text{ass3}\}} \tau_i\, y_i$

(available budget) $\sum_{i \in I} b_i\, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0,1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- ▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- ▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

Objective function: maximize (total revenue) $r_{\text{beetle}} \cdot x_{\text{beetle}} + r_{\text{cabrio}} \cdot x_{\text{cabrio}}$

C

(total raw material available) $\rho_{\text{beetle}} x_{\text{beetle}} + \rho_{\text{cabrio}} x_{\text{cabrio}} \leq R$

(time spent in manufacturing) $\sum_{c \in C} t_{c,\text{man}} x_c - \tau_{\text{man1}}\, y_{\text{man1}} \leq T_{\text{man}}$

(time spent in assembly) $\sum_{c \in C} t_{c,\text{ass}}\, x_c - \sum_{i \in \{\text{ass1},\text{ass2},\text{ass3}\}} \tau_i\, y_i \leq T_{\text{ass}}$

(available budget) $\sum_{i \in I} b_i\, y_i \leq B$

V

(0/1 decision variables) $y_i \in \{0,1\}$ for all $i \in I$

$x_c \geq 0$ for all $c \in C$

(integrality of variables) x_c integer for all $c \in C$

- ▷ New set: investment decisions $I = \{\text{man1}, \text{ass1}, \text{ass2}, \text{ass3}\}$
- ▷ New parameters: investment capital $b_{\text{man1}} = 10, b_{\text{ass1}} = 10, b_{\text{ass2}} = 25, b_{\text{ass3}} = 40$

  additional capacities $\tau_{\text{man1}} = 19, \tau_{\text{ass1}} = 20, \tau_{\text{ass2}} = 32, \tau_{\text{ass3}} = 45$

optimal IP solution

# beetles	4
# cabrios	15
manufact. investment option	1
assembly investment option 1	1
assembly investment option 2	1
assembly investment option 3	0

optimal IP solution	
# beetles	4
# cabrios	15
manufact. investment option	1
assembly investment option 1	1
assembly investment option 2	1
assembly investment option 3	0

➡ revenue: $250000

constraints	**used up**	**available**
manufact. time	65	50 +19
assembly time	121	70 +20 +32
raw material	7600	8000
budget	10T + 10T + 25T	50T

optimal IP solution

# beetles	4
# cabrios	15
manufact. investment option	1
assembly investment option 1	1
assembly investment option 2	1
assembly investment option 3	0

➡ revenue: $250000

constraints	**used up**	**available**
manufact. time	65	50 +19
assembly time	121	70 +20 +32
raw material	7600	8000
budget	10T + 10T + 25T	50T

optimum of the LP relaxation

# beetles	0
# cabrios	19.197151
manu. invest.	0.399550
assembly invest. option 1	1
assembly invest. option 2	1
assembly invest. option 3	0.275112

➡ revenue: $268760.12

constraints	**used up**	**available**
manufact. time	57.59145	57.59145
assembly time	134.38	134.38004
raw material	7678.8604	8000
budget	50T	50T

LP-relaxation

maximize/minimize $\sum_{j=1}^{n} c_j x_j$ **Objective function**

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all $i = 1, \ldots, m$ **C**

$\ell_j \leq x_j \leq u_j$ for all $j = 1, \ldots, n$ **V**

➡ Integer Program: x_j integer for all $j = 1, \ldots, n$

➡ Mixed Integer Program: x_j integer for all $j = 1, \ldots, \ell$ $(\ell < n)$

➡ Binary variables: $x_j \in \{0, 1\}$ for all $j = 1, \ldots, \ell$

- Increase an existing capacity (yes/no decision)

- ▷ Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0, 1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity

- ▷ Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ (...used up capacity...) $\leq T + \tau \cdot y$ (T: existing capacity, τ: additional capacity)

- ▷ Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ (...used up capacity...) $- \tau \cdot y \ \leq \ T$ (T: existing capacity, τ: additional capacity)

- ▷ Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ (...used up capacity...) $- \tau \cdot y \ \leq \ T$ (T: existing capacity, τ: additional capacity)
- ▷ Install a number of (capacity-increasing) devices

- ▷ Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ (...used up capacity...) $- \tau \cdot y \;\leq\; T$ (T: existing capacity, τ: additional capacity)
- ▷ Install a number of (capacity-increasing) devices
  - ➡ integer variable $z \in \mathbb{Z}$, meaning: $z =$ number of devices

MIP modelling: capacity constraints

▷ Increase an existing capacity (yes/no decision)

➡ binary variable $y \in \{0, 1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity

➡ (...used up capacity...) $- \tau \cdot y \ \leq \ T$ (T: existing capacity, τ: additional capacity)

▷ Install a number of (capacity-increasing) devices

➡ integer variable $z \in \mathbb{Z}$, meaning: $z =$ number of devices

➡ (...used up capacity...) $- \tau \cdot z \ \leq \ T$ (τ: additional capacity per device)

- Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0, 1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ (...used up capacity...) $- \tau \cdot y \;\leq\; T$ (T: existing capacity, τ: additional capacity)
- Install a number of (capacity-increasing) devices
  - ➡ integer variable $z \in \mathbb{Z}$, meaning: $z =$ number of devices
  - ➡ (...used up capacity...) $- \tau \cdot z \;\leq\; T$ (τ: additional capacity per device)
- Choose between various options to increase capacity

- ▷ Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0, 1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ (...used up capacity...) $- \tau \cdot y \ \leq \ T$ (T: existing capacity, τ: additional capacity)
- ▷ Install a number of (capacity-increasing) devices
  - ➡ integer variable $z \in \mathbb{Z}$, meaning: $z =$ number of devices
  - ➡ (...used up capacity...) $- \tau \cdot z \ \leq \ T$ (τ: additional capacity per device)
- ▷ Choose between various options to increase capacity
  - ➡ binary variables $y_1, \ldots, y_n \in \{0, 1\}$, meaning: $y_i = 1 \Leftrightarrow$ choose option i

MIP modelling: capacity constraints

- ▷ Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ (...used up capacity...) $- \tau \cdot y \;\leq\; T$ (T: existing capacity, τ: additional capacity)
- ▷ Install a number of (capacity-increasing) devices
  - ➡ integer variable $z \in \mathrm{z}$, meaning: $z =$ number of devices
  - ➡ (...used up capacity...) $- \tau \cdot z \;\leq\; T$ (τ: additional capacity per device)
- ▷ Choose between various options to increase capacity
  - ➡ binary variables $y_1, \ldots, y_n \in \{0,1\}$, meaning: $y_i = 1 \Leftrightarrow$ choose option i
  - ➡ (...used up capacity...) $- \sum_{i=1}^{n} \tau_i \cdot y_i \;\leq\; T$ (τ_i: add. capacity available through option i)

- Increase an existing capacity (yes/no decision)
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ invest to increase capacity
  - ➡ $(\ldots\text{used up capacity}\ldots) - \tau \cdot y \;\leq\; T$ (T: existing capacity, τ: additional capacity)
- Install a number of (capacity-increasing) devices
  - ➡ integer variable $z \in \mathbb{Z}$, meaning: $z =$ number of devices
  - ➡ $(\ldots\text{used up capacity}\ldots) - \tau \cdot z \;\leq\; T$ (τ: additional capacity per device)
- Choose between various options to increase capacity
  - ➡ binary variables $y_1, \ldots, y_n \in \{0,1\}$, meaning: $y_i = 1 \Leftrightarrow$ choose option i
  - ➡ $(\ldots\text{used up capacity}\ldots) - \sum_{i=1}^{n} \tau_i \cdot y_i \;\leq\; T$ (τ_i: add. capacity available through option i)
- Common extension: only one of the available options can be chosen
  - ➡ add set packing constraints for the y_i's...

▷ Set packing constraints:

➡ choose at most one of the binary variables $y_1, \ldots, y_n$

- Set packing constraints:
  - ➡ choose at most one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \;\leq\; 1$

- Set packing constraints:
  - ➡ choose at most one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \leq 1$
- Set covering constraints:
  - ➡ choose at least one of the binary variables $y_1, \ldots, y_n$

- Set packing constraints:
  - ➡ choose at most one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \leq 1$
- Set covering constraints:
  - ➡ choose at least one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \geq 1$

- Set packing constraints:
  - ➡ choose at most one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \leq 1$
- Set covering constraints:
  - ➡ choose at least one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \geq 1$
- Set partitioning constraints:
  - ➡ choose exactly one of the binary variables $y_1, \ldots, y_n$

- Set packing constraints:
  - ➡ choose at most one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \leq 1$
- Set covering constraints:
  - ➡ choose at least one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \geq 1$
- Set partitioning constraints:
  - ➡ choose exactly one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n = 1$

- ▷ Set packing constraints:
  - ➡ choose at most one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \ \leq \ 1$
- ▷ Set covering constraints:
  - ➡ choose at least one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \ \geq \ 1$
- ▷ Set partitioning constraints:
  - ➡ choose exactly one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \ = \ 1$
- ▷ Similarly for arbitrary (integer!) quantity on the RHS:
  - ➡ choose at most/at least/exactly k of the binary variables $y_1, \ldots, y_n$

- Set packing constraints:
  - ➡ choose at most one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \leq 1$
- Set covering constraints:
  - ➡ choose at least one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \geq 1$
- Set partitioning constraints:
  - ➡ choose exactly one of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n = 1$
- Similarly for arbitrary (integer!) quantity on the RHS:
  - ➡ choose at most/at least/exactly k of the binary variables $y_1, \ldots, y_n$
  - ➡ $y_1 + y_2 + \ldots + y_n \leq k$ / $\geq k$ / $= k$

▷ Implication: if decision A is taken, then also decision B has to be taken

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_{\mathsf{A}}, y_{\mathsf{B}} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_{\mathsf{A}}, y_{\mathsf{B}} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_{\mathsf{A}} \;\leq\; y_{\mathsf{B}}$

▷ Implication: if decision A is taken, then also decision B has to be taken

➡ binary variables $y_\mathsf{A}, y_\mathsf{B} \in \{0, 1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$

➡ $y_\mathsf{A} \;\leq\; y_\mathsf{B}$

▷ Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken

➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_\mathsf{B} \in \{0, 1\}$, same meaning as above

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_\mathsf{A}, y_\mathsf{B} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_\mathsf{A} \;\leq\; y_\mathsf{B}$
- ▷ Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_\mathsf{B} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \;\leq\; n \cdot y_\mathsf{B}$

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_\mathsf{A}, y_\mathsf{B} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_\mathsf{A} \;\leq\; y_\mathsf{B}$
- ▷ Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_\mathsf{B} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \;\leq\; n \cdot y_\mathsf{B}$ ➡ possibly bad if n is large (big-M constraints)

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_{\mathsf{A}}, y_{\mathsf{B}} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_{\mathsf{A}} \;\leq\; y_{\mathsf{B}}$
- ▷ Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_{\mathsf{B}} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \;\leq\; n \cdot y_{\mathsf{B}}$ ➡ possibly bad if n is large (big-M constraints)
- ▷ Triggering: if some value exceeds a given threshold, then B happens

- Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_{\mathsf{A}}, y_{\mathsf{B}} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_{\mathsf{A}} \leq y_{\mathsf{B}}$
- Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_{\mathsf{B}} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \leq n \cdot y_{\mathsf{B}}$ ➡ possibly bad if n is large (big-M constraints)
- Triggering: if some value exceeds a given threshold, then B happens
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ B happens

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_{\mathsf{A}}, y_{\mathsf{B}} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_{\mathsf{A}} \le y_{\mathsf{B}}$
- ▷ Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_{\mathsf{B}} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \le n \cdot y_{\mathsf{B}}$ ➡ possibly bad if n is large (big-M constraints)
- ▷ Triggering: if some value exceeds a given threshold, then B happens
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ B happens
  - ➡ (...linear expression for value...) $\le T + M \cdot y$ (T: threshold, M: big enough value)

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_\mathsf{A}, y_\mathsf{B} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_\mathsf{A} \;\leq\; y_\mathsf{B}$
- ▷ Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_\mathsf{B} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \;\leq\; n \cdot y_\mathsf{B}$ ➡ possibly bad if n is large (big-M constraints)
- ▷ Triggering: if some value exceeds a given threshold, then B happens
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ B happens
  - ➡ (...linear expression for value...) $\leq\; T + M \cdot y$ (T: threshold, M: big enough value)
- ▷ Special case: if all of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ are taken, then B has to be taken

- Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_\mathsf{A}, y_\mathsf{B} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_\mathsf{A} \ \leq \ y_\mathsf{B}$
- Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_\mathsf{B} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \ \leq \ n \cdot y_\mathsf{B}$ ➡ possibly bad if n is large (big-M constraints)
- Triggering: if some value exceeds a given threshold, then B happens
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ B happens
  - ➡ (...linear expression for value...) $\leq \ T + M \cdot y$ (T: threshold, M: big enough value)
- Special case: if all of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ are taken, then B has to be taken
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \ \leq \ n - 1 + y_\mathsf{B}$

- ▷ Implication: if decision A is taken, then also decision B has to be taken
  - ➡ binary variables $y_{\mathsf{A}}, y_{\mathsf{B}} \in \{0,1\}$, meaning: $y_* = 1 \Leftrightarrow$ take decision $*$
  - ➡ $y_{\mathsf{A}} \;\leq\; y_{\mathsf{B}}$
- ▷ Similarly: if any one of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ is taken, then also B has to be taken
  - ➡ binary variables $y_{\mathsf{A}_1}, \ldots, y_{\mathsf{A}_n}, y_{\mathsf{B}} \in \{0,1\}$, same meaning as above
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \;\leq\; n \cdot y_{\mathsf{B}}$ ➡ possibly bad if n is large (big-M constraints)
- ▷ Triggering: if some value exceeds a given threshold, then B happens
  - ➡ binary variable $y \in \{0,1\}$, meaning: $y = 1 \Leftrightarrow$ B happens
  - ➡ (...linear expression for value...) $\leq\; T + M \cdot y$ (T: threshold, M: big enough value)
- ▷ Special case: if all of decisions $\mathsf{A}_1, \ldots, \mathsf{A}_n$ are taken, then B has to be taken
  - ➡ $y_{\mathsf{A}_1} + \ldots + y_{\mathsf{A}_n} \;\leq\; n - 1 + y_{\mathsf{B}}$
- ▷ Even more complicated logical relations are possible by combining constraints...

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling
- Branch & Bound, Cutting Planes; More Examples; Combinatorial Optimization
- Combinatorial Optimization: Examples, Graphs, Algorithms
- Complexity Theory
- Nonlinear Optimization
- Scheduling
- Lot Sizing
- Multicriteria Optimization
- Oral exam