Mathematical Tools for Engineering and Management

Lecture 8

7 Dec 2011

- ▷ Models, Data and Algorithms
- ▷ Linear Optimization
- ▷ Mathematical Background: Polyhedra, Simplex-Algorithm
- ▷ Sensitivity Analysis; (Mixed) Integer Programming
- ▷ MIP Modelling
- ▷ MIP Modelling: More Examples; Branch & Bound
- ▷ Cutting Planes; Combinatorial Optimization: Examples, Graphs, Algorithms
- ▷ TSP-Heuristics
- ▷ Complexity Theory
- ▷ Nonlinear Optimization
- ▷ Scheduling
- ▷ Lot Sizing
- ▷ Multicriteria Optimization
- ▷ Oral exam

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▷ Try to prove quality of solution ➡ approximation algorithms

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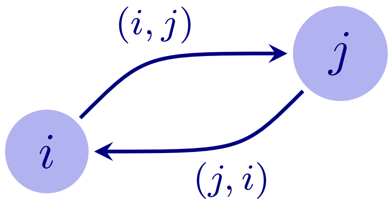

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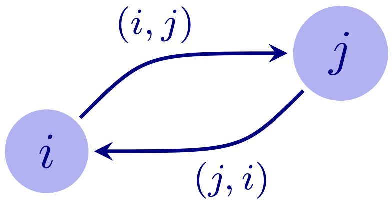

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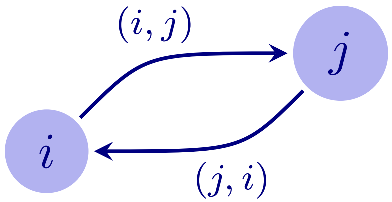

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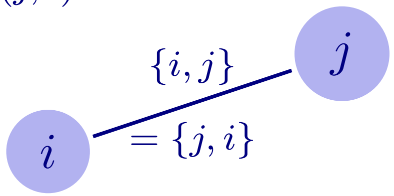

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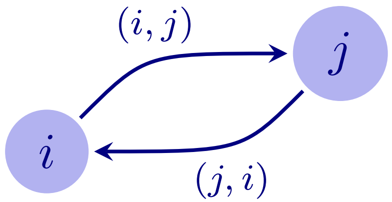

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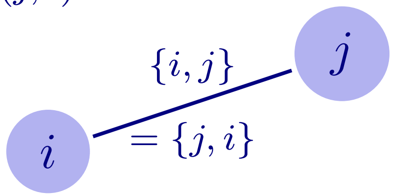
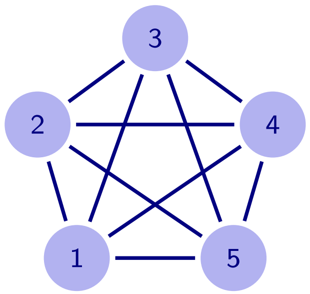


▷ Examples:

complete graph
on n vertices: K_n



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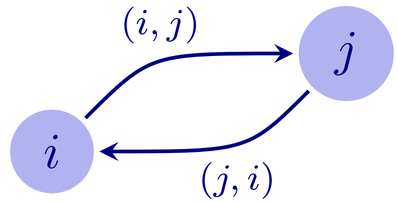

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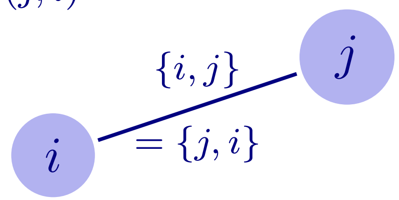

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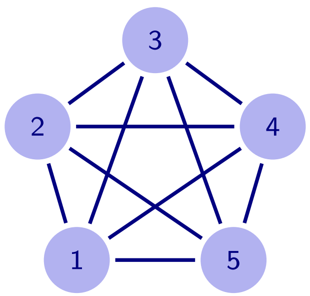
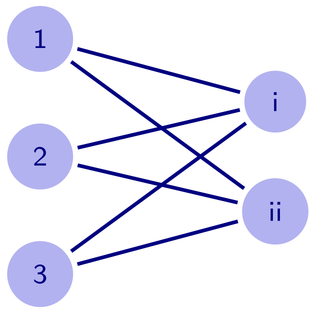


K_5

complete bipartite graph:
$K_{m,n}$



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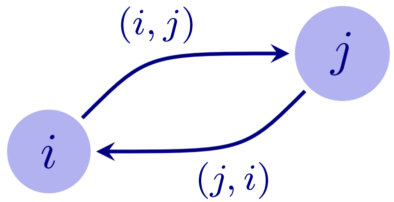

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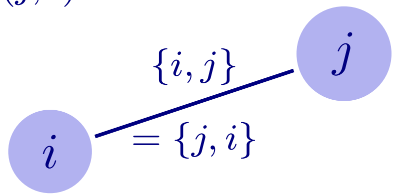
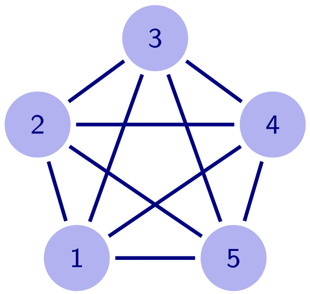


- Examples:

complete graph on n vertices: K_n



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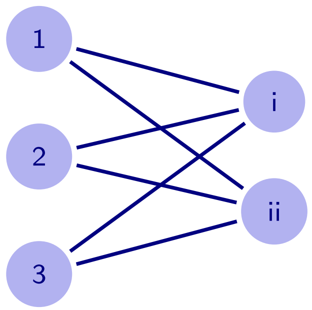
complete bipartite graph: $K_{m,n}$



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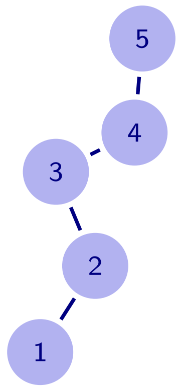
path: P_n



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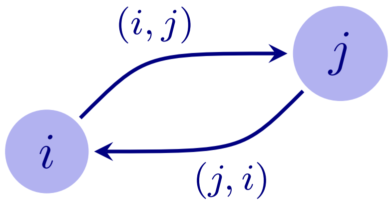

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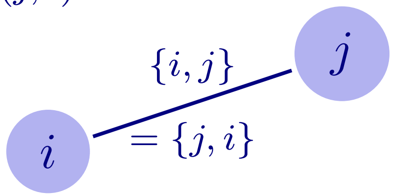

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complete graph on n vertices: K_n

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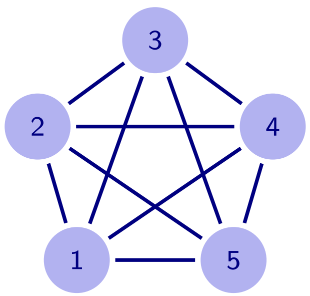
path: P_n

cycle on n vertices: C_n



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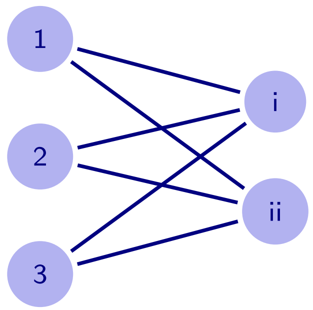

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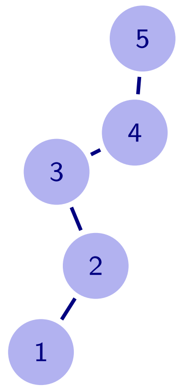

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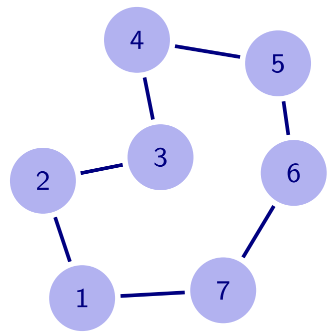

C_7

A tree T is a graph with the following properties:

- T contains no cycles
- T is connected (i.e. every two vertices can be connected by a path in T)
- There is exactly one more vertex than there are edges (i.e. $|V| = |E| + 1$)

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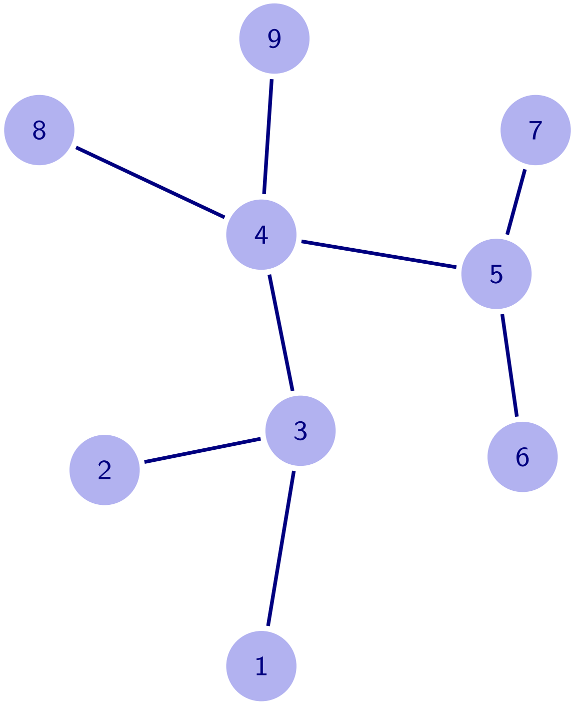

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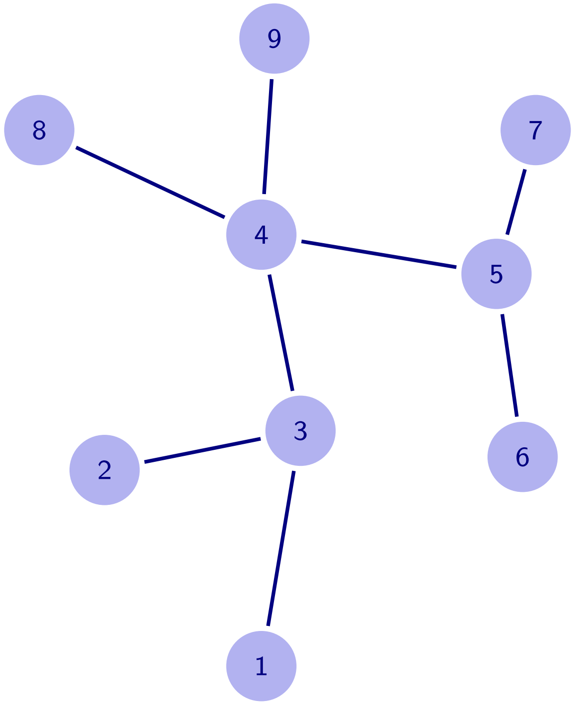

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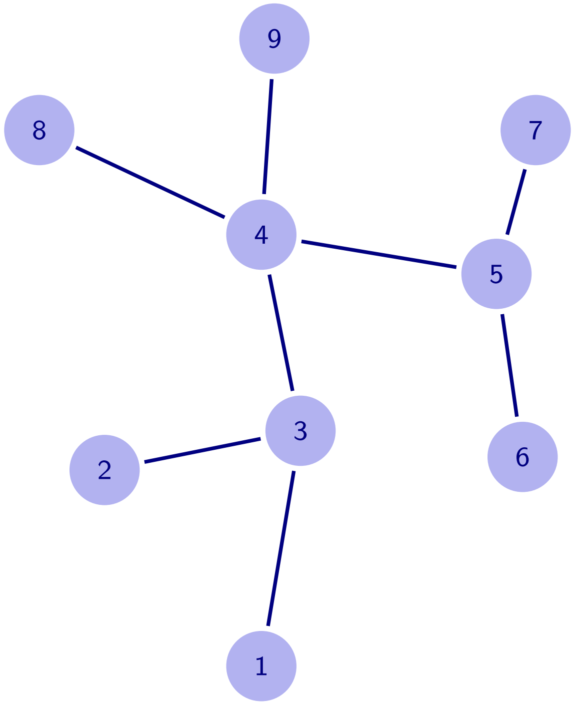

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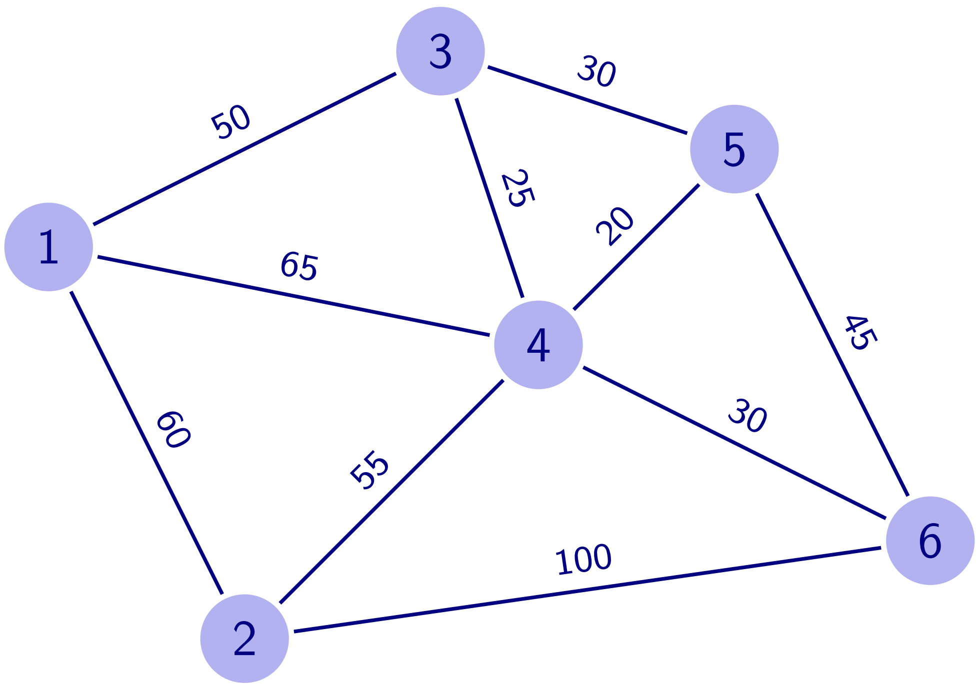

▷ Given a graph $G = (V, E)$ with non-negative edge-weights w_e for all $e \in E$...

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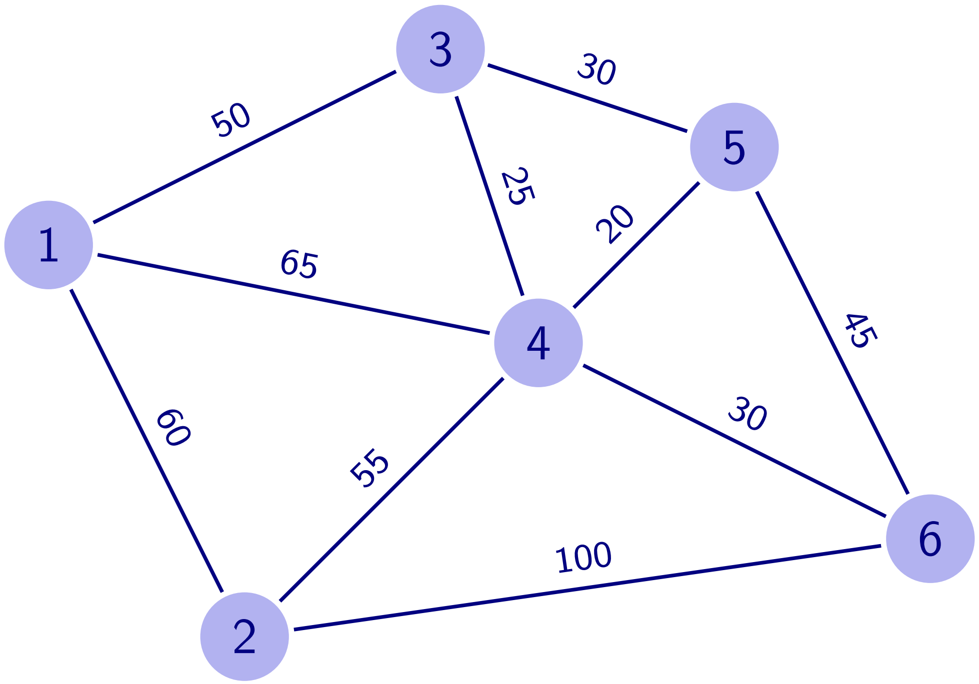

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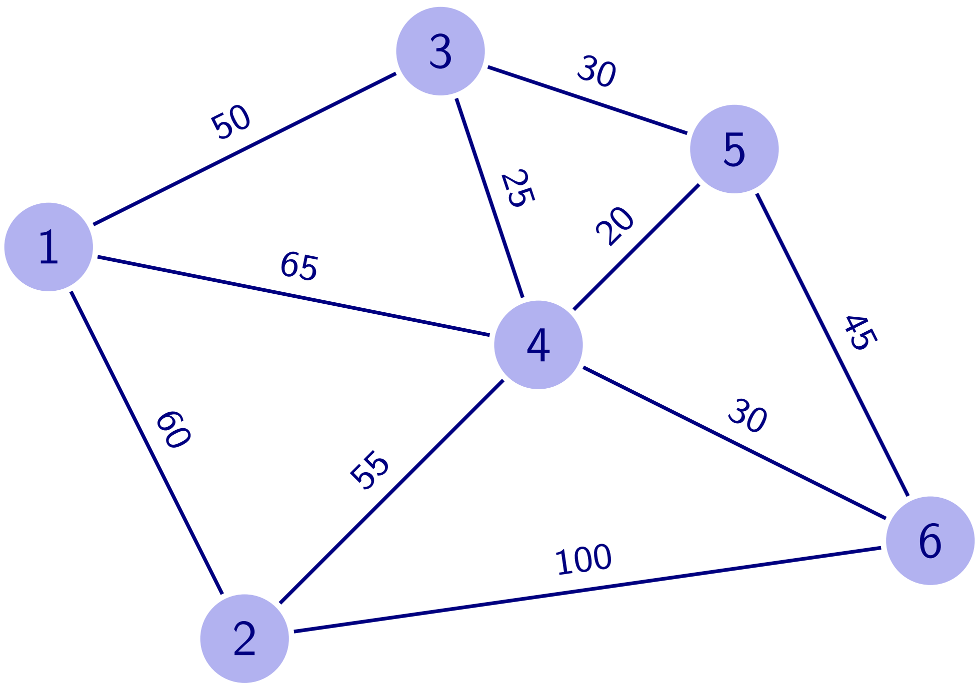

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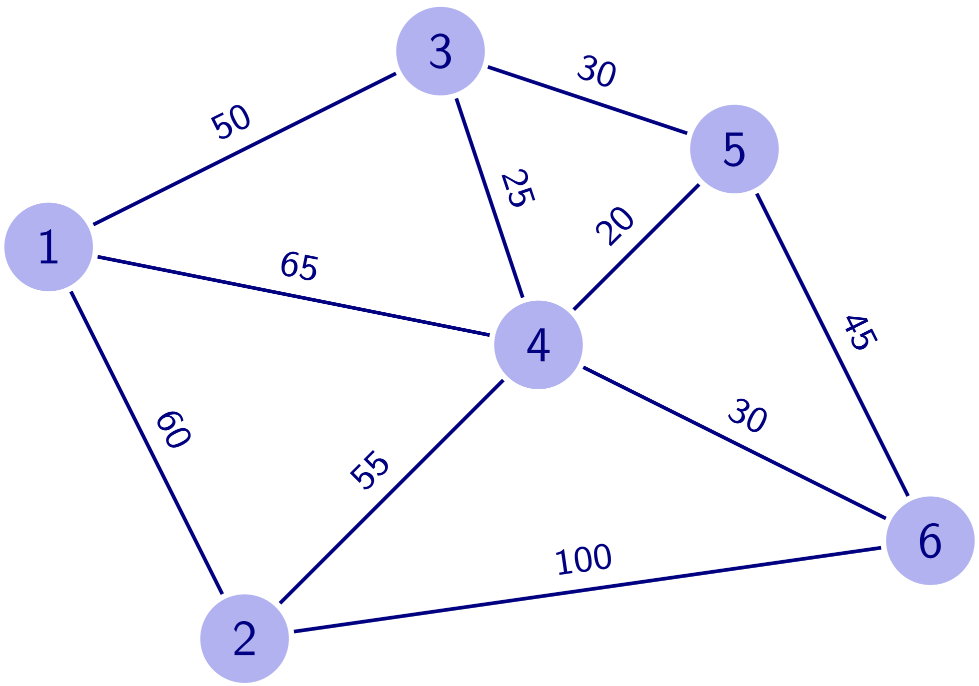

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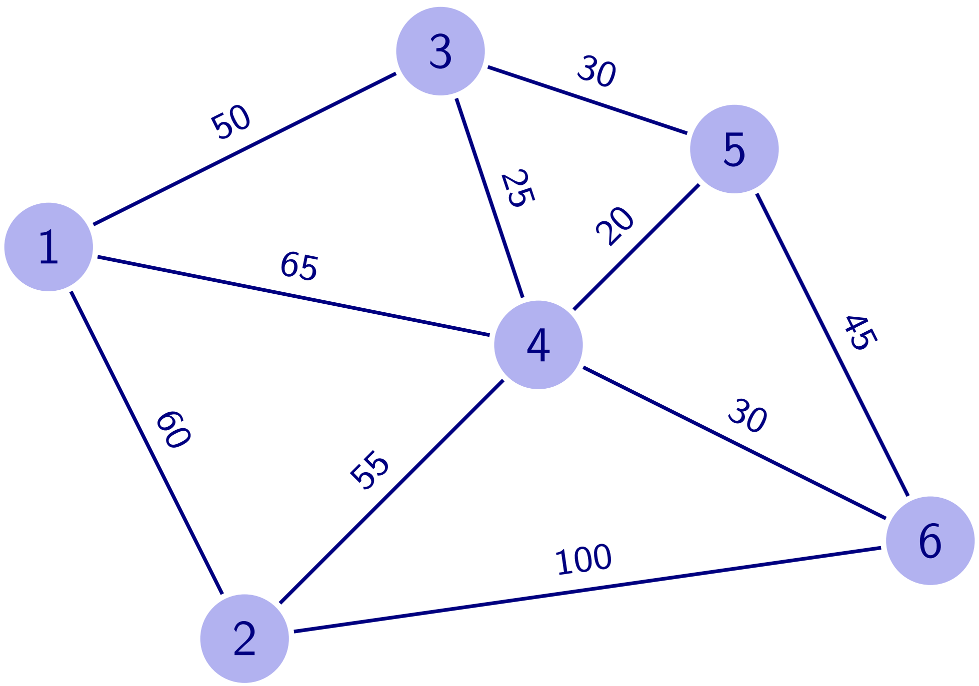

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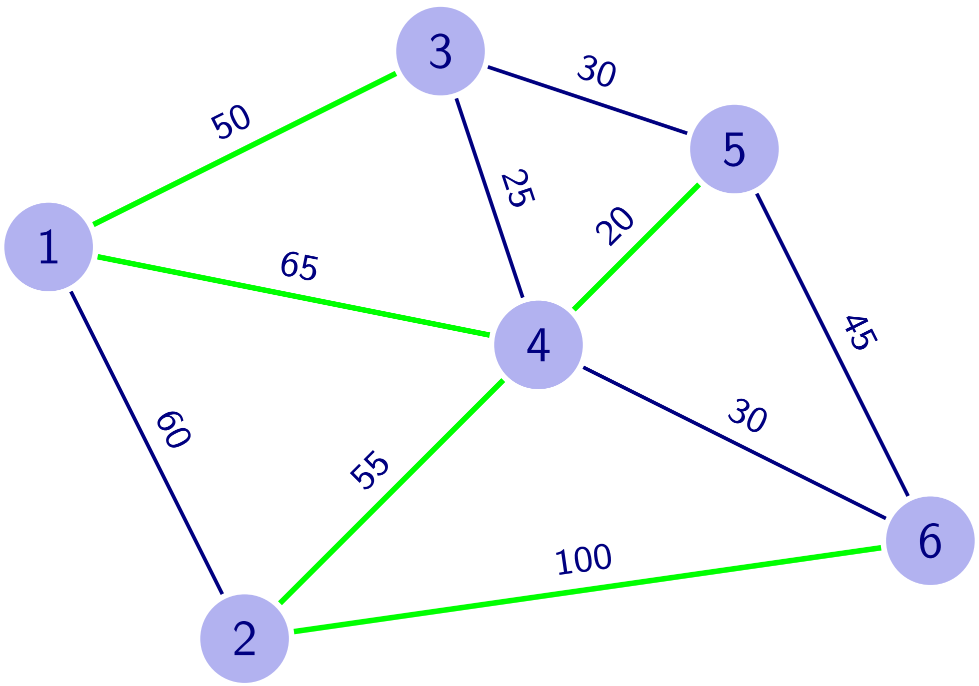

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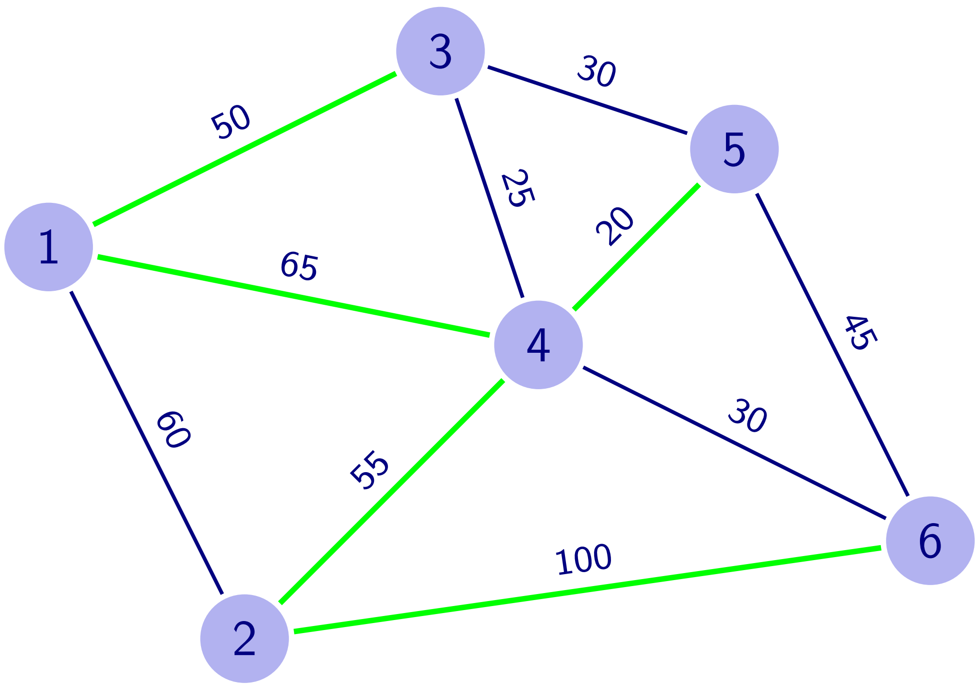

➡ total weight: 290

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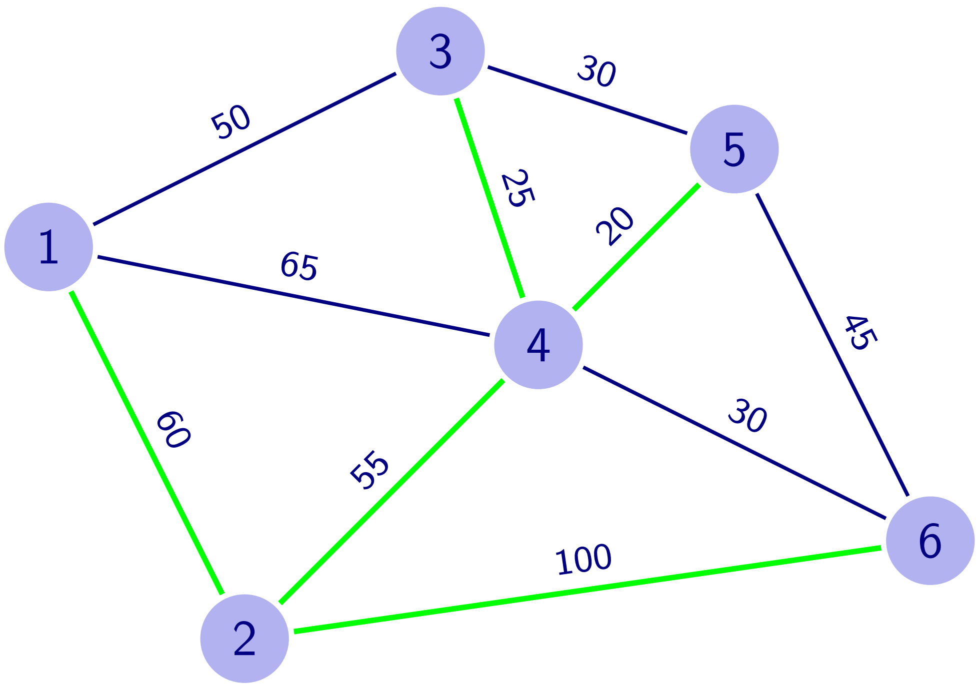

➡ total weight: 260

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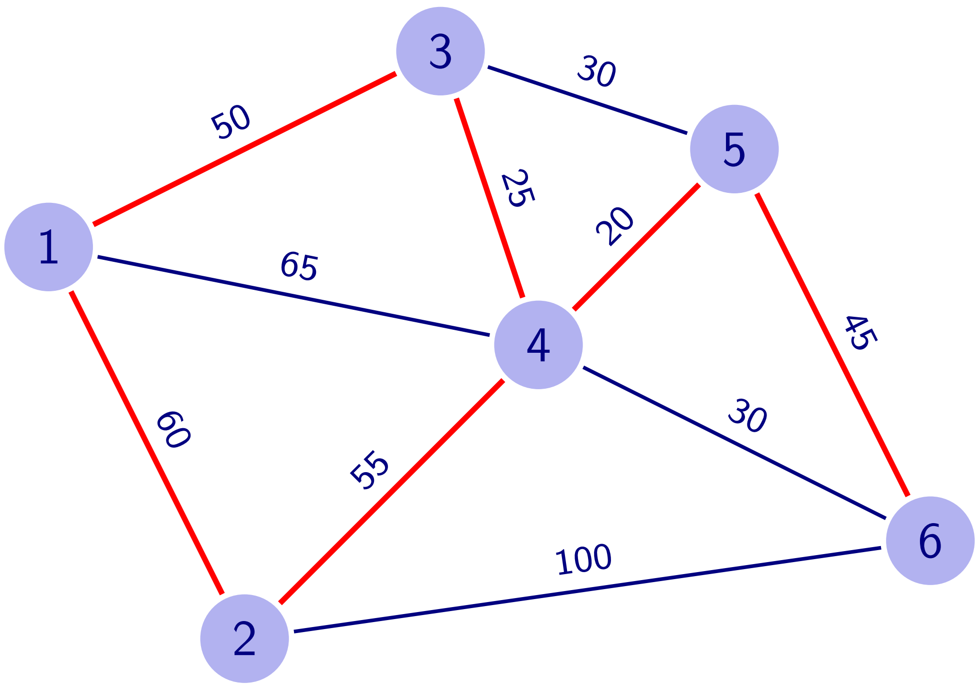

➡ not allowed: not a tree!

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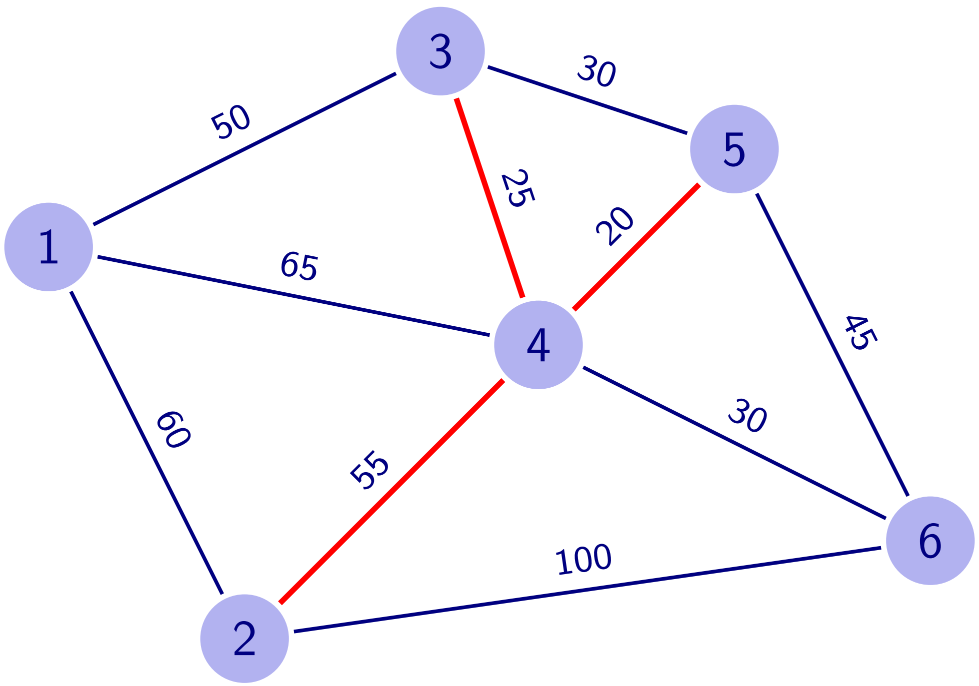

➡ not allowed: misses vertices!

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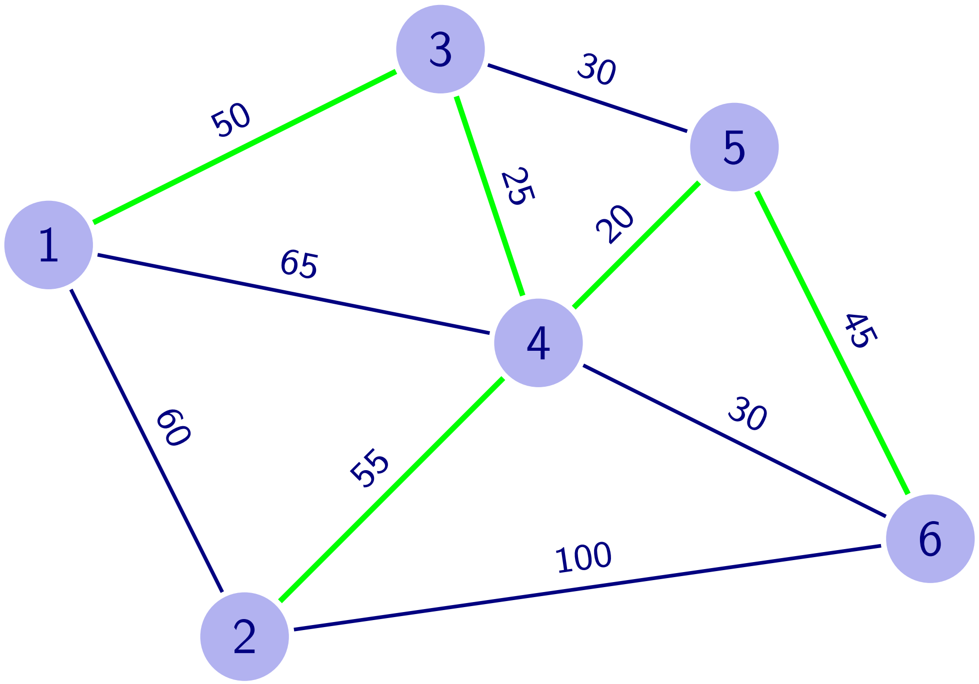

➡ total weight: 195

▷ Given a graph $G = (V, E)$ with non-negative edge-weights w_e for all $e \in E$...

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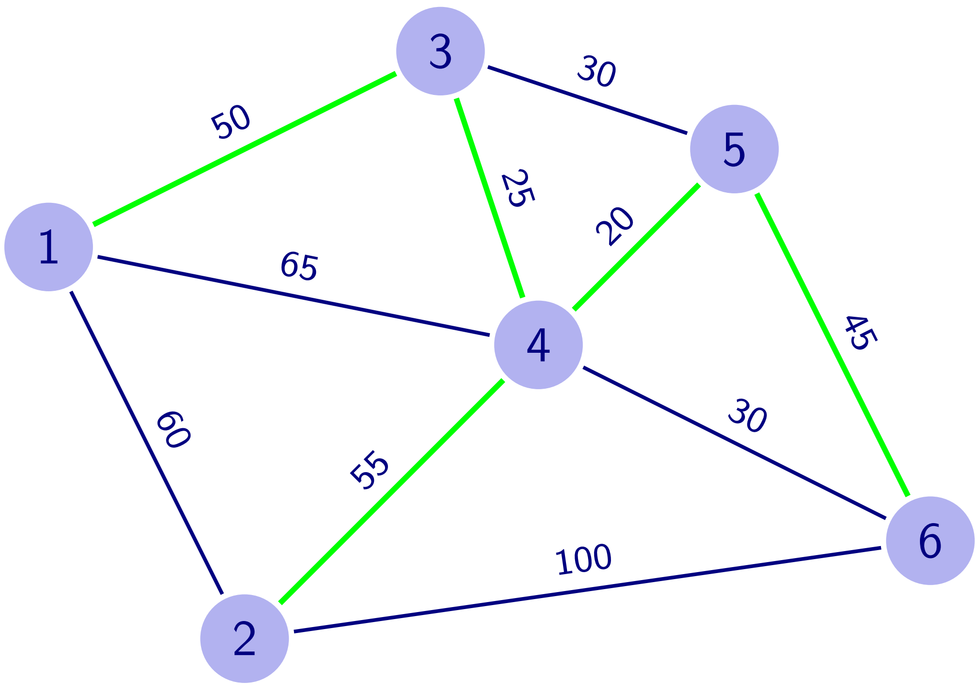

➡ total weight: 195

▷ Real-world problem:

Connect a set of given computers to form a local network, at minimal cost

- Idea: select cheap edges, as long as they don't result in a cycle (greedy)

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- Until all vertices are in the tree T...
- ...determine cheapest remaining potential edge: $\rightarrow\ e$

▷ Idea: select cheap edges, as long as they don't result in a cycle (greedy)

- Set of potential edges $:= E$, tree $T :=$ empty
- Until all vertices are in the tree T...
- ...determine cheapest remaining potential edge: $\rightarrow\ e$
- ...if adding edge e to the tree T does not result in a cycle:

▷ Idea: select cheap edges, as long as they don't result in a cycle (greedy)

- Set of potential edges $:= E$, tree $T :=$ empty
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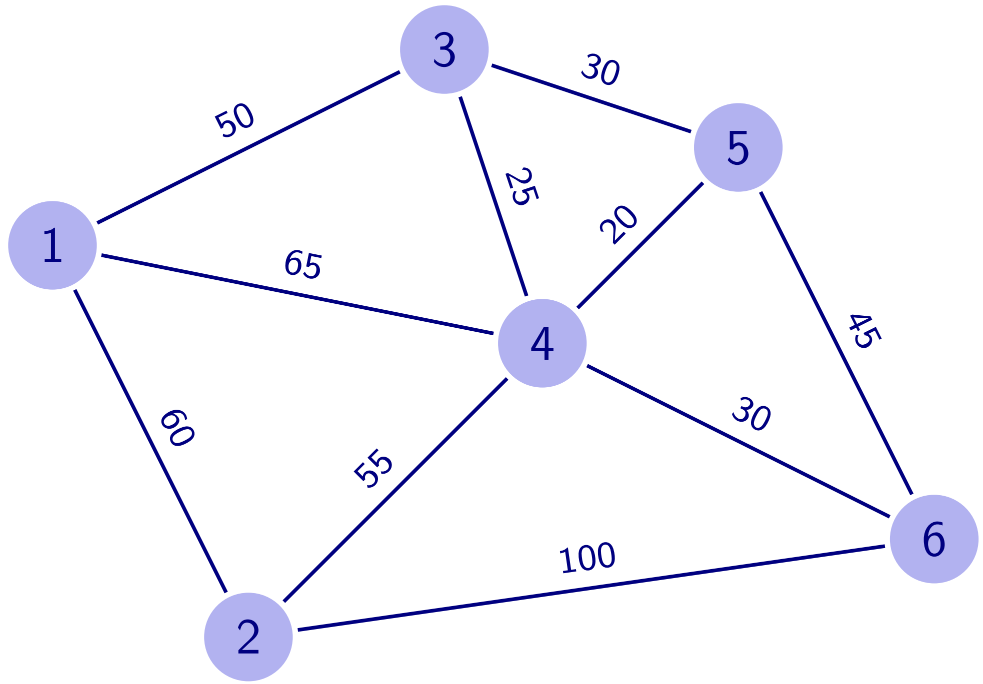

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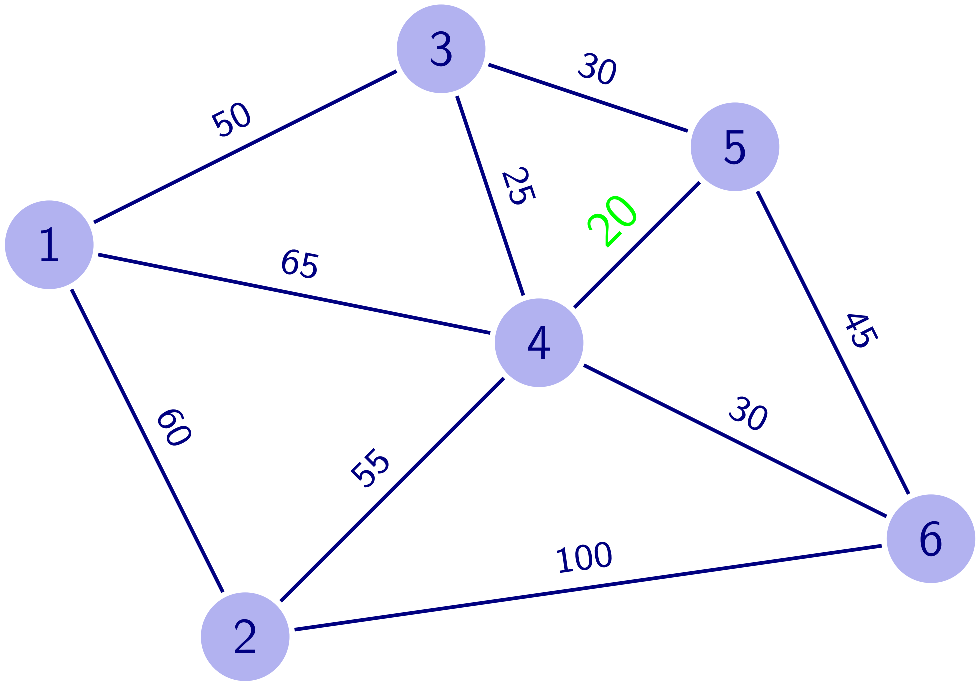

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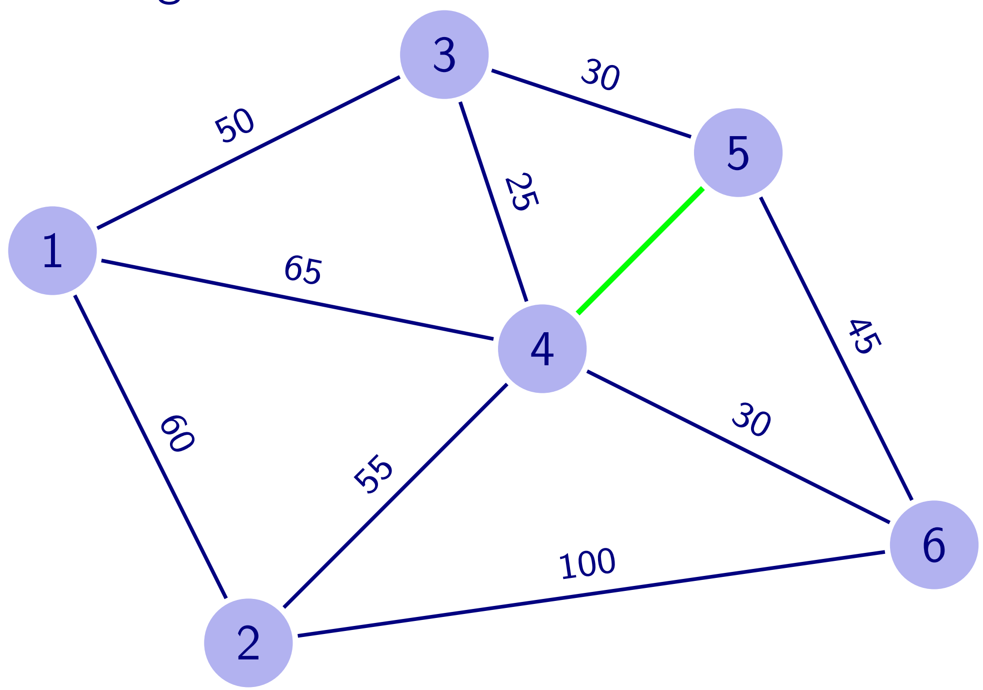
total weight: 20



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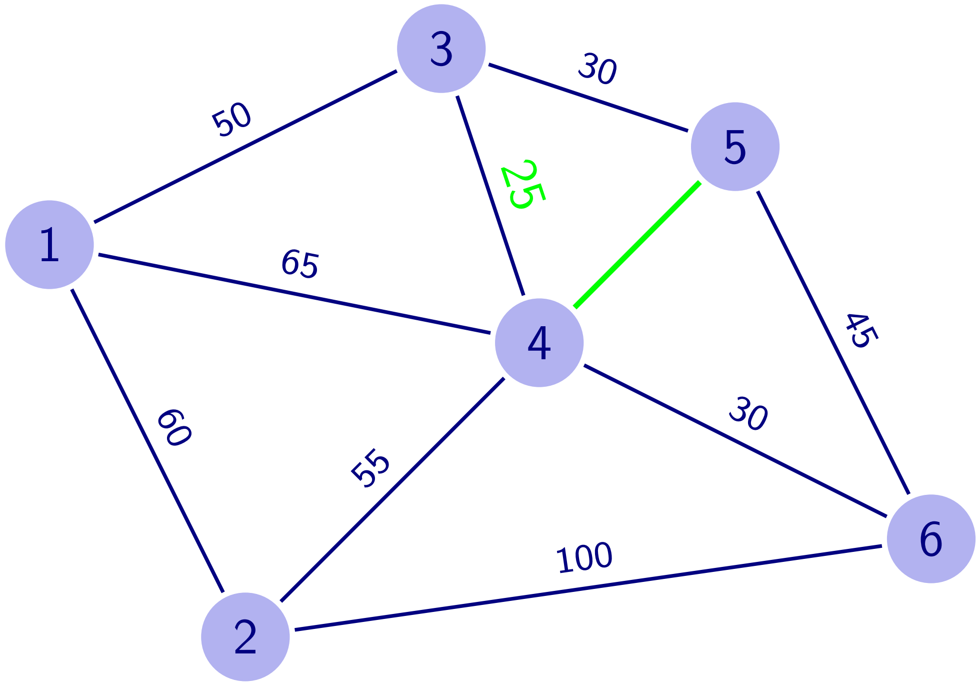

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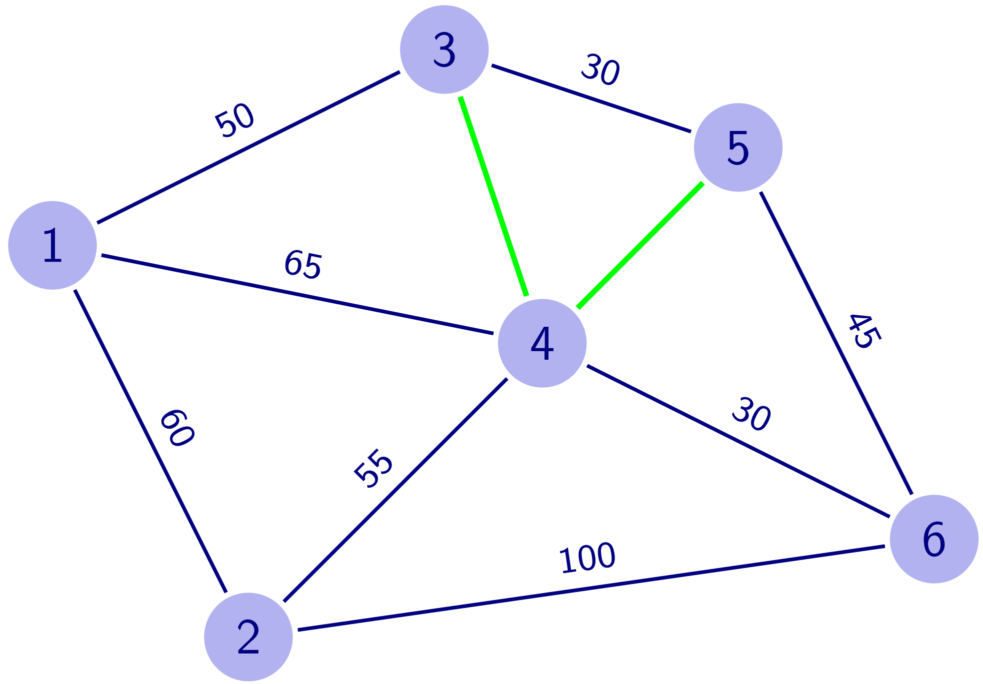

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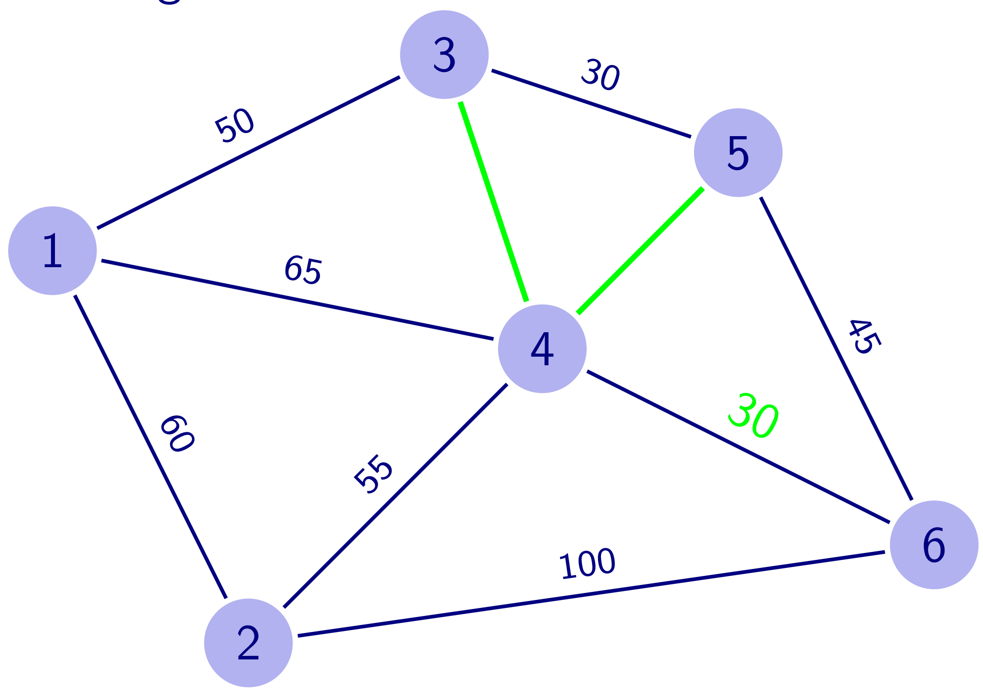

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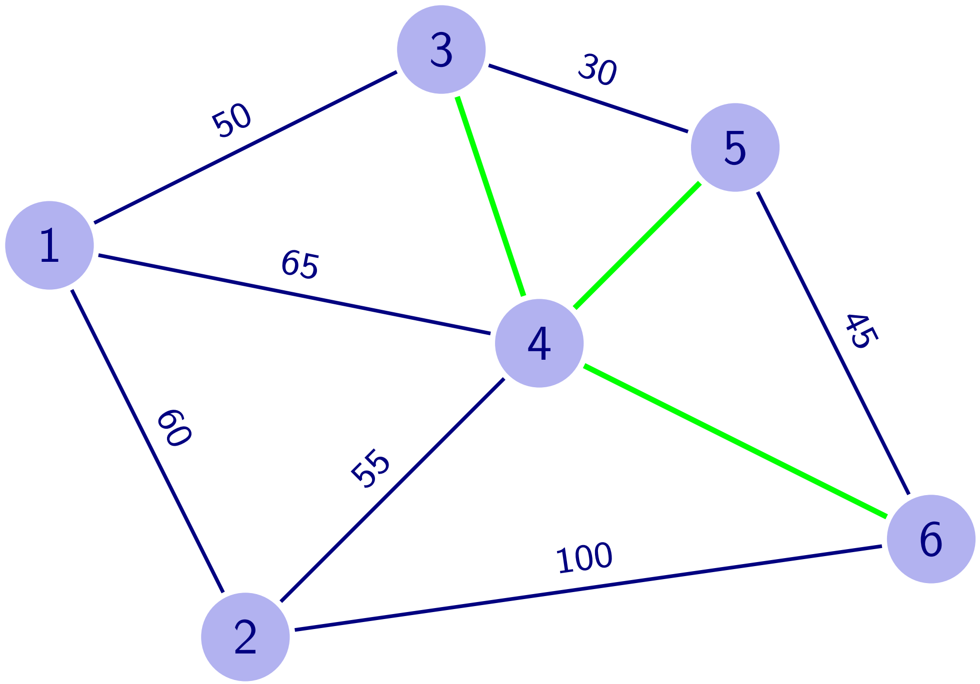
total weight: 75



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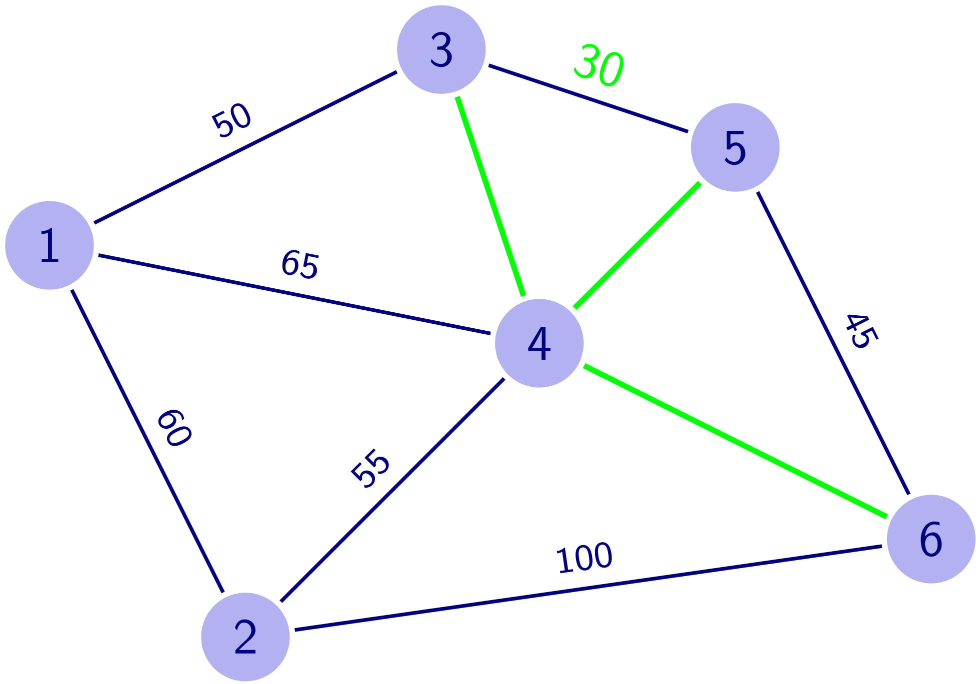

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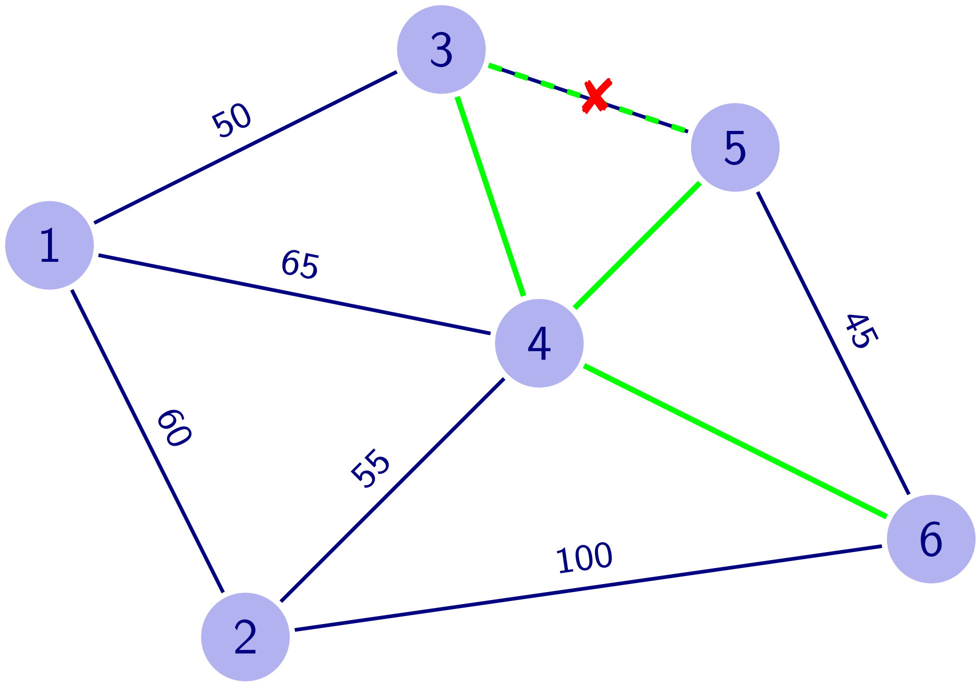

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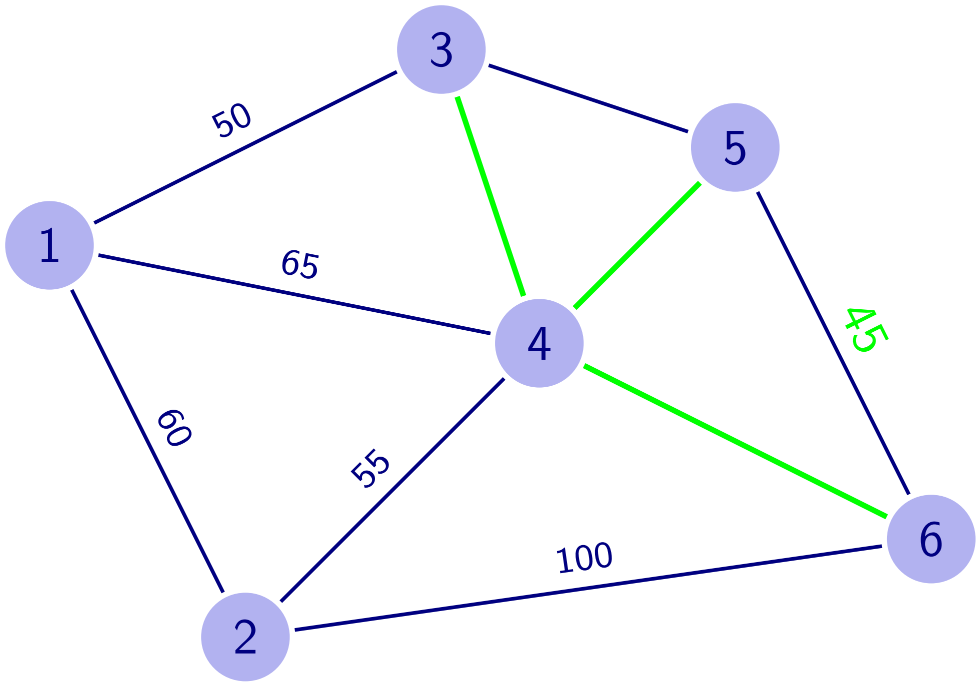

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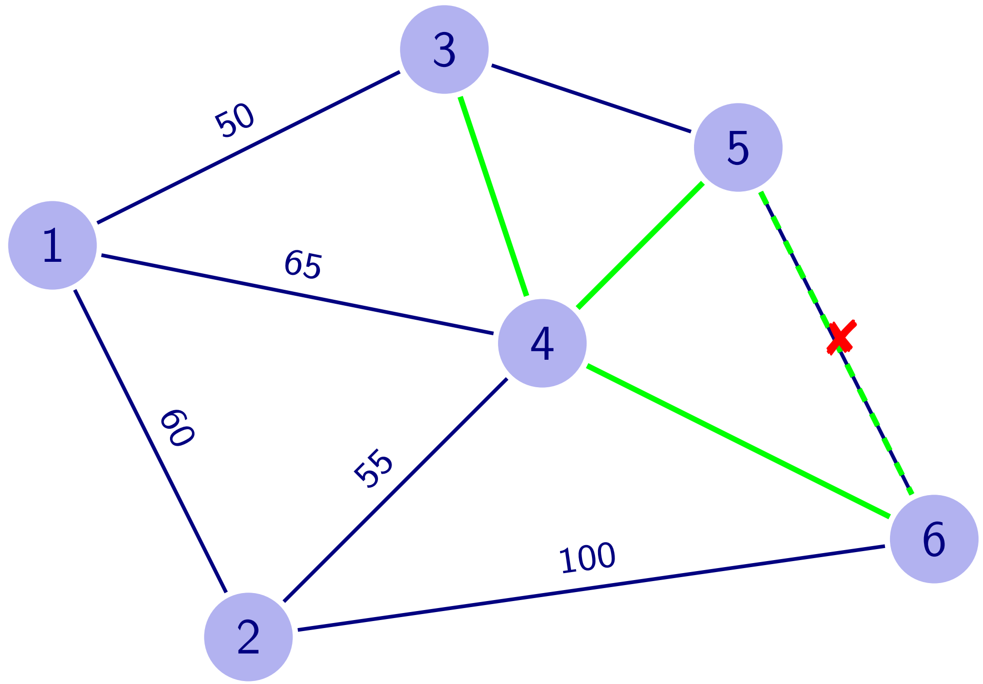

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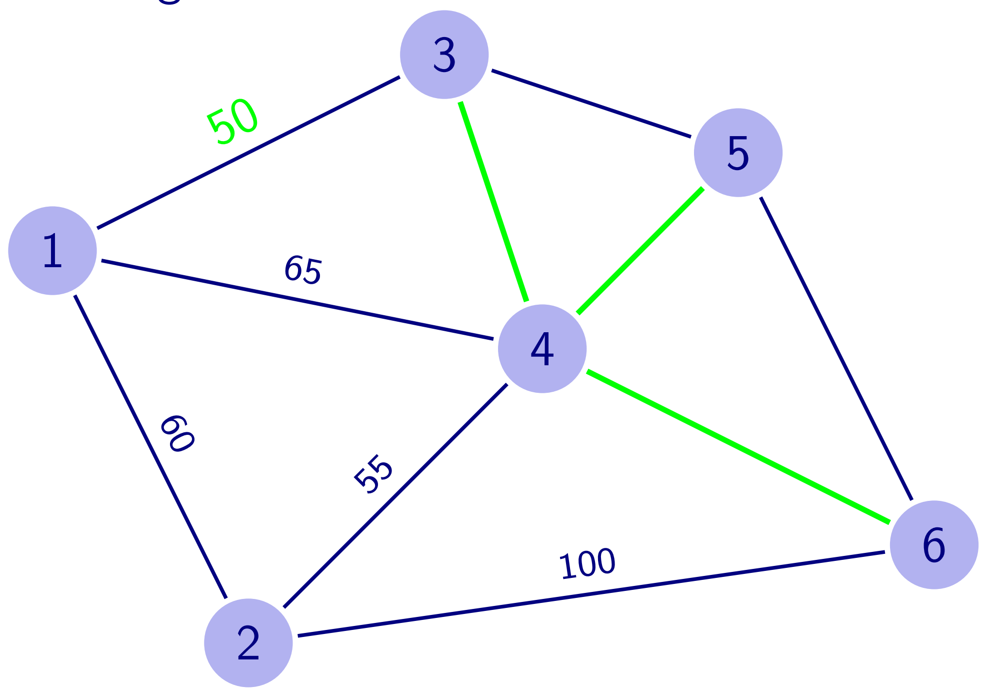

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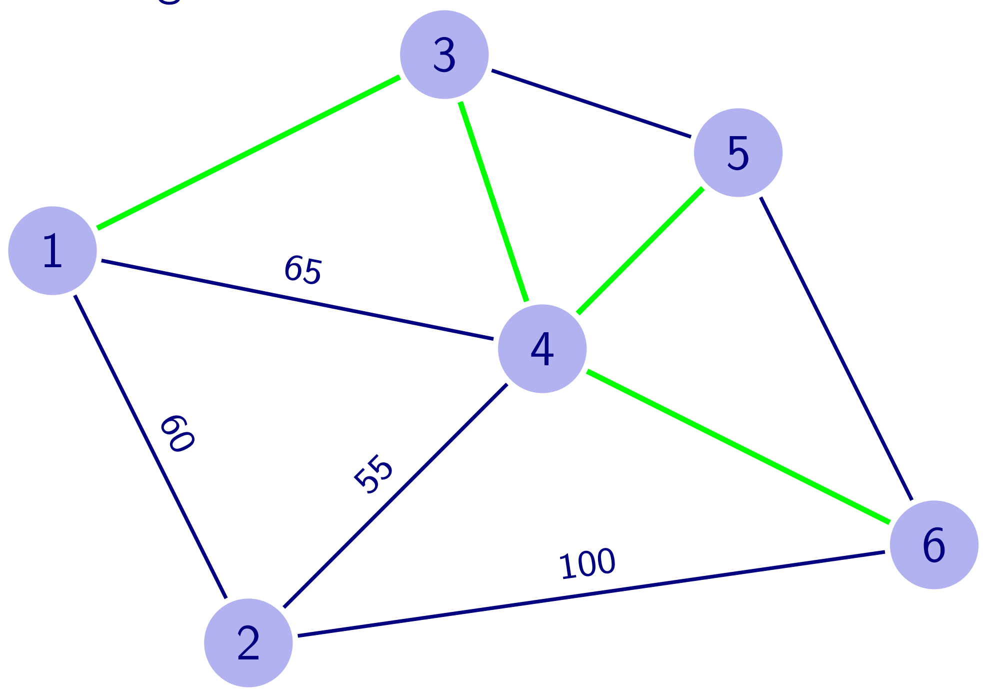
total weight: 125



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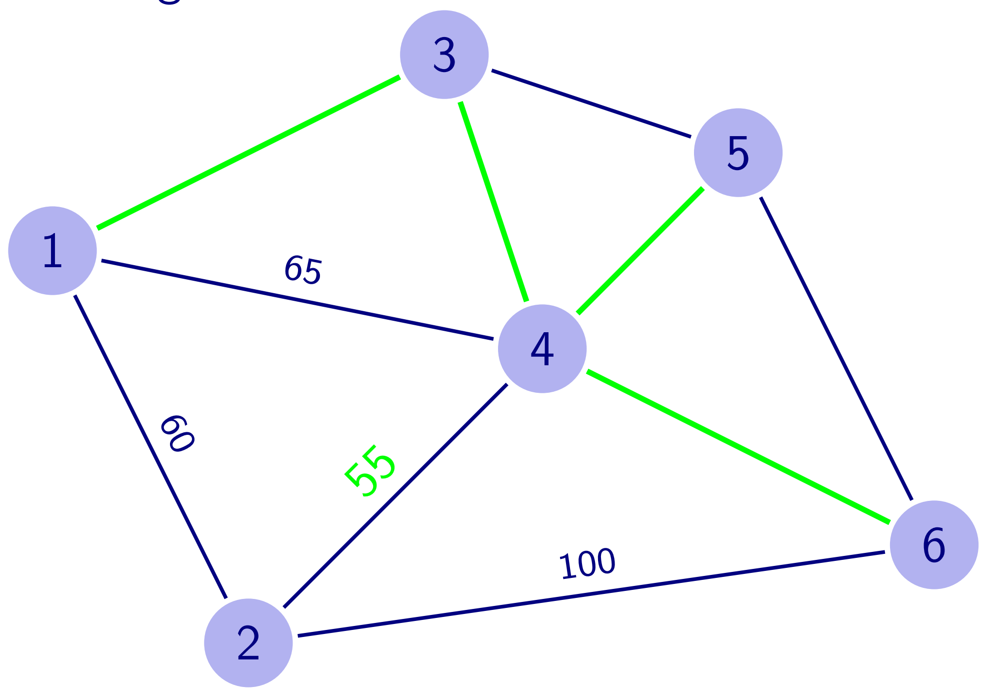

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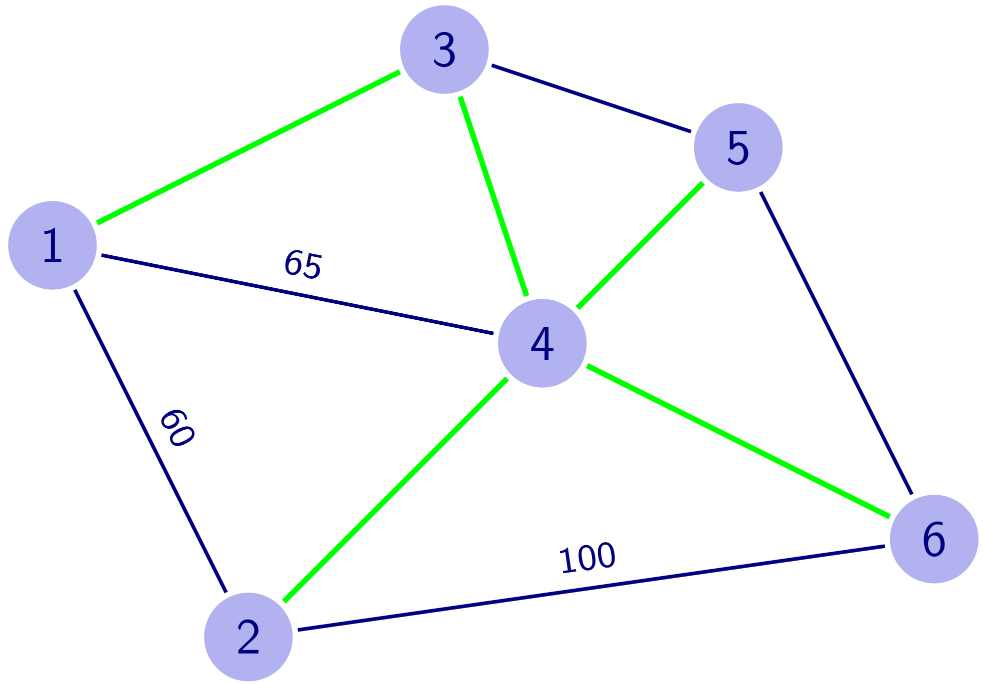

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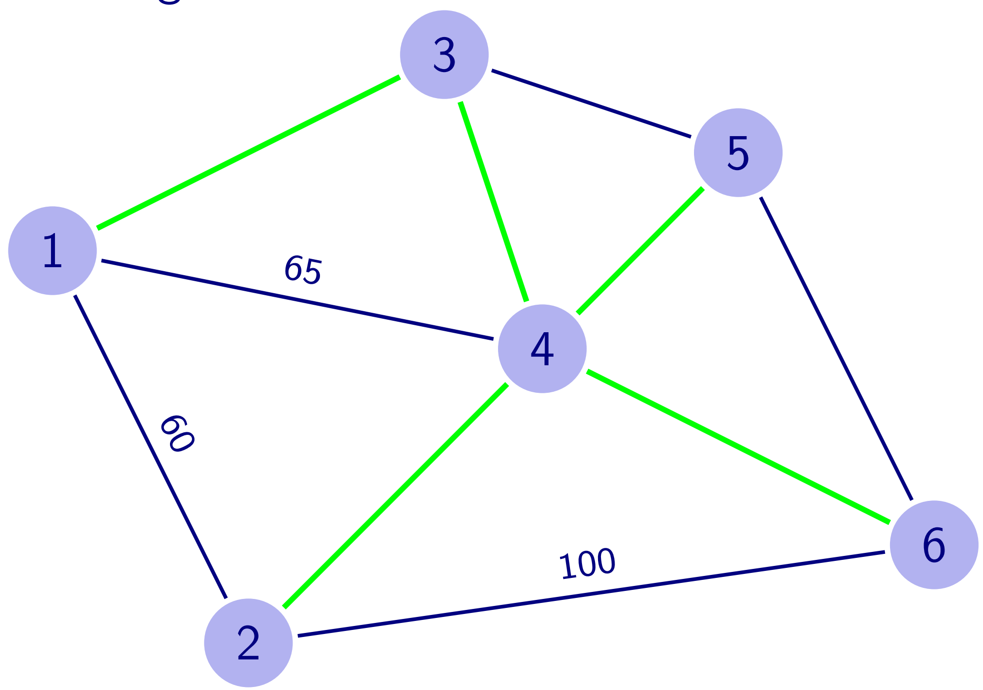

total weight: 180

➡ T contains all vertices

➡ done!

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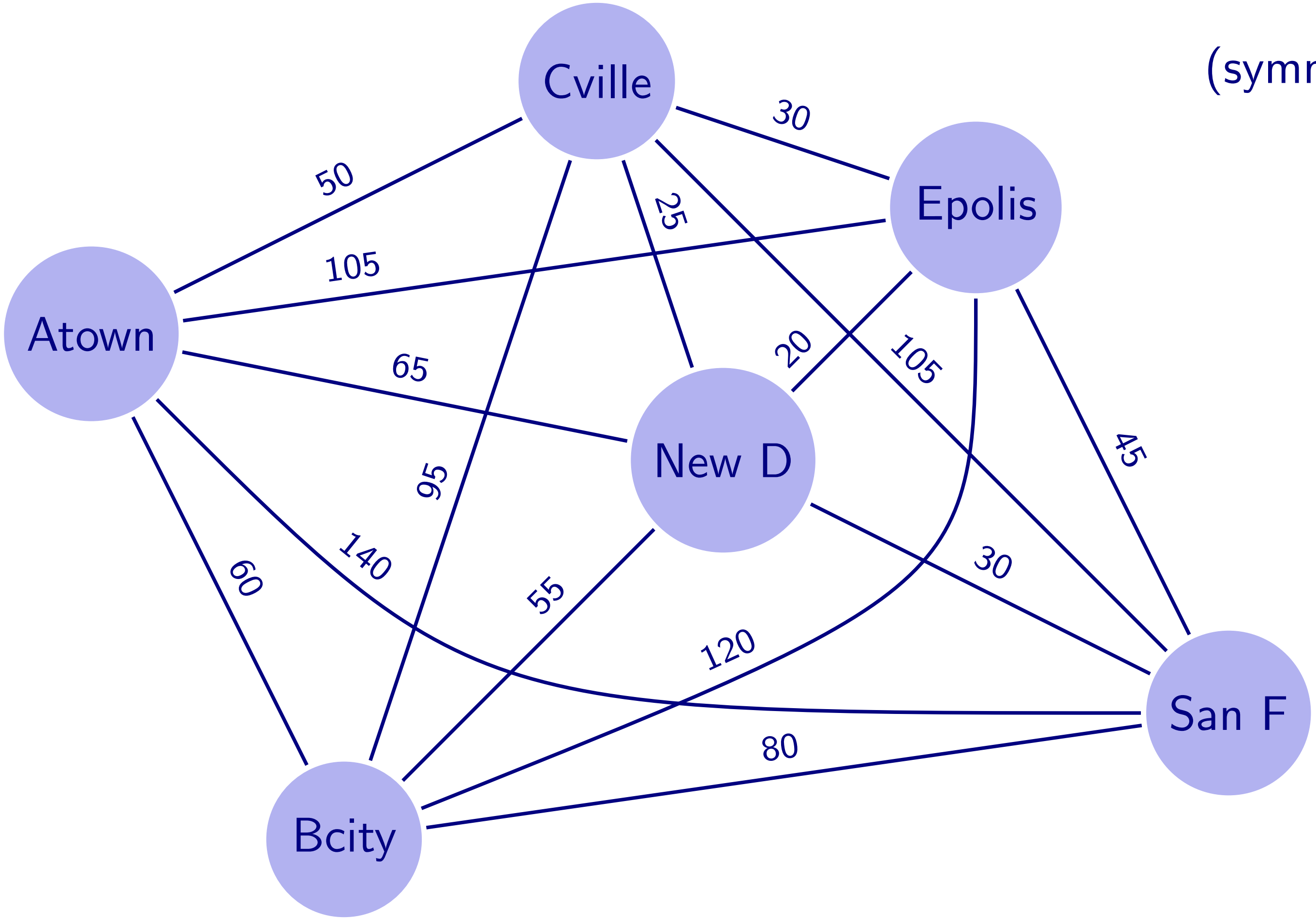

(symmetric TSP)

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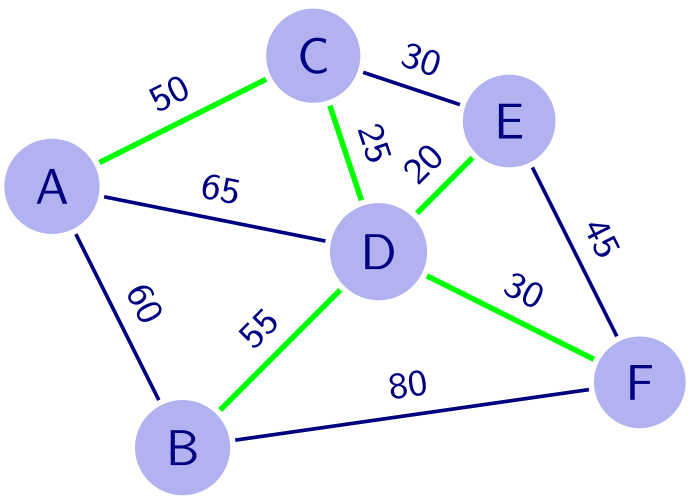

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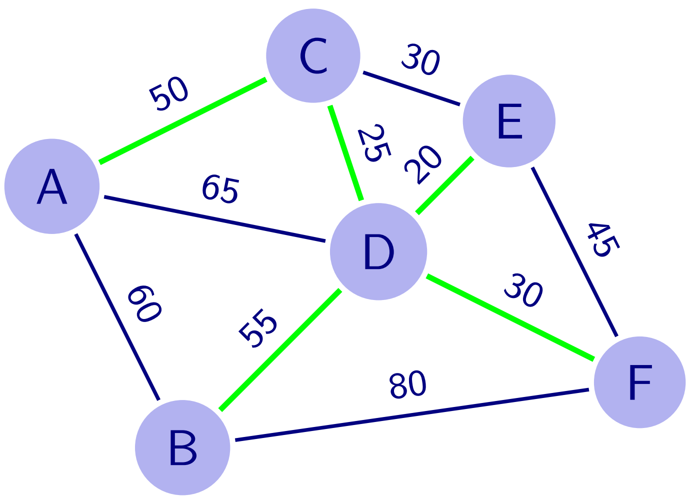

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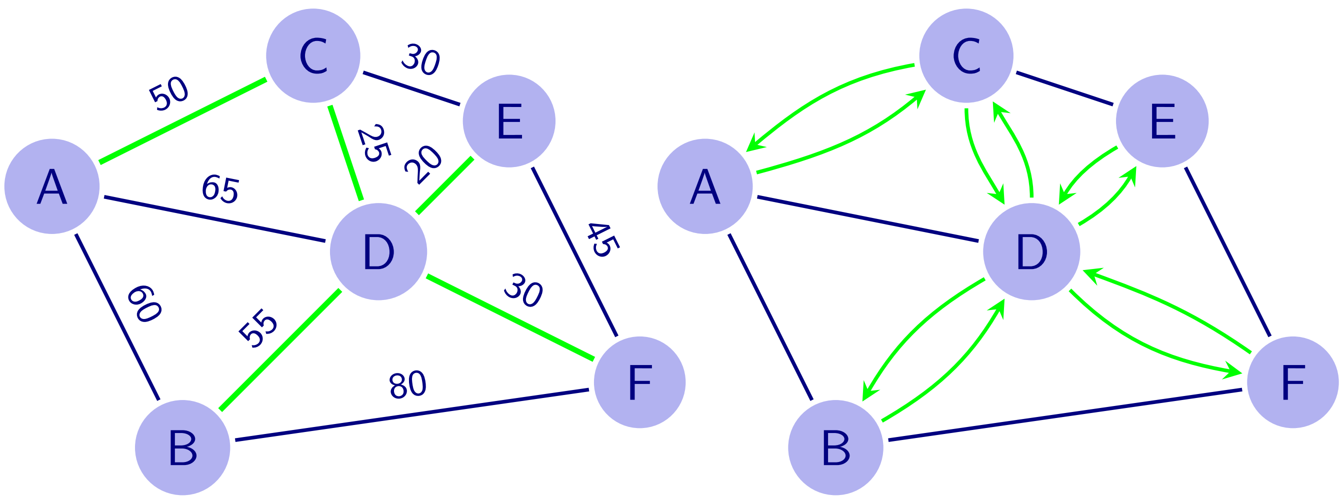

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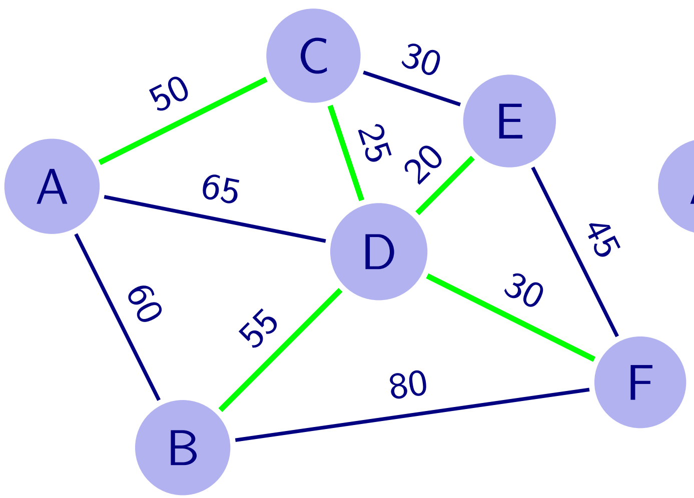

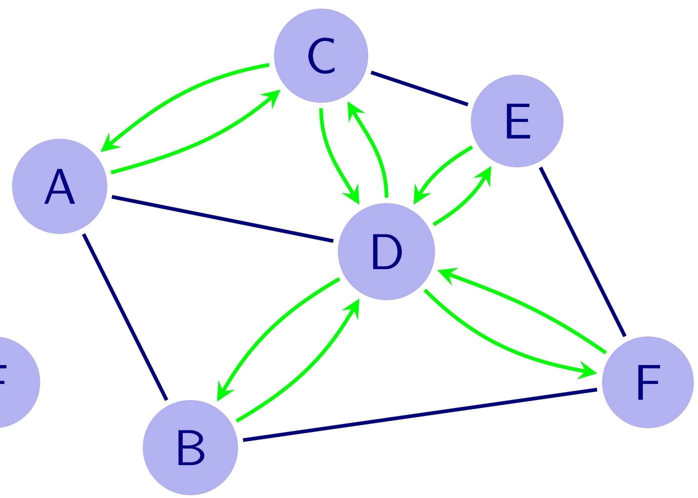

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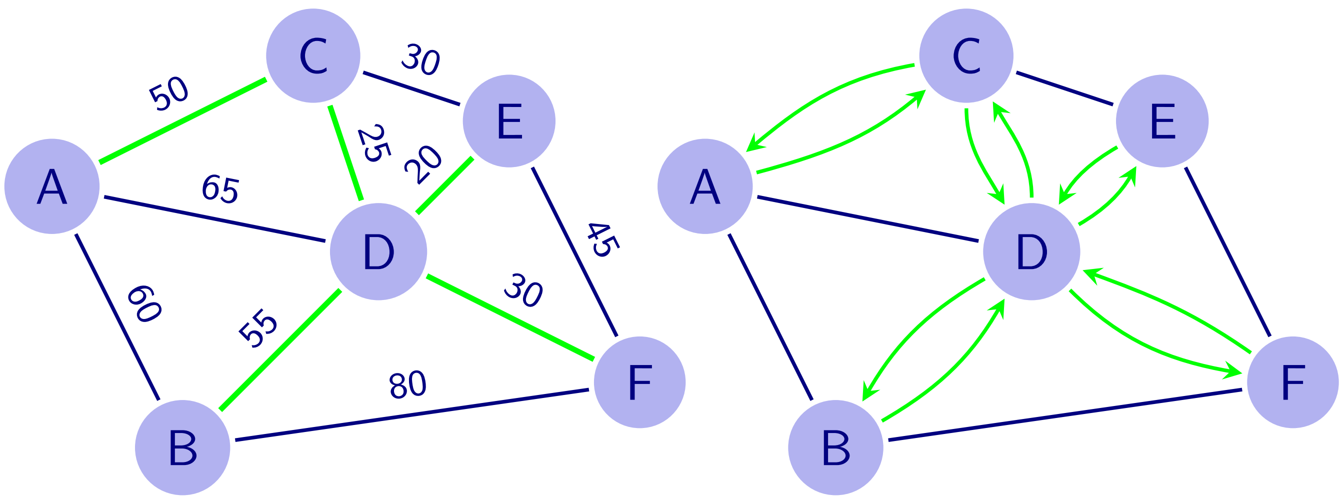

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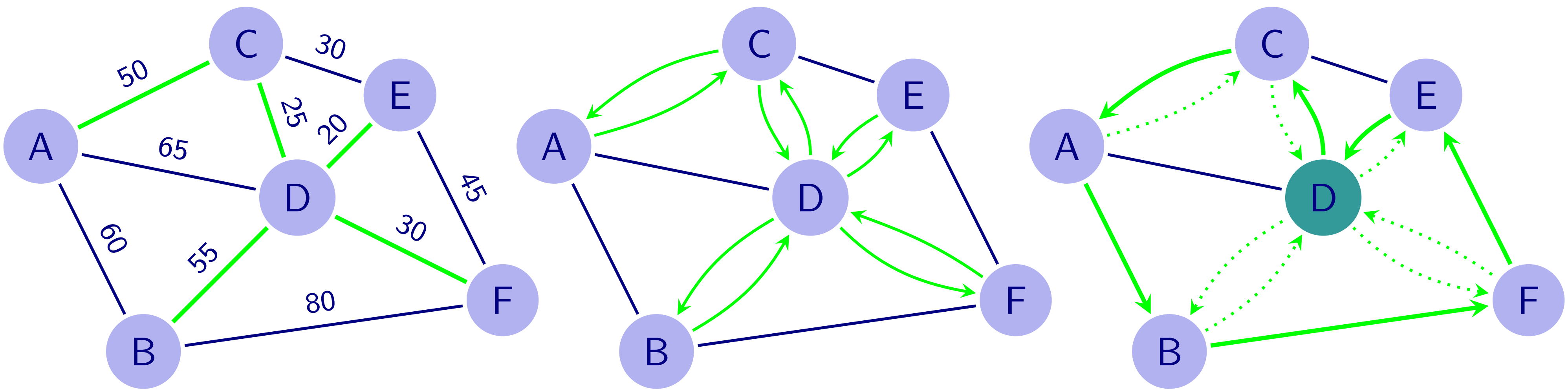

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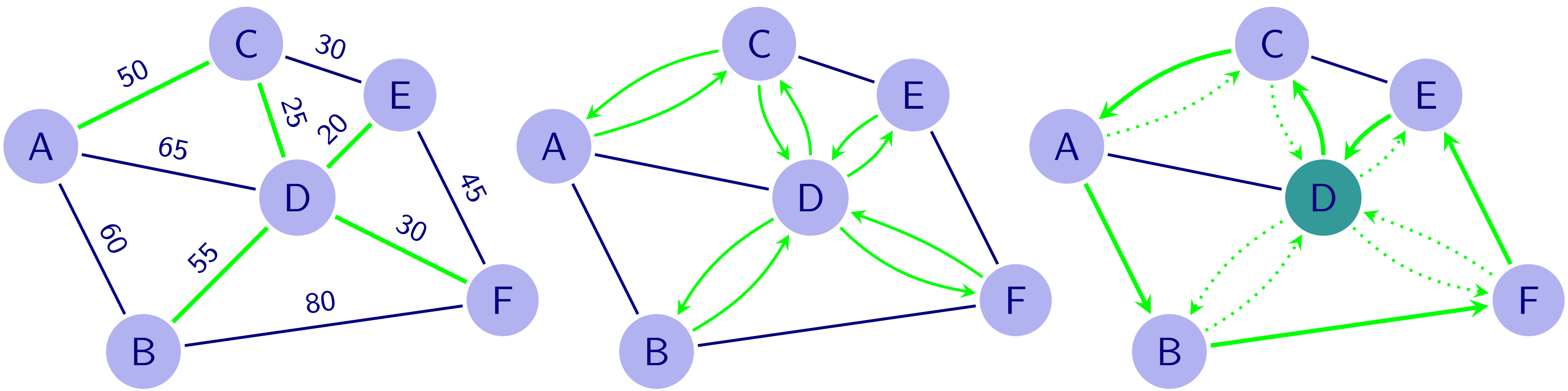

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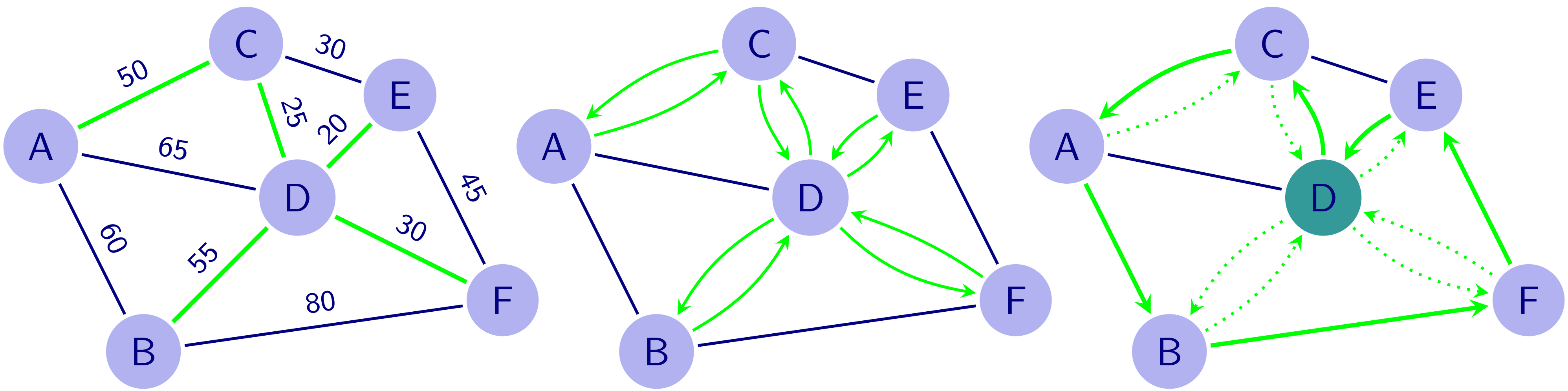

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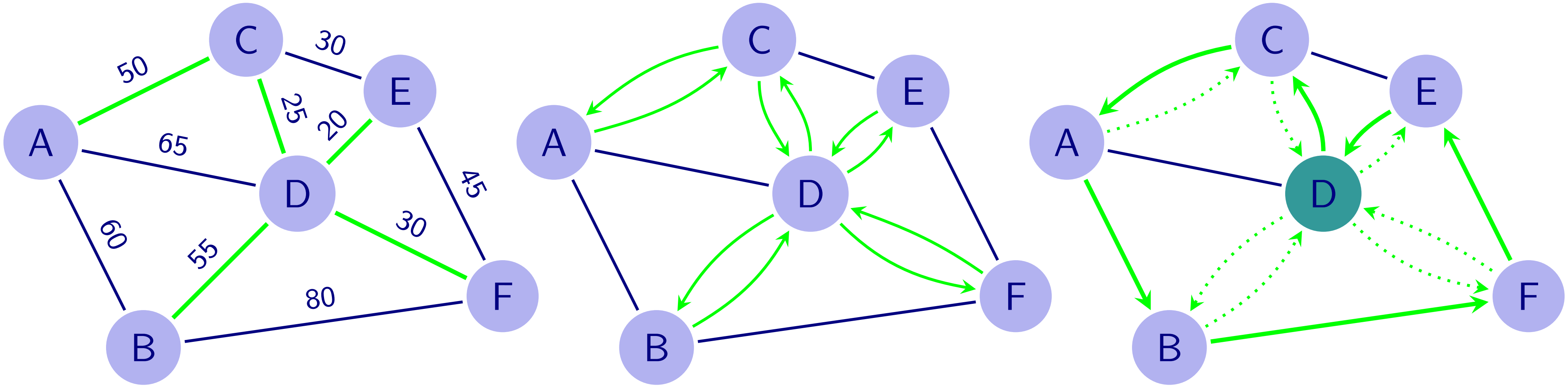

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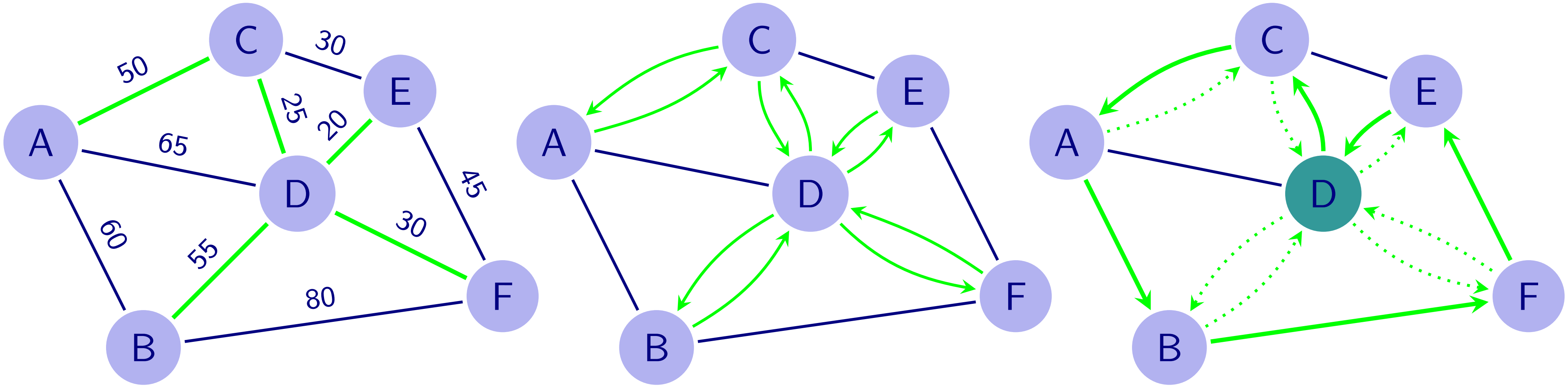

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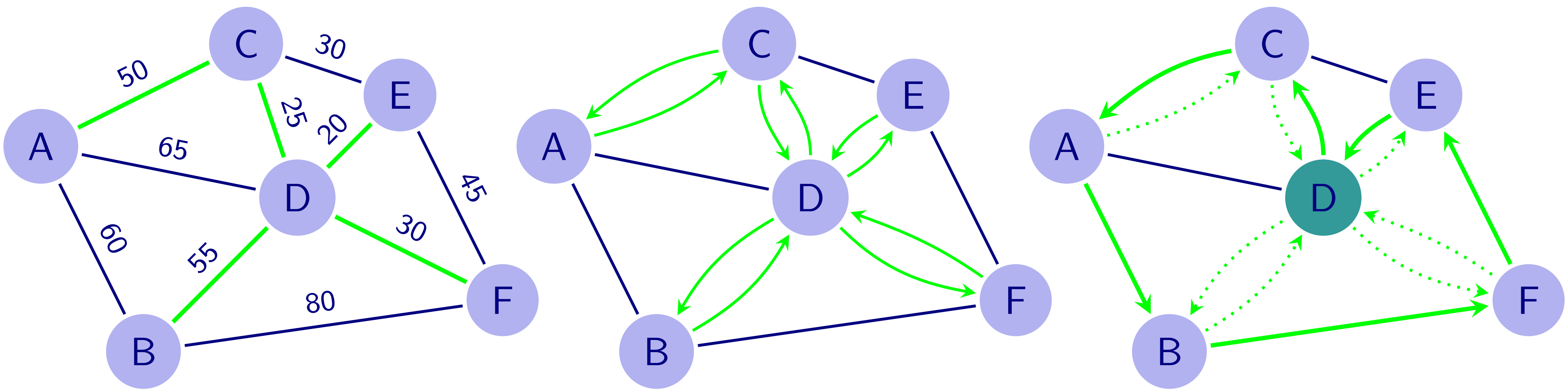

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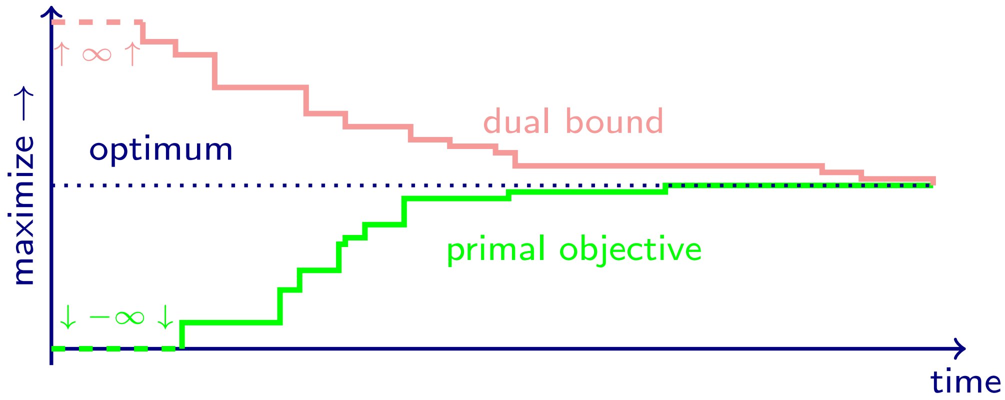

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling
- MIP Modelling: More Examples; Branch & Bound
- Cutting Planes; Combinatorial Optimization: Examples, Graphs, Algorithms
- TSP-Heuristics
- Network Flows, Complexity Theory
- Nonlinear Optimization
- Scheduling
- Lot Sizing
- Multicriteria Optimization
- Oral exam