Mathematical Tools for Engineering and Management

Lecture 9

14 Dec 2011

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling
- MIP Modelling: More Examples; Branch & Bound
- Cutting Planes; Combinatorial Optimization: Examples, Graphs, Algorithms
- TSP-Heuristics
- Network Flows
- Shortest Path Problem, Complexity Theory
- Nonlinear Optimization
- Scheduling, Lot Sizing
- Multicriteria Optimization
- Oral exam

- Consider a data network with

Example: data flow

- Consider a data network with central offices , routers  and users 

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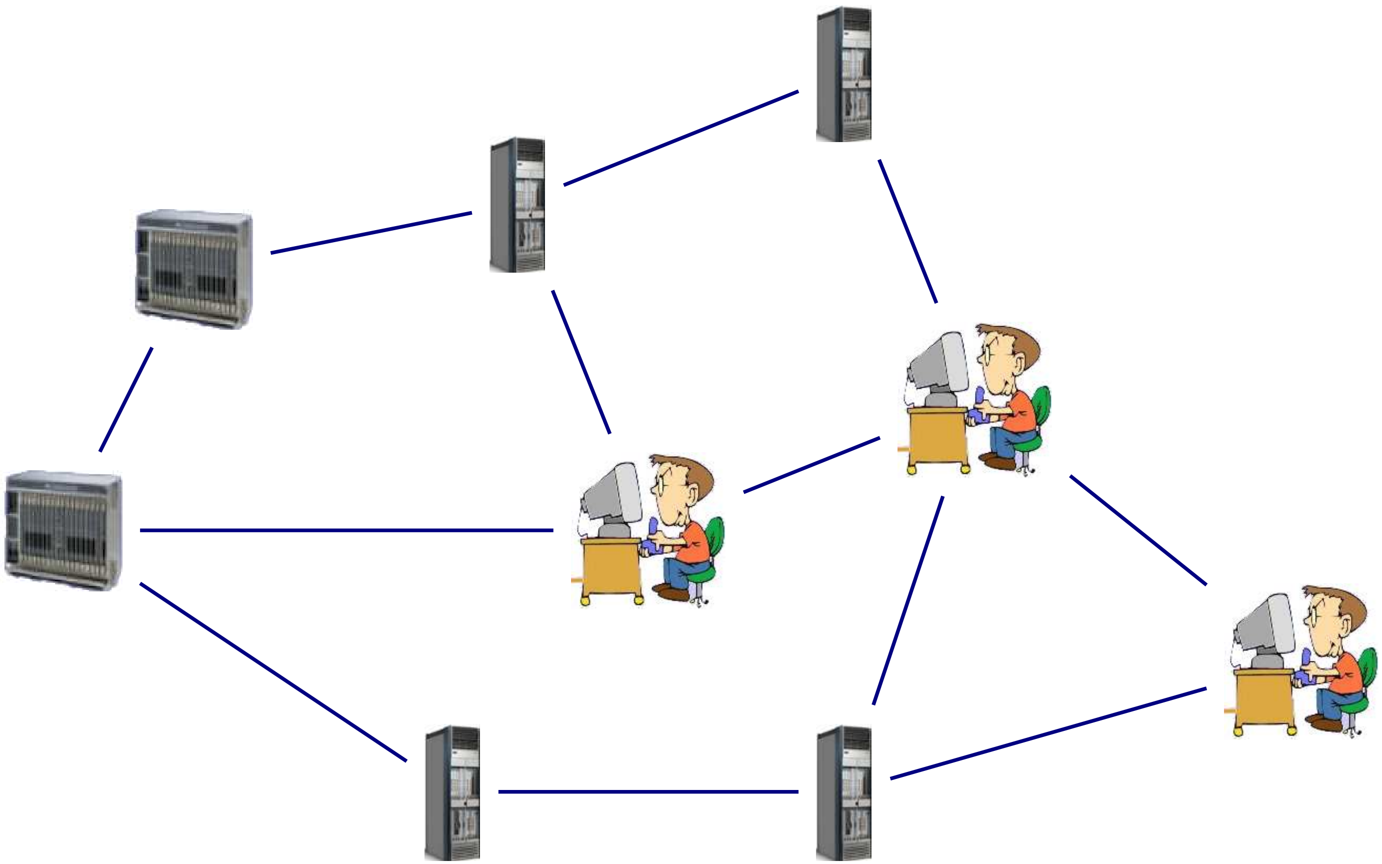

Example: data flow

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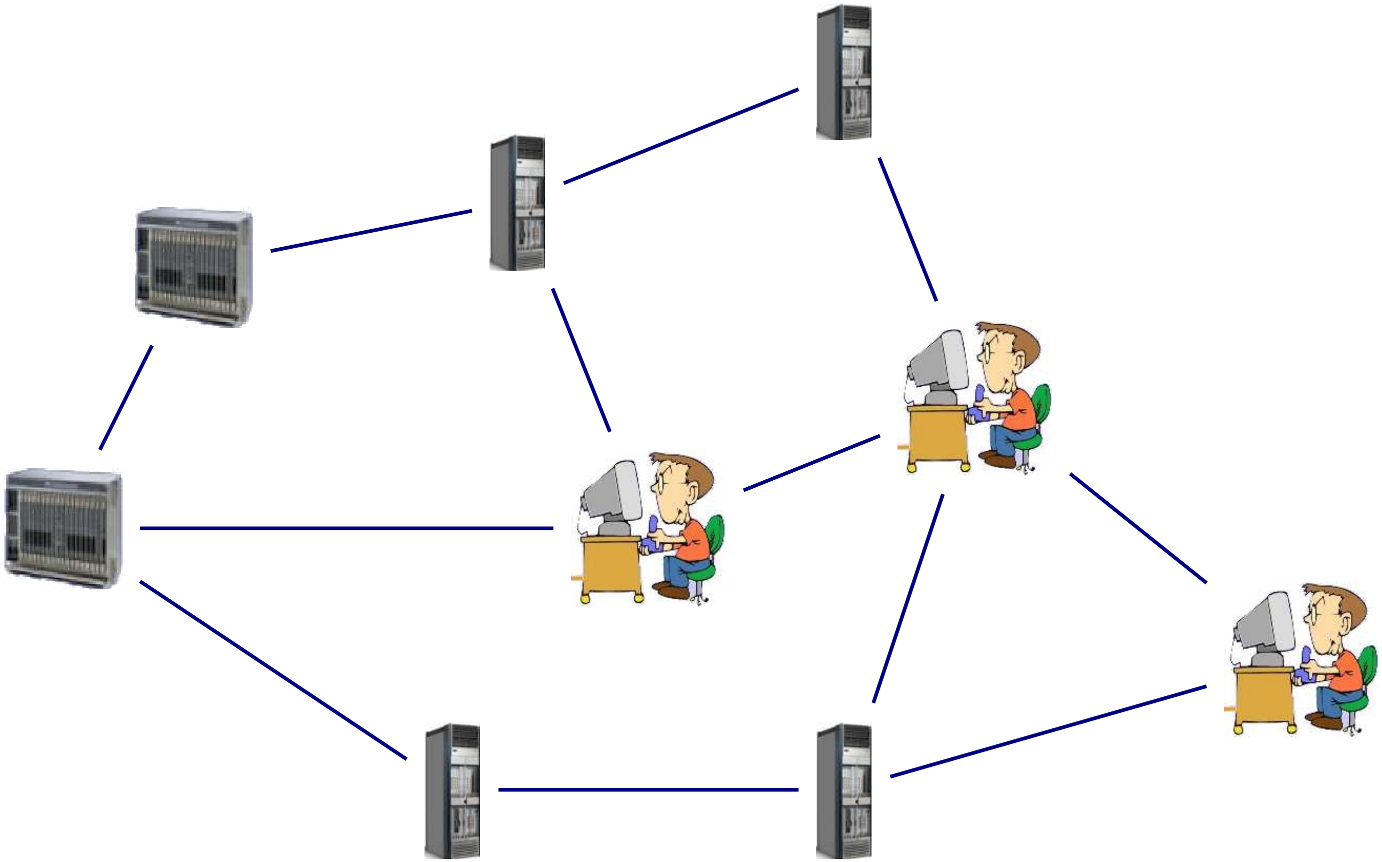

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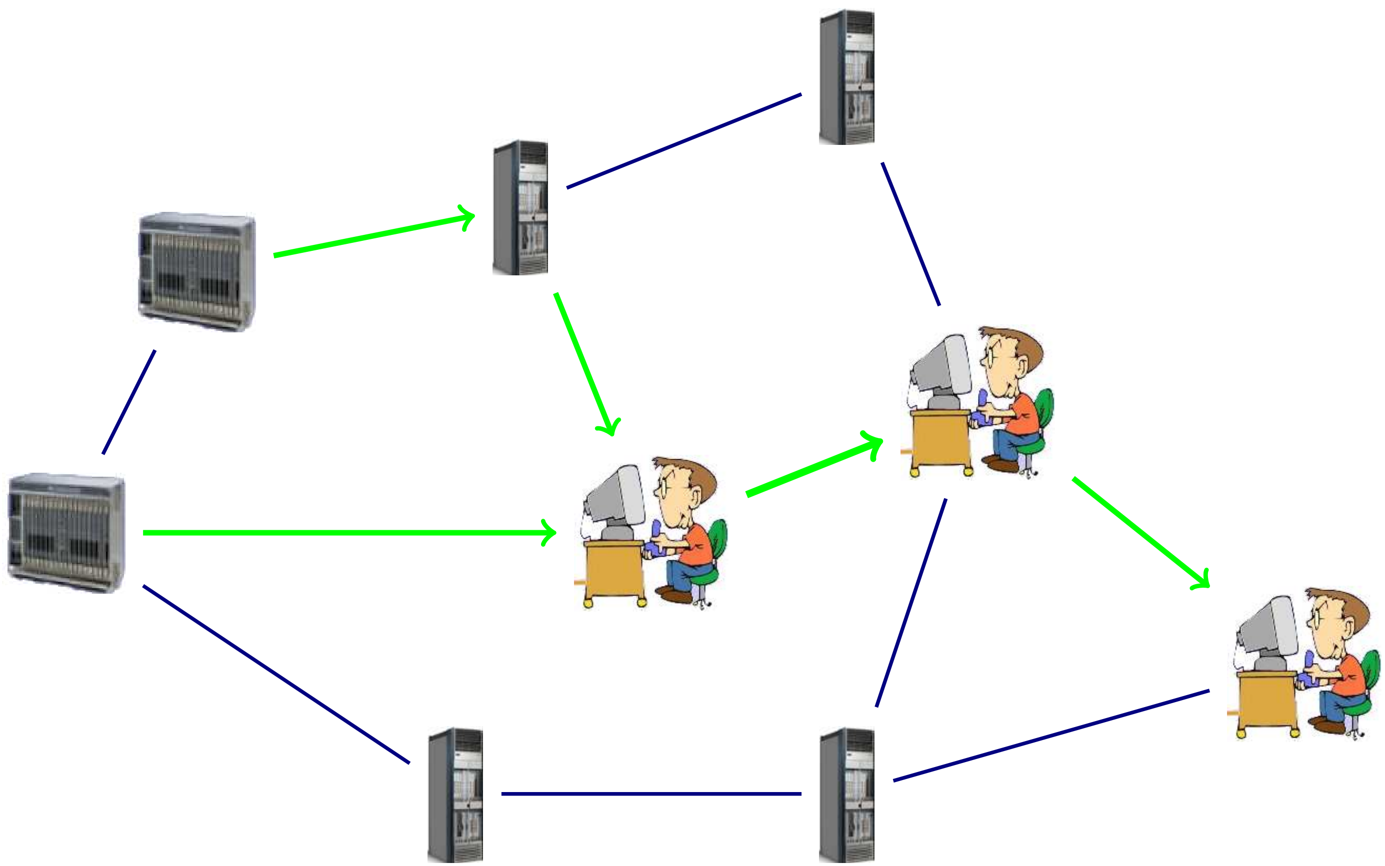

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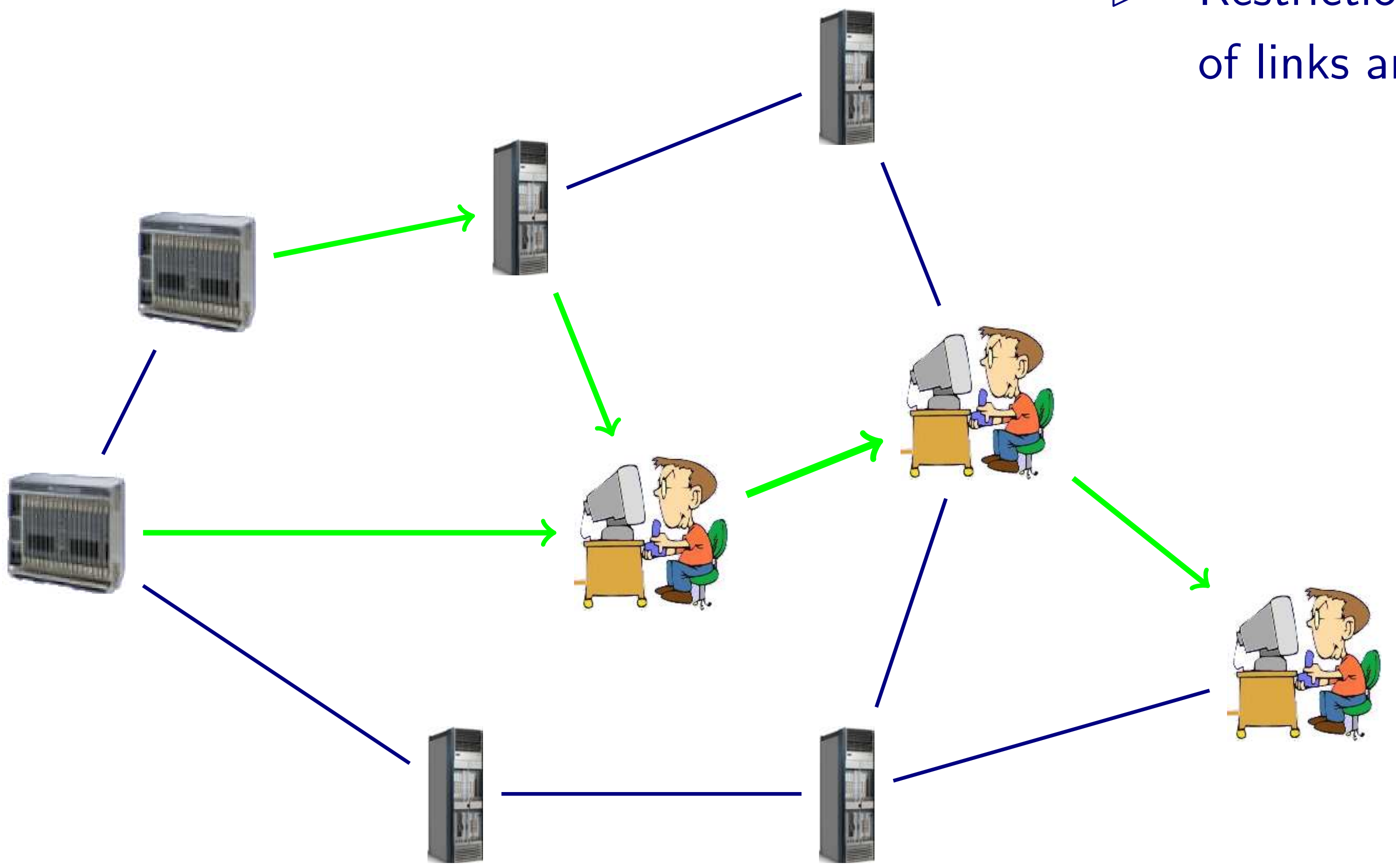

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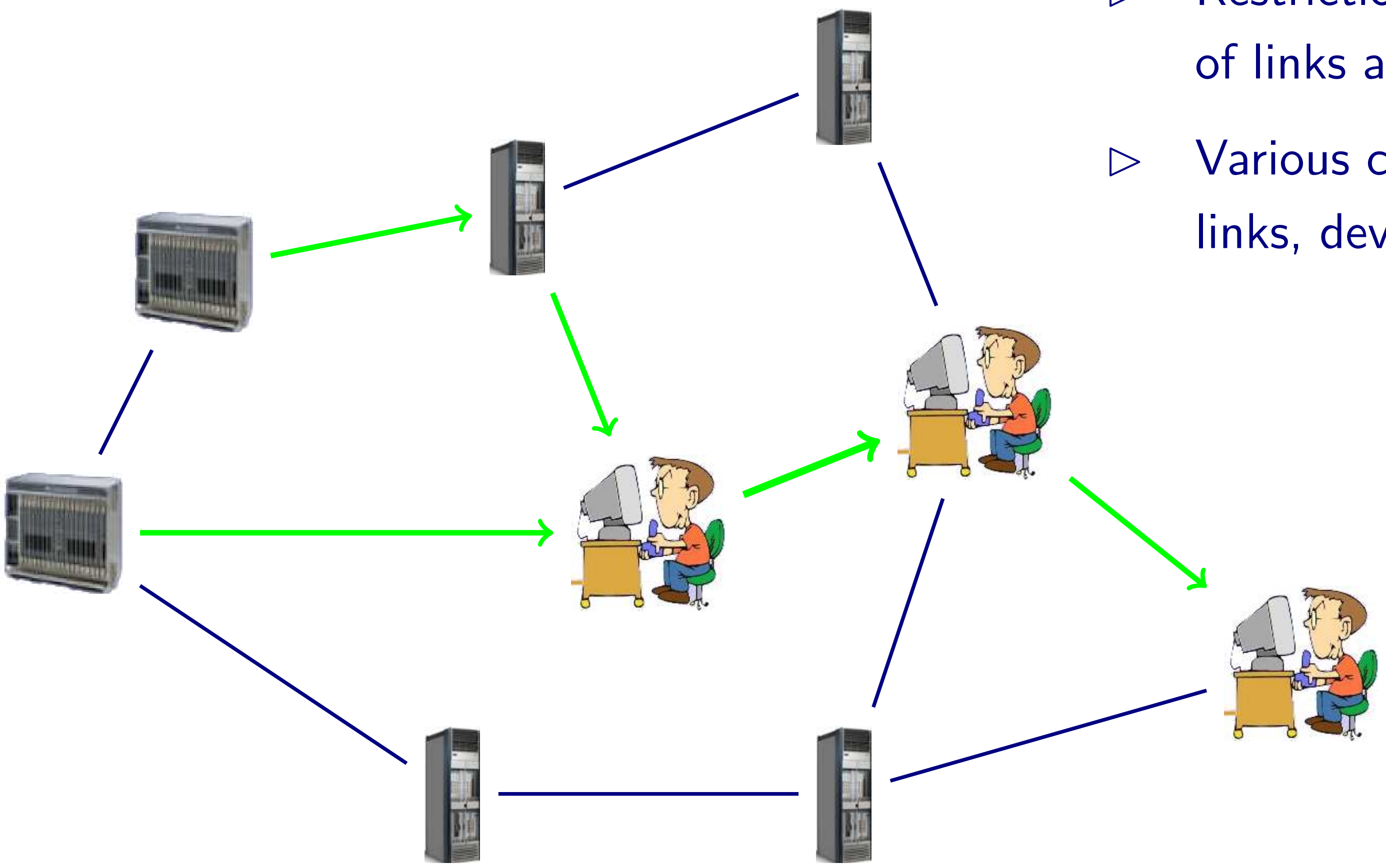

- ▷ Restrictions given by capacities of links and devices
- ▷ Various costs depending on links, devices and data volume

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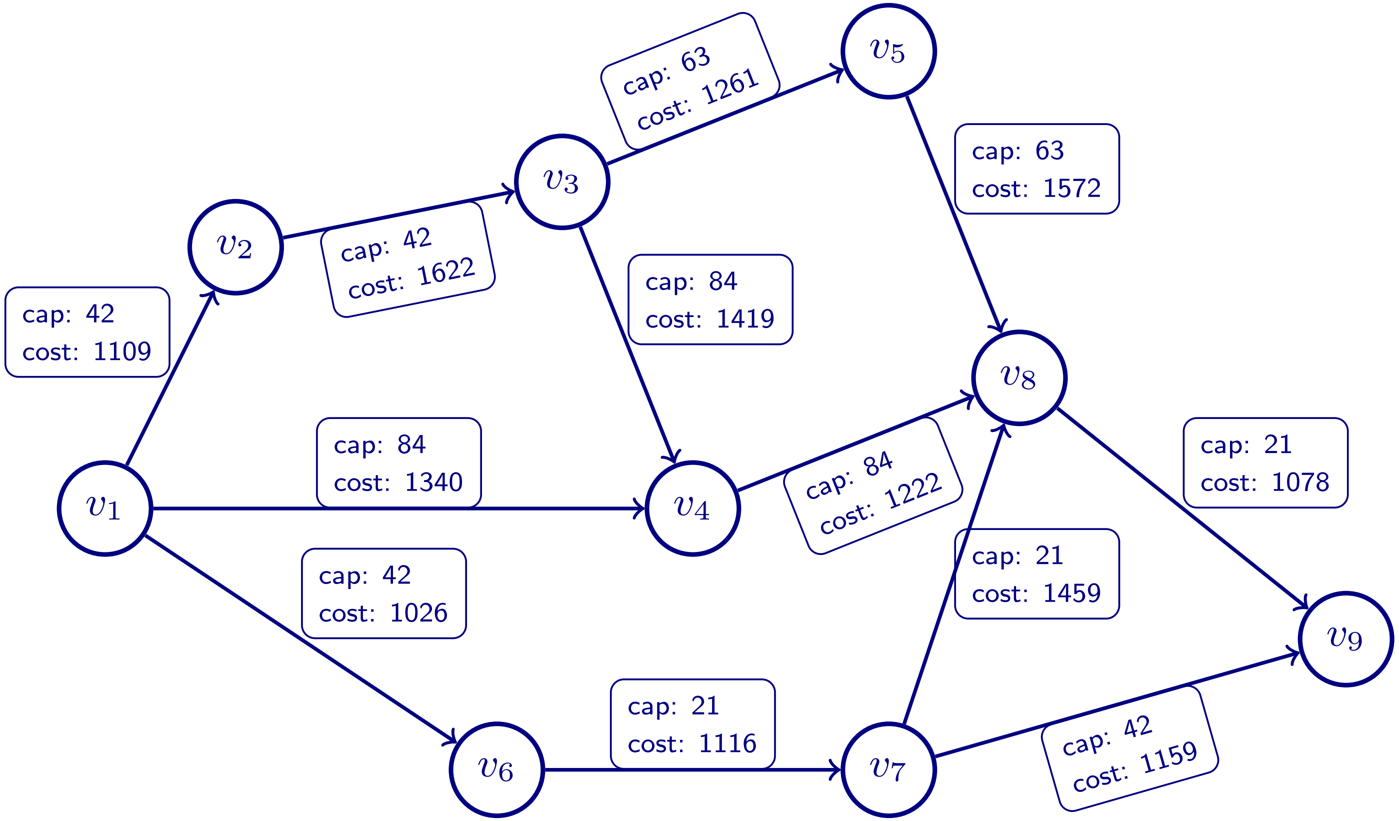

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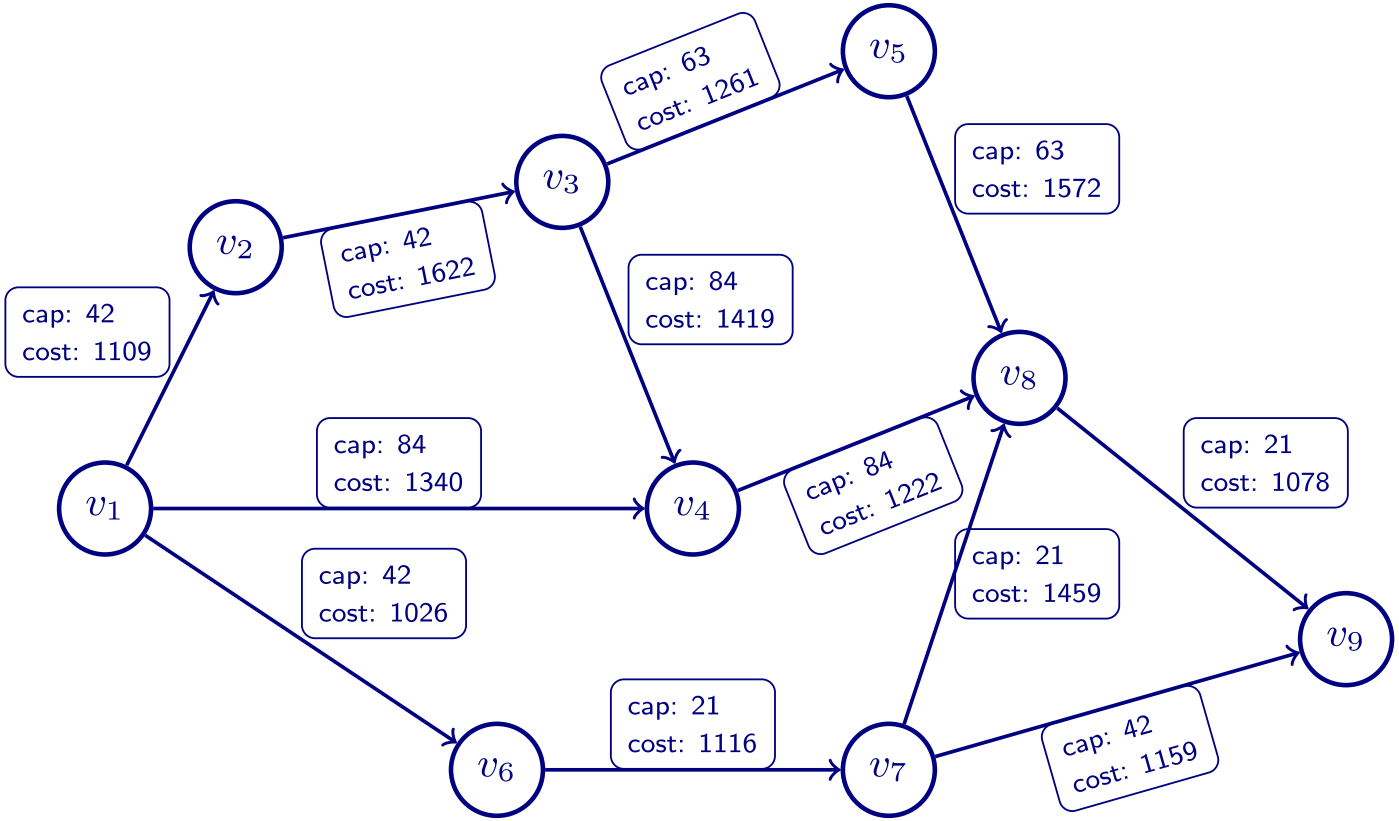

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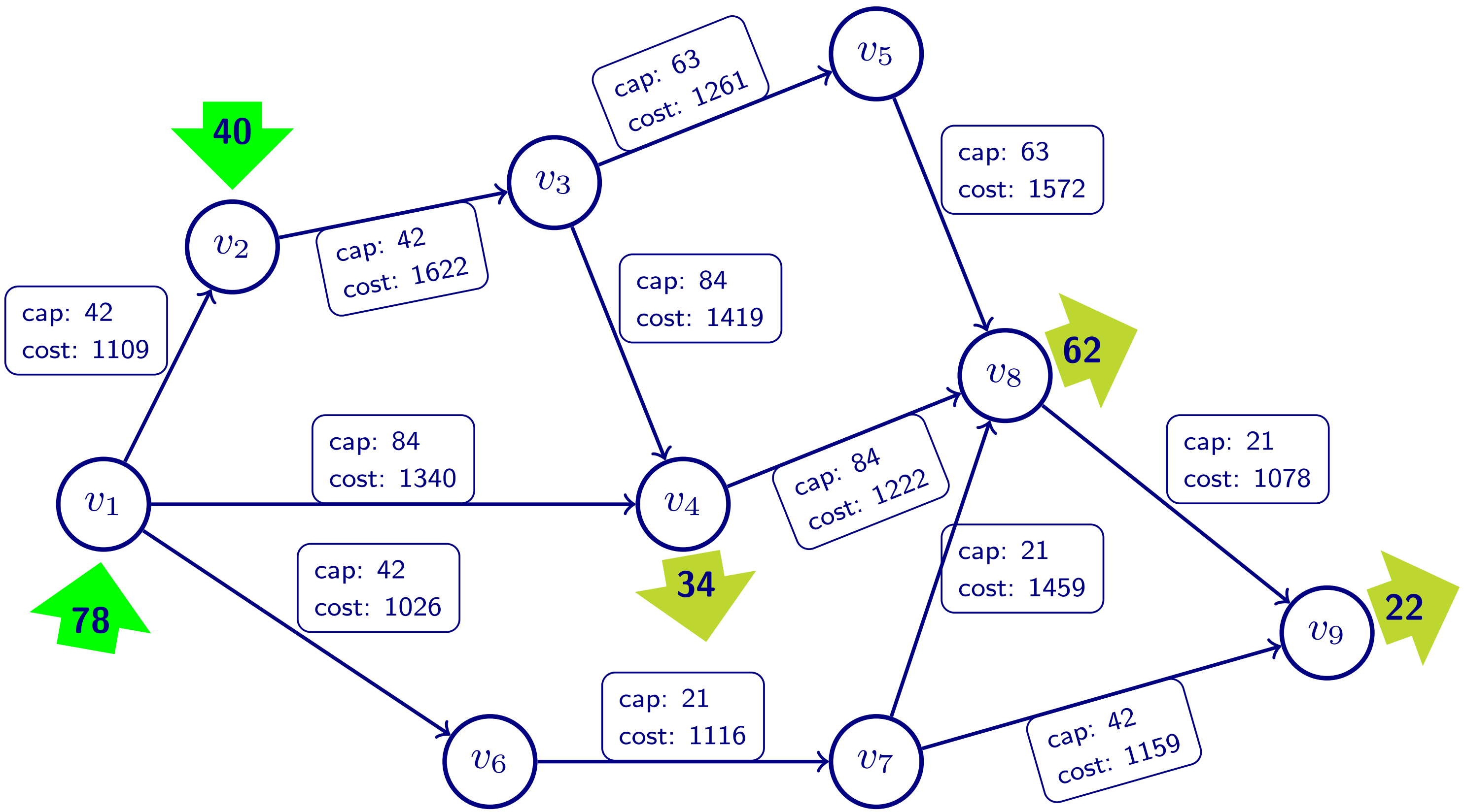

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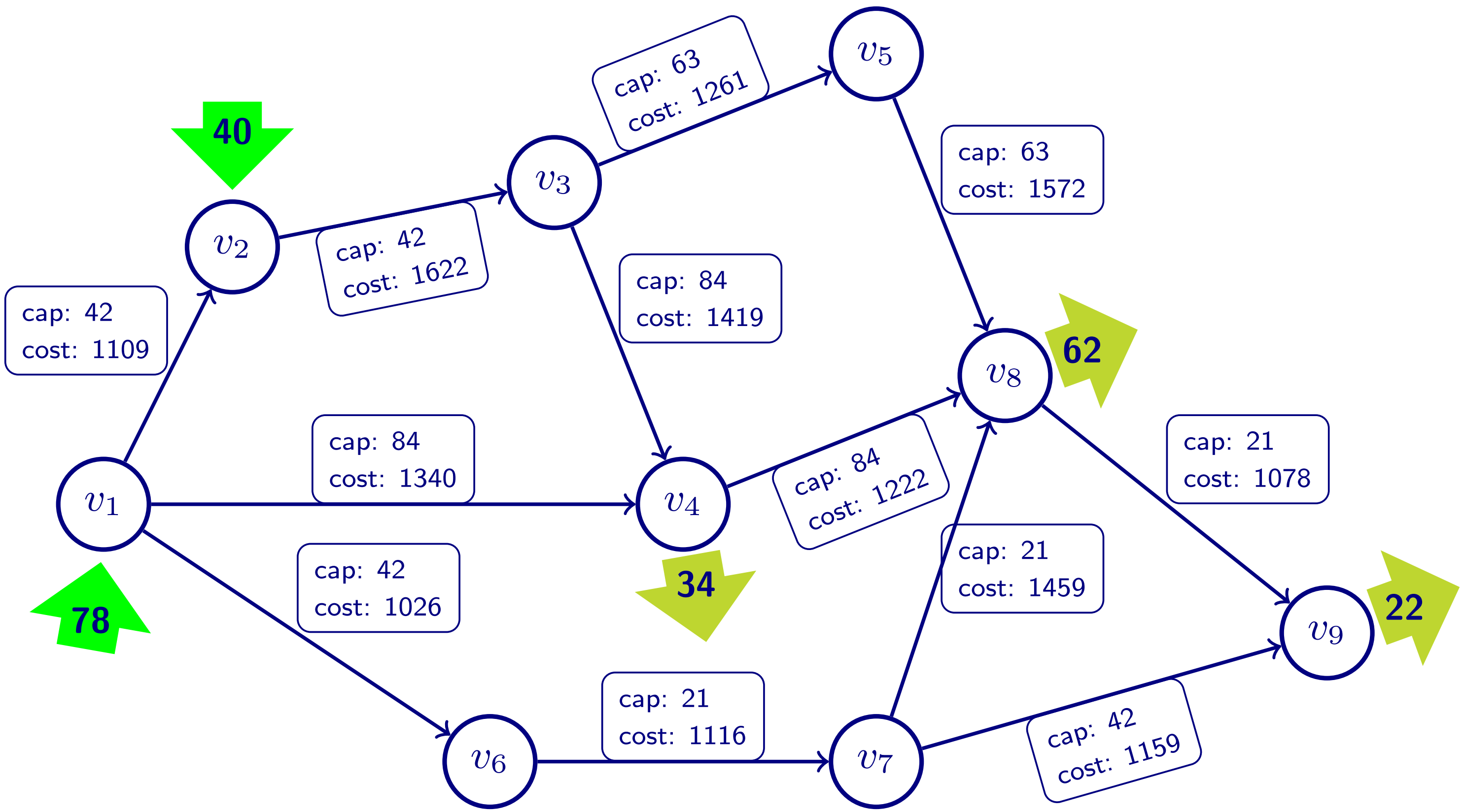

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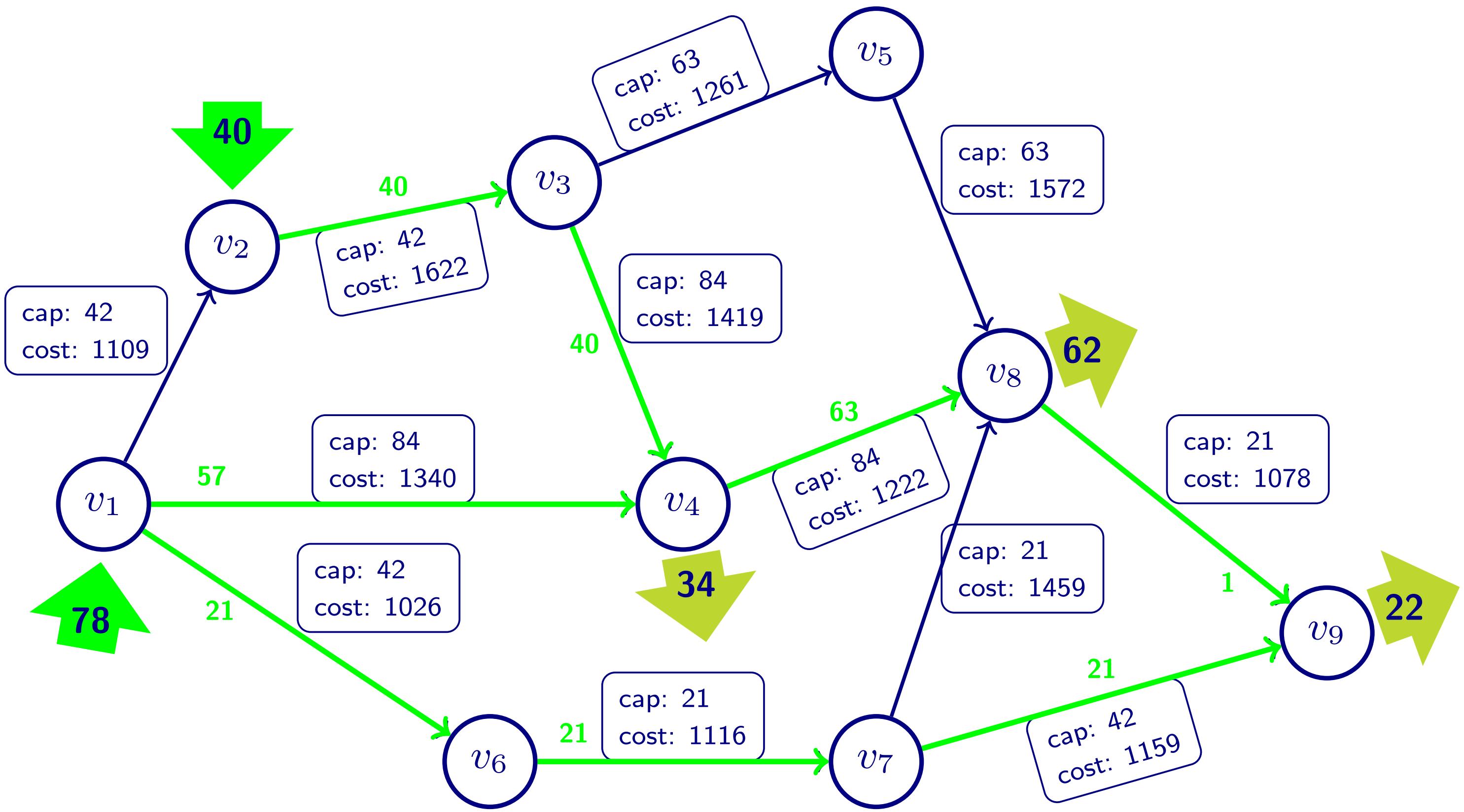

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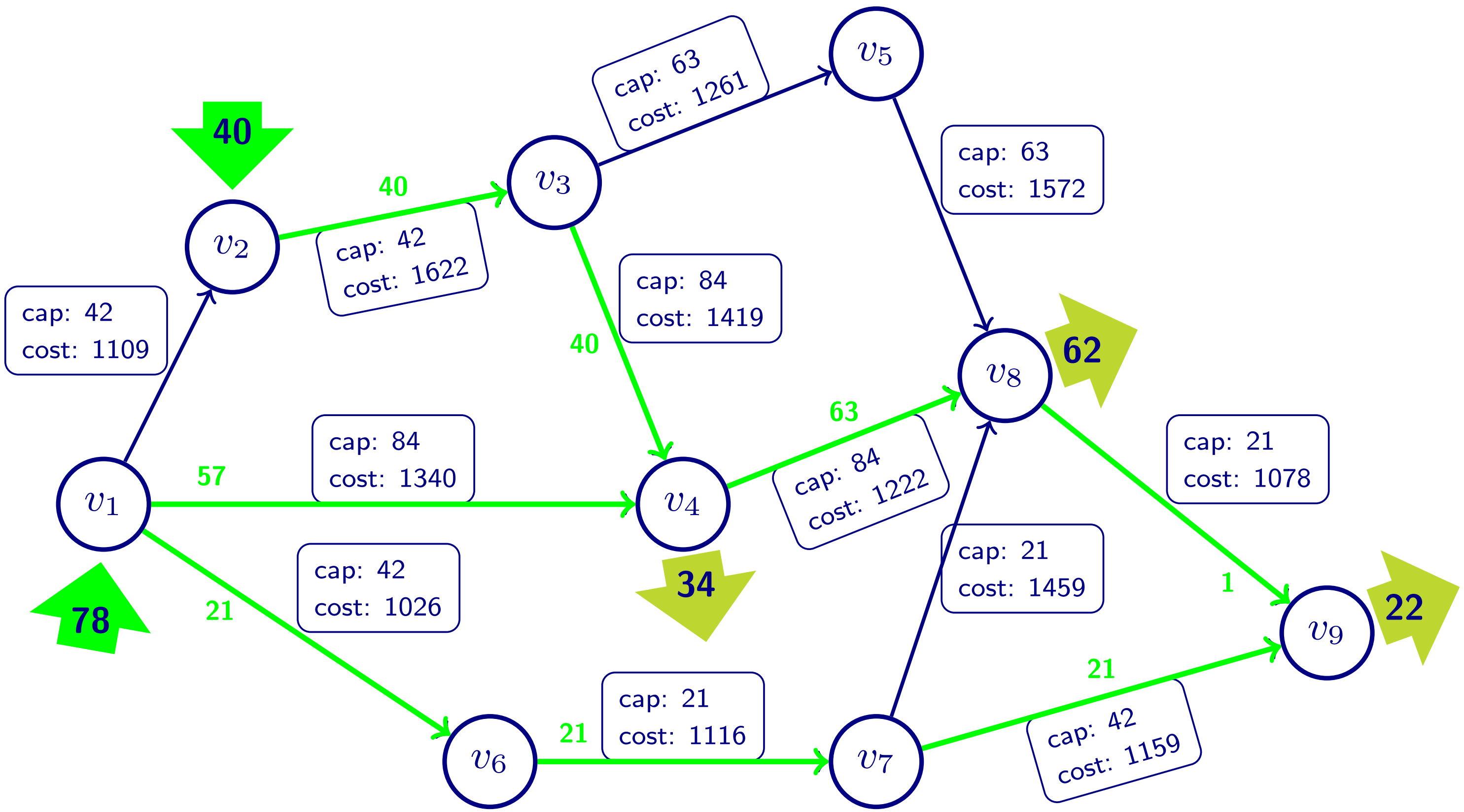
- ...compute a flow through the network satisfying the demand, respecting the capacities, with minimal total cost



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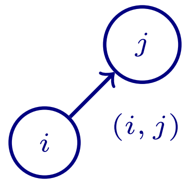

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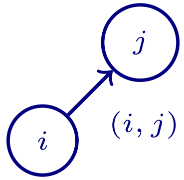

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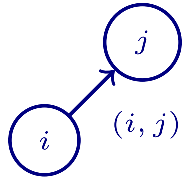

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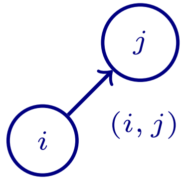

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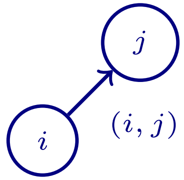

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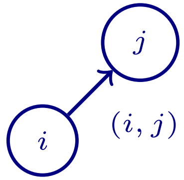

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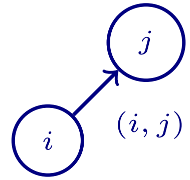

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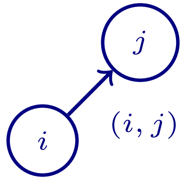

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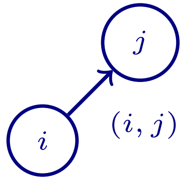

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$c_{(i,j)}$ ➡ cost per flow unit on arc (i, j)

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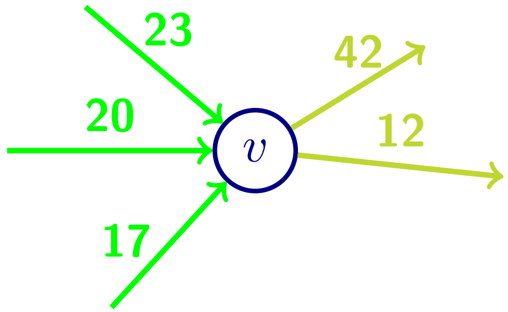

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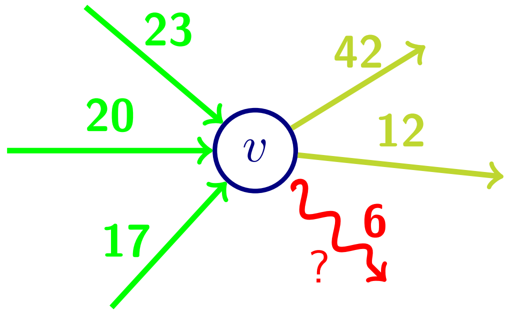

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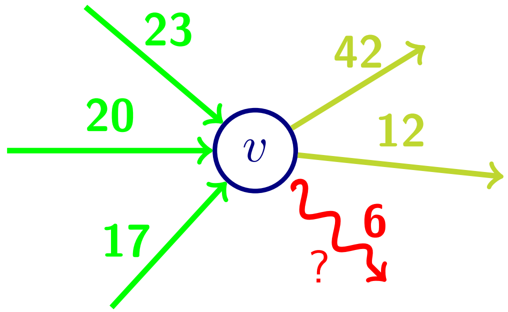

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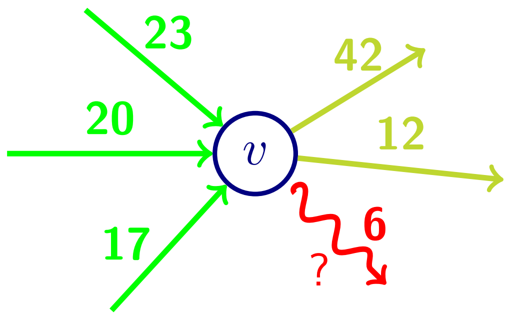

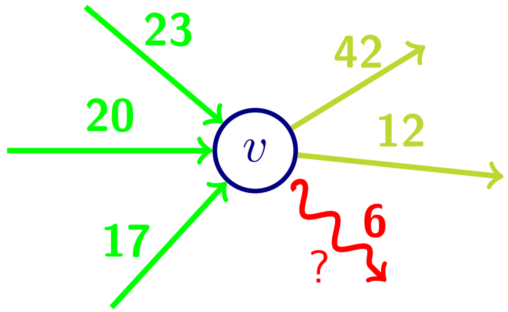

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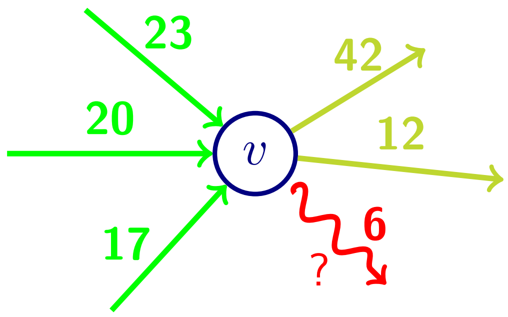

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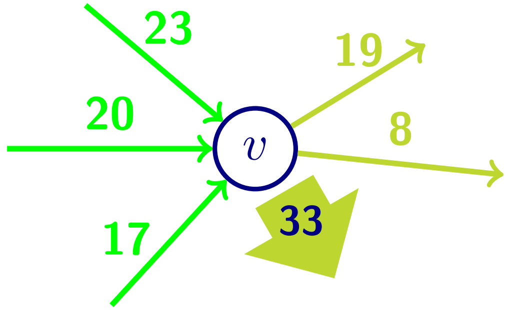

Flow conservation constraints

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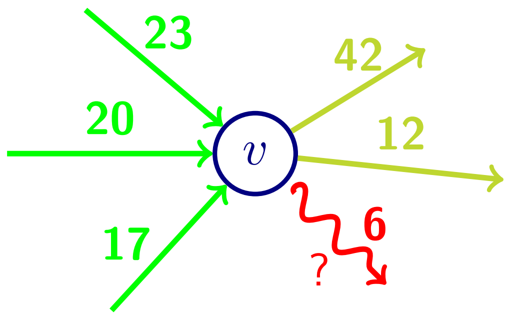

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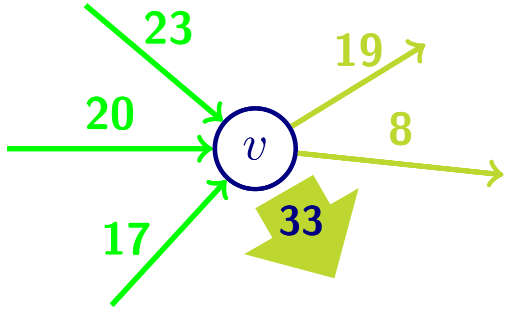

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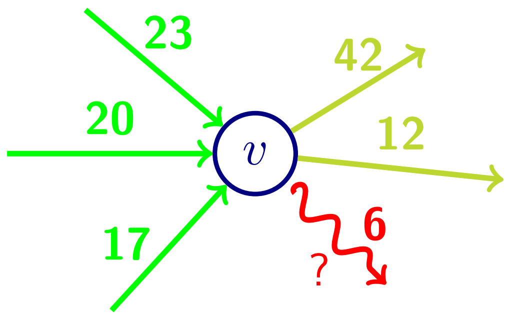

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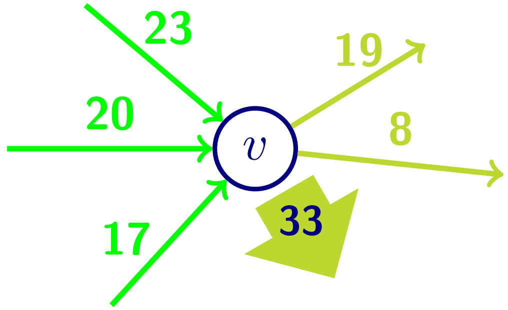

- In general:

$$\sum_{(v,j)\in A} f_{(v,j)} - \sum_{(i,v)\in A} f_{(i,v)} = \begin{cases} -d_v & \text{if } v \text{ is a demand node} \\ s_v & \text{if } v \text{ is a supply node} \\ 0 & \text{otherwise} \end{cases}$$

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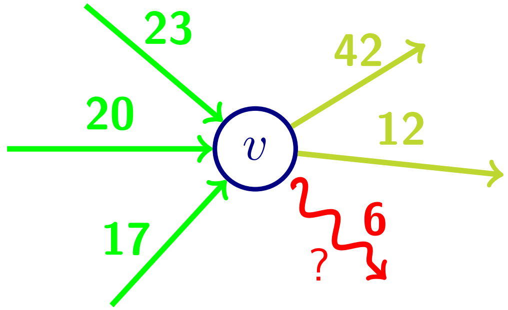

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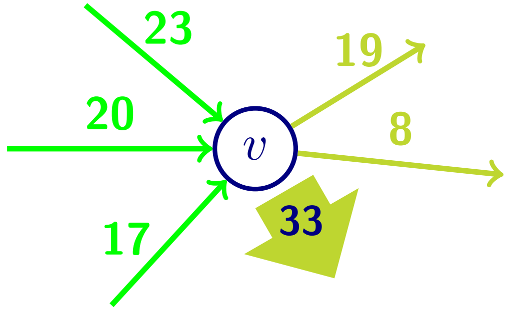

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P Link capacities: $u_{(i,j)} \geq 0$ for all arcs $(i,j) \in A$

Costs: $c_{(i,j)}$ for all arcs $(i,j) \in A$

Demand of demand nodes: $d_v \geq 0$

Supply of supply nodes: $s_v \geq 0$

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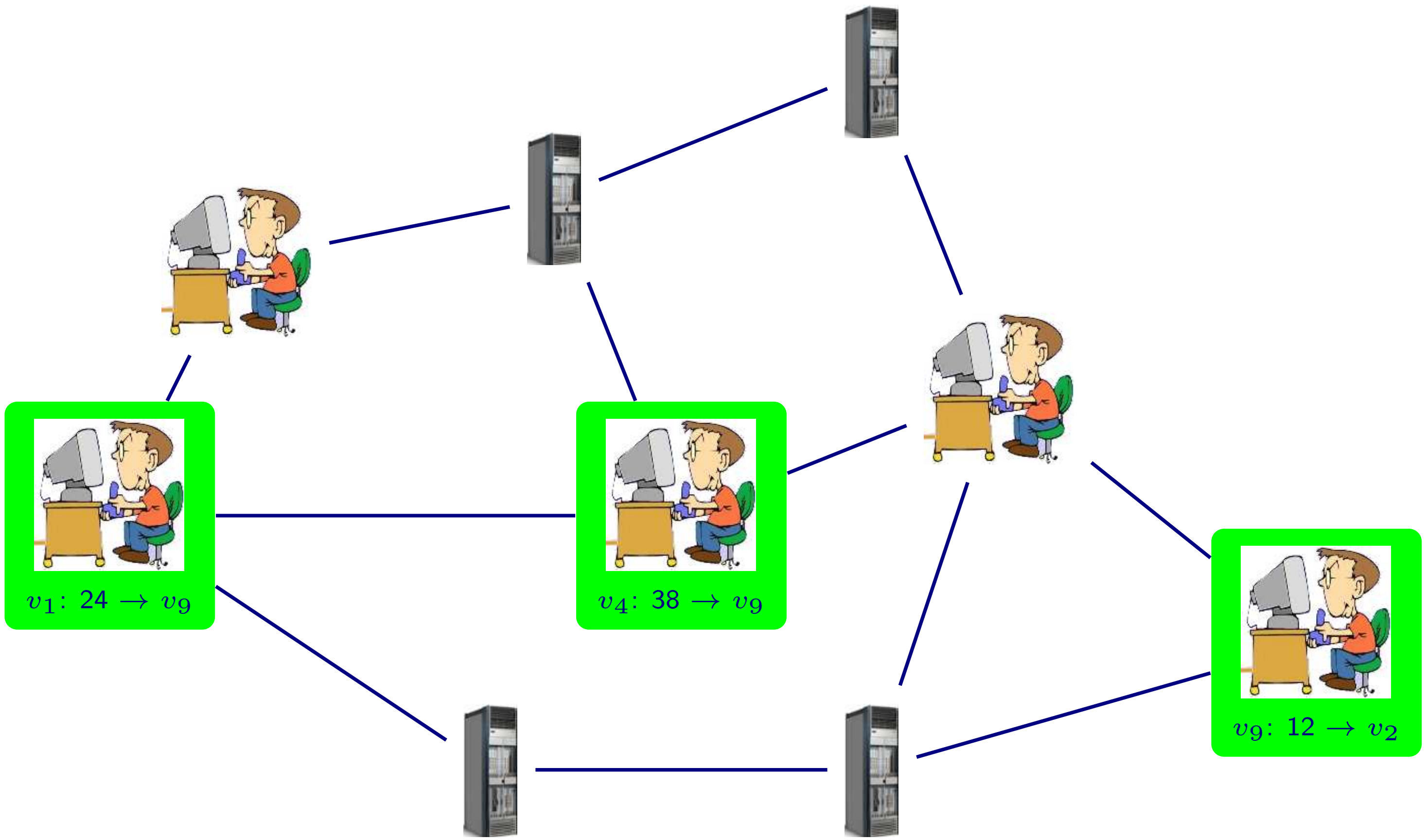

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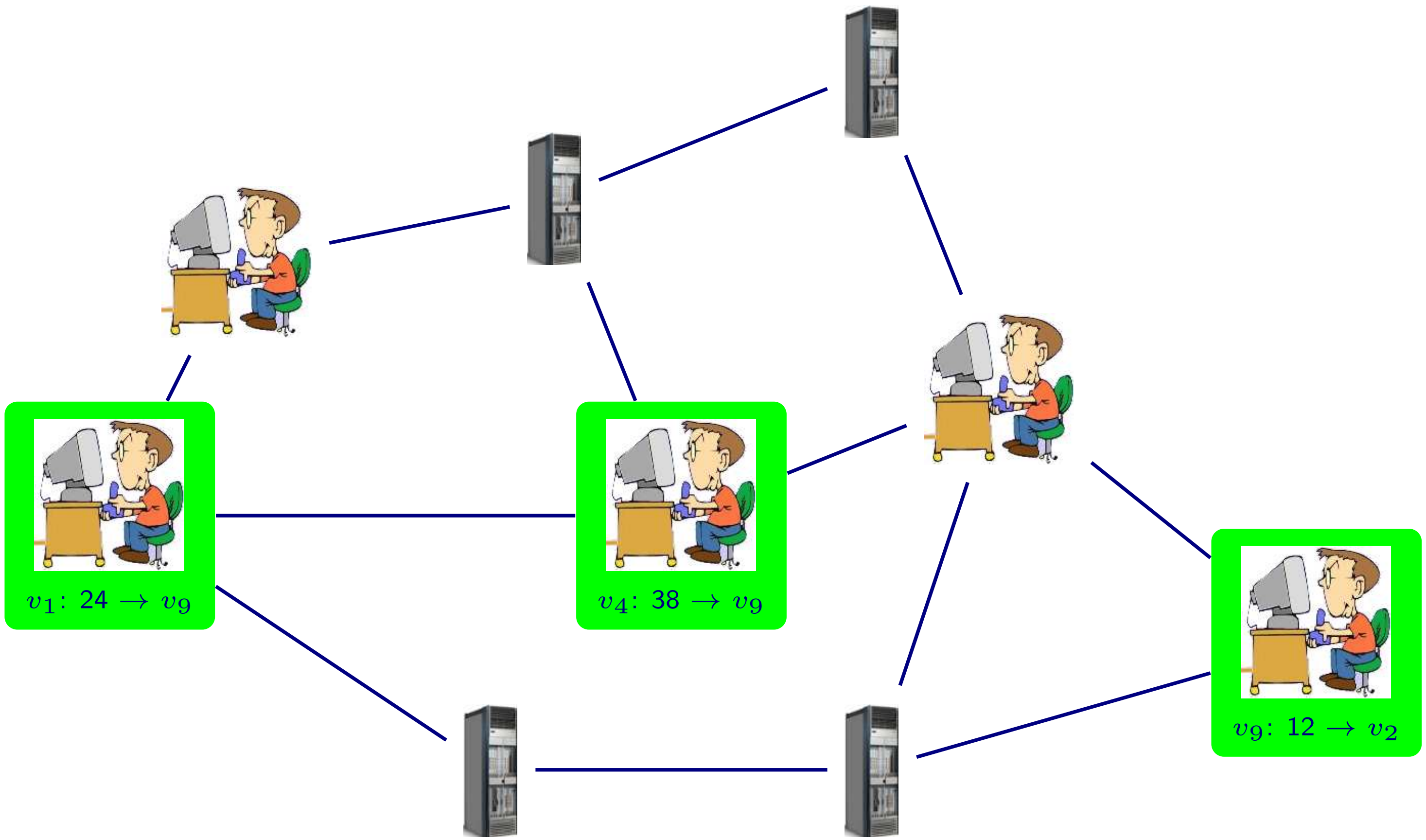

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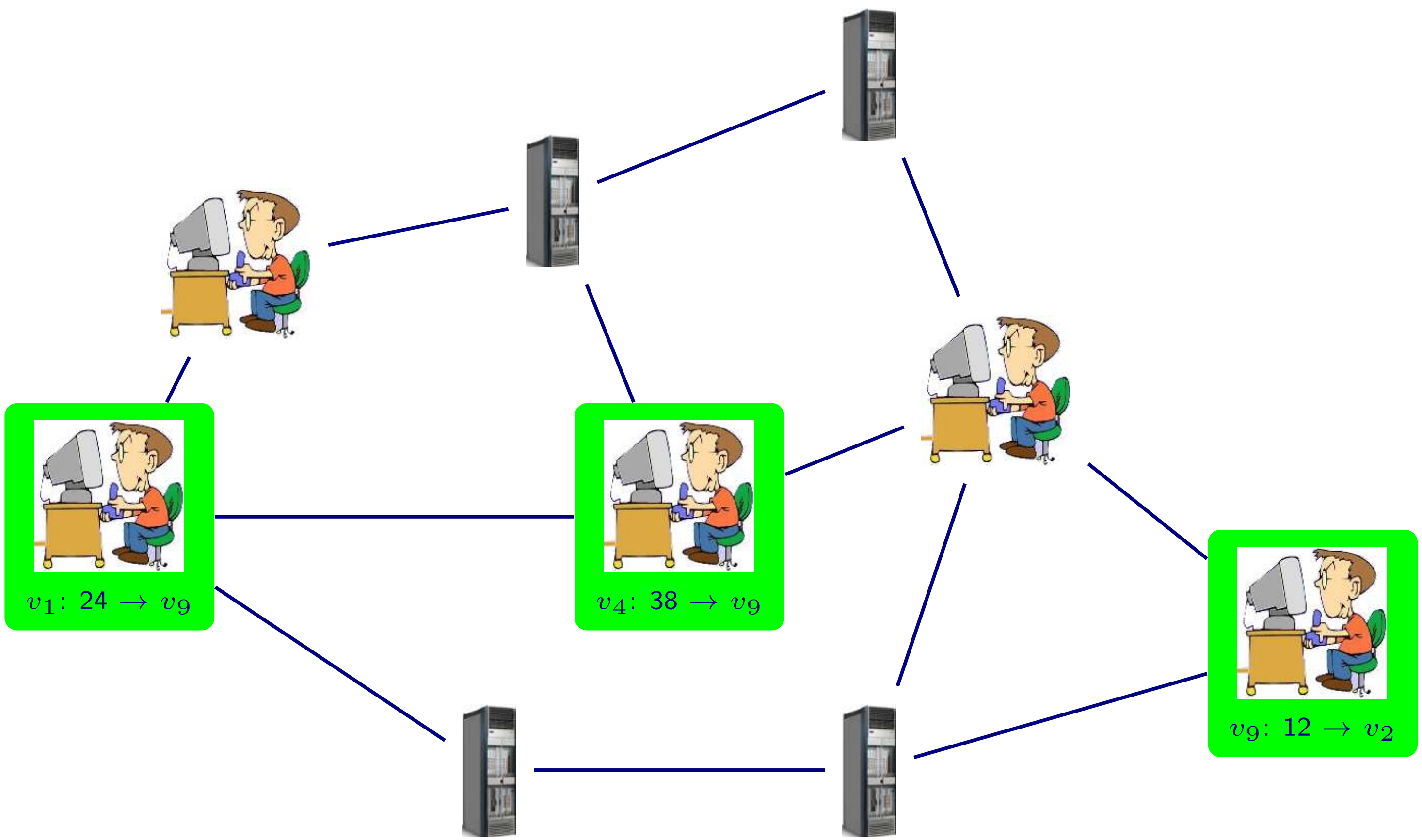

Example: IP routing

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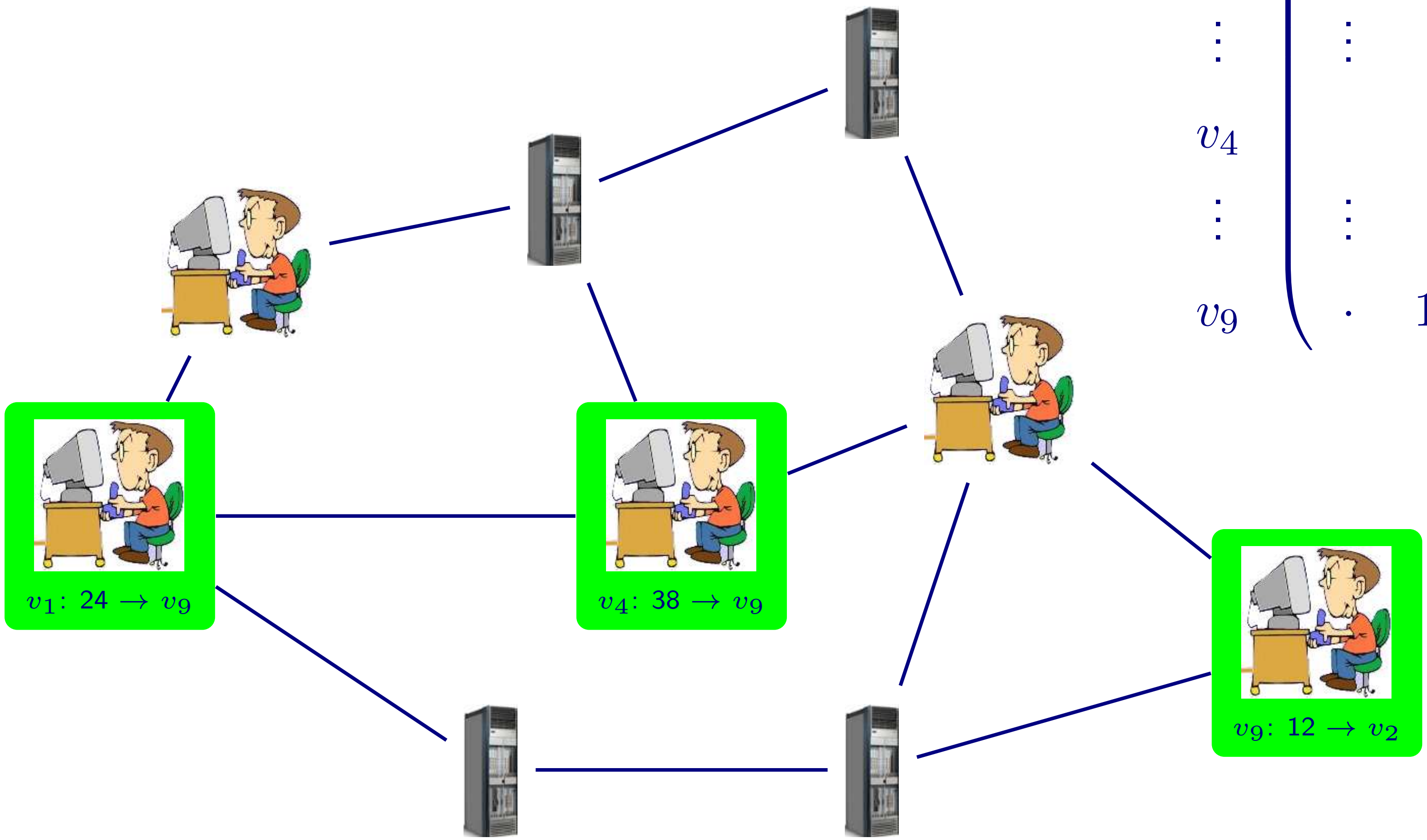
  - ➡ Separate (disaggregated) flow for every pair of nodes
  - ➡ Demand matrix with entries $d_{u,v} \geq 0$

$$
\begin{array}{c} \\ v_1 \\ \vdots \\ v_4 \\ \vdots \\ v_9 \end{array}
\begin{array}{c}
\begin{array}{ccccc} v_1 & v_2 & \cdots & & v_9 \end{array} \\
\begin{pmatrix}
0 & \cdots & & \cdots & 24 \\
\vdots & 0 & & & \vdots \\
 & & & & 38 \\
\vdots & & & \ddots & \vdots \\
\cdot & 12 & & \cdots & 0
\end{pmatrix}
\end{array}
$$



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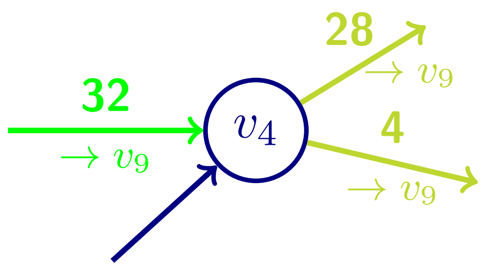

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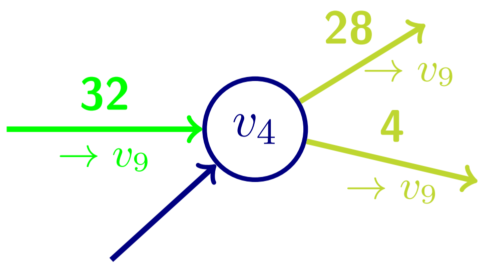

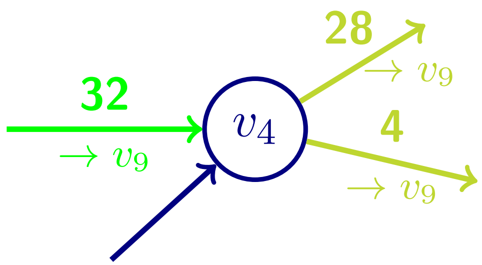

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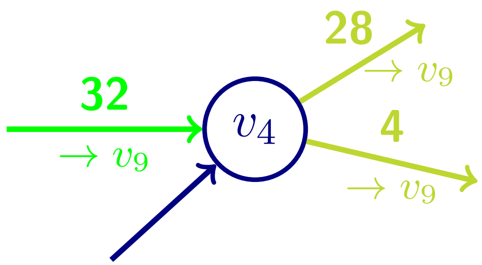
- Idea: replace flow variables $f_{(i,j),u\to v}$ with binary decision variables:

$y_{(i,j),u\to v} \in \{0,1\}$ ➡ $y_{(i,j),u\to v} = 1 \Leftrightarrow$ complete flow from u to v uses arc (i,j)



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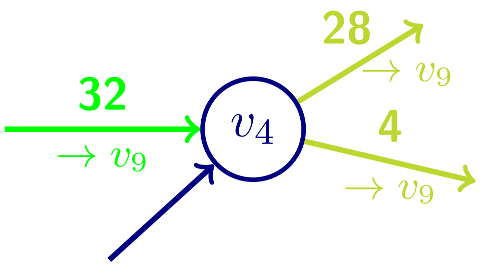

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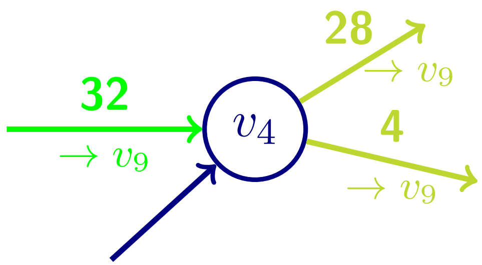

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- ▷ Similarly: k-splittable flow – allow splitting into at most/exactly/at least k parts at each node

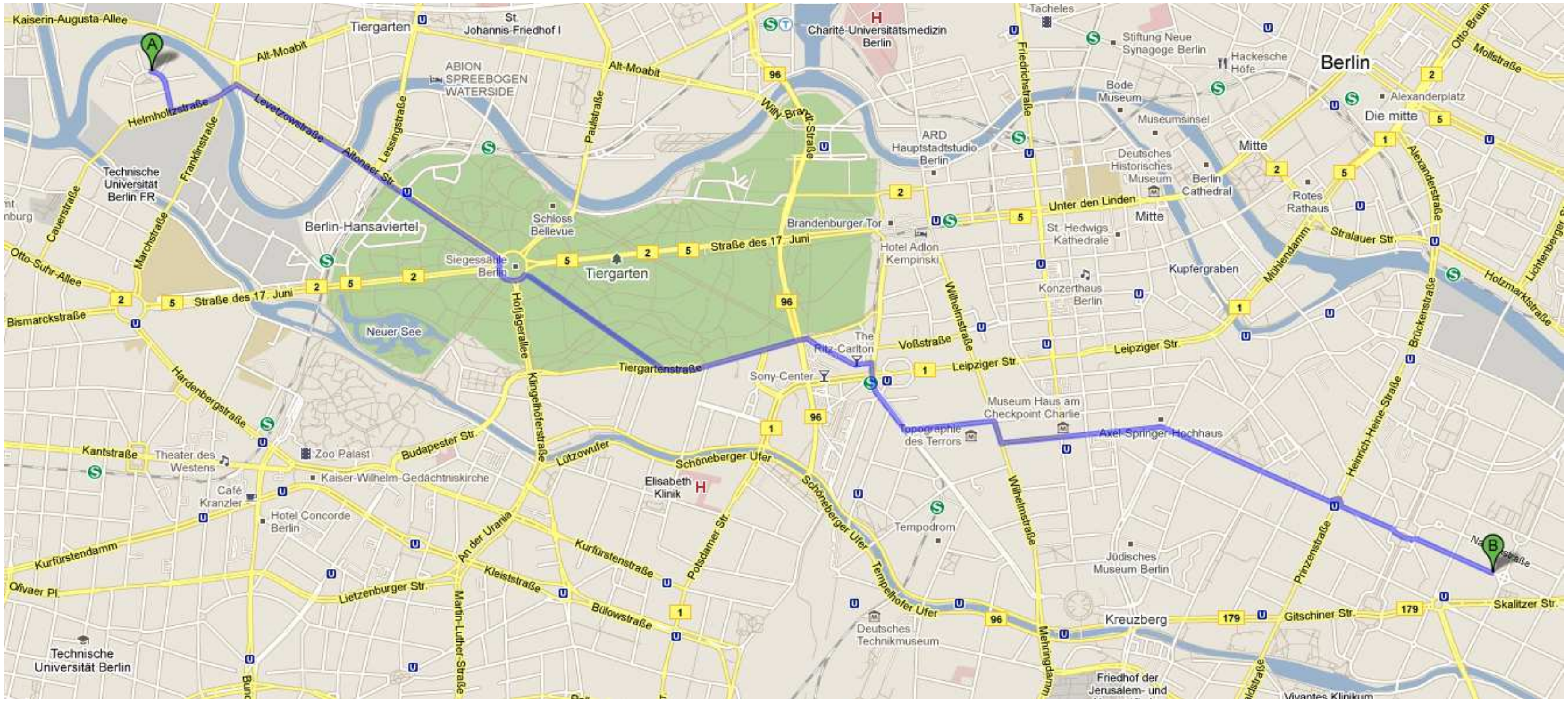
- Problem: find a shortest path from PTZ to Kreuzburger

$\rightsquigarrow$

Example: Shortest Path

- Problem: find a shortest path from PTZ to Kreuzburger

⇝



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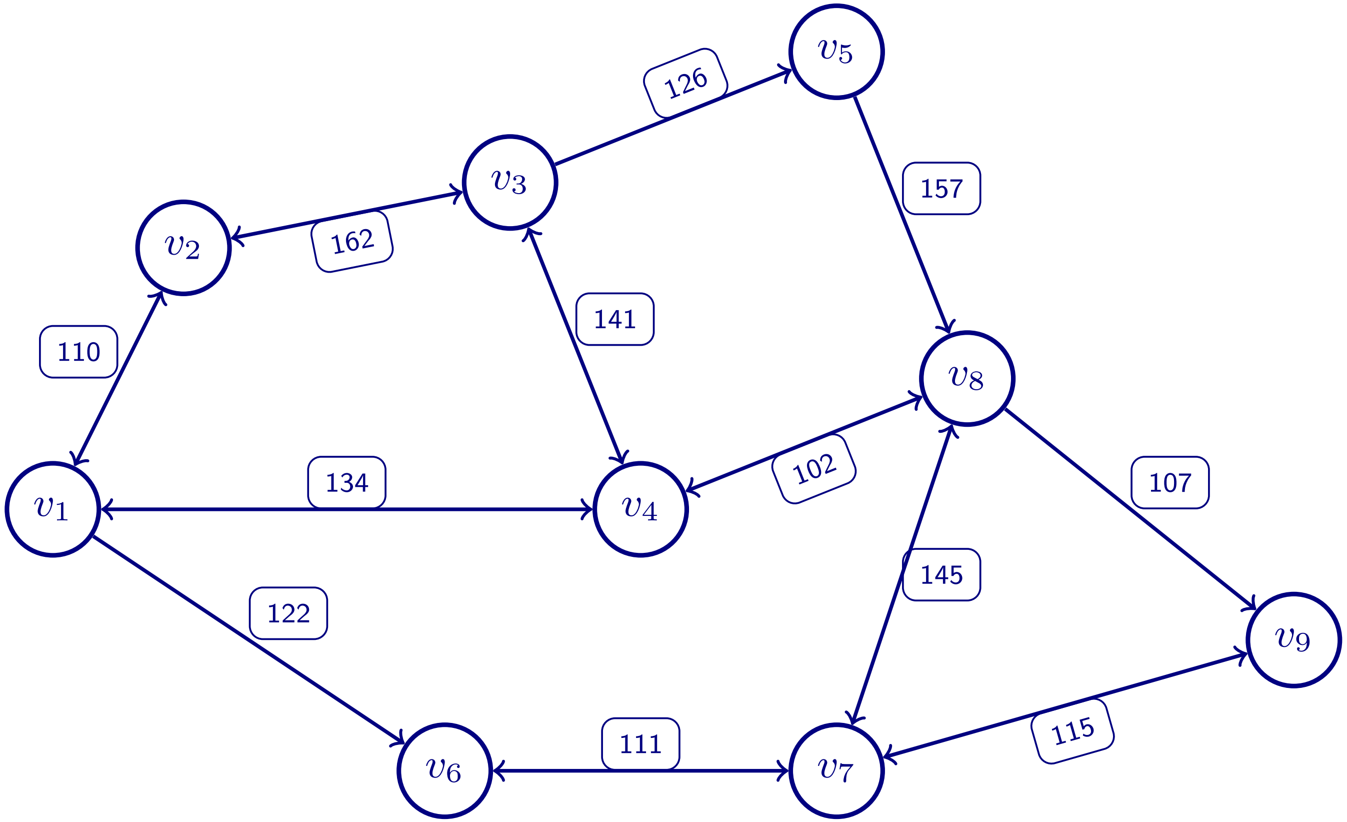

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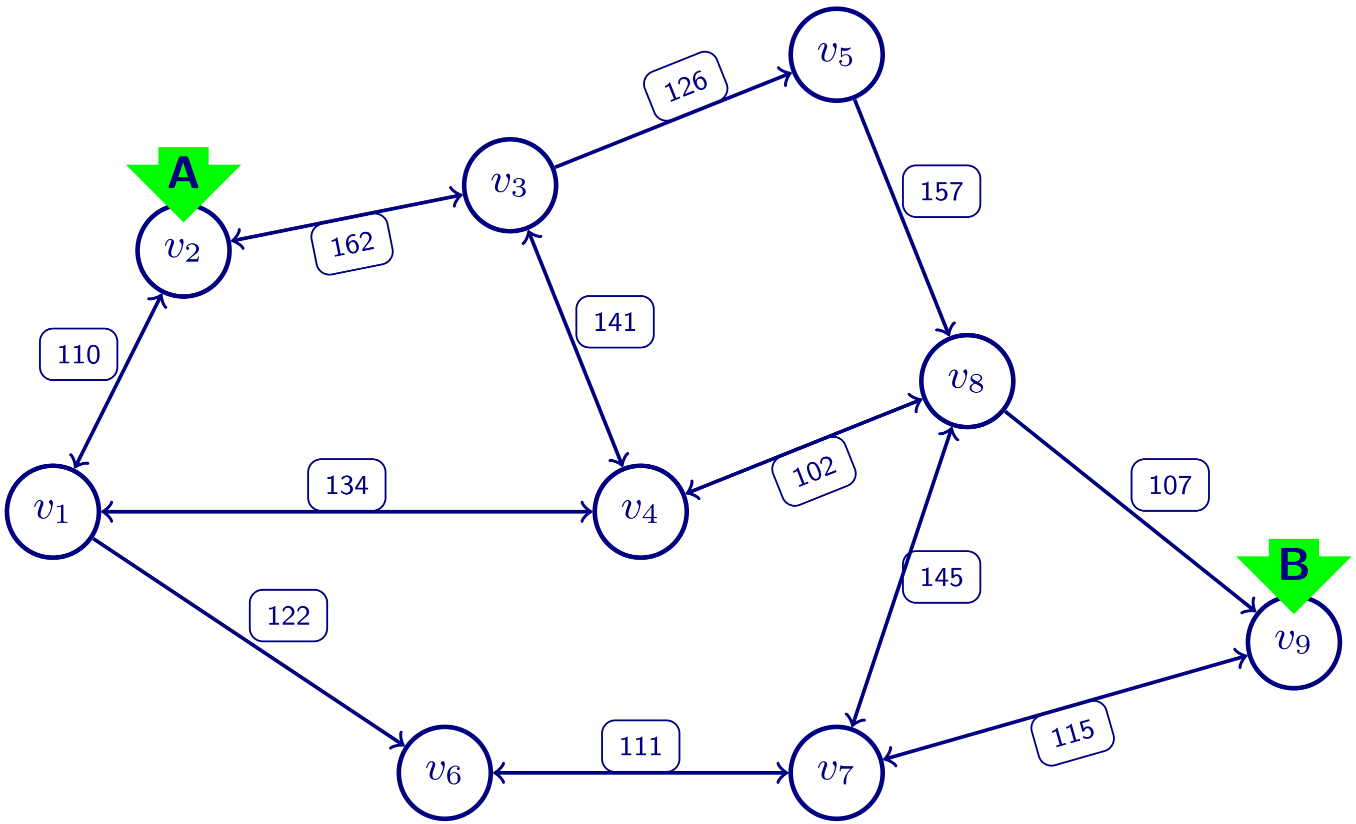

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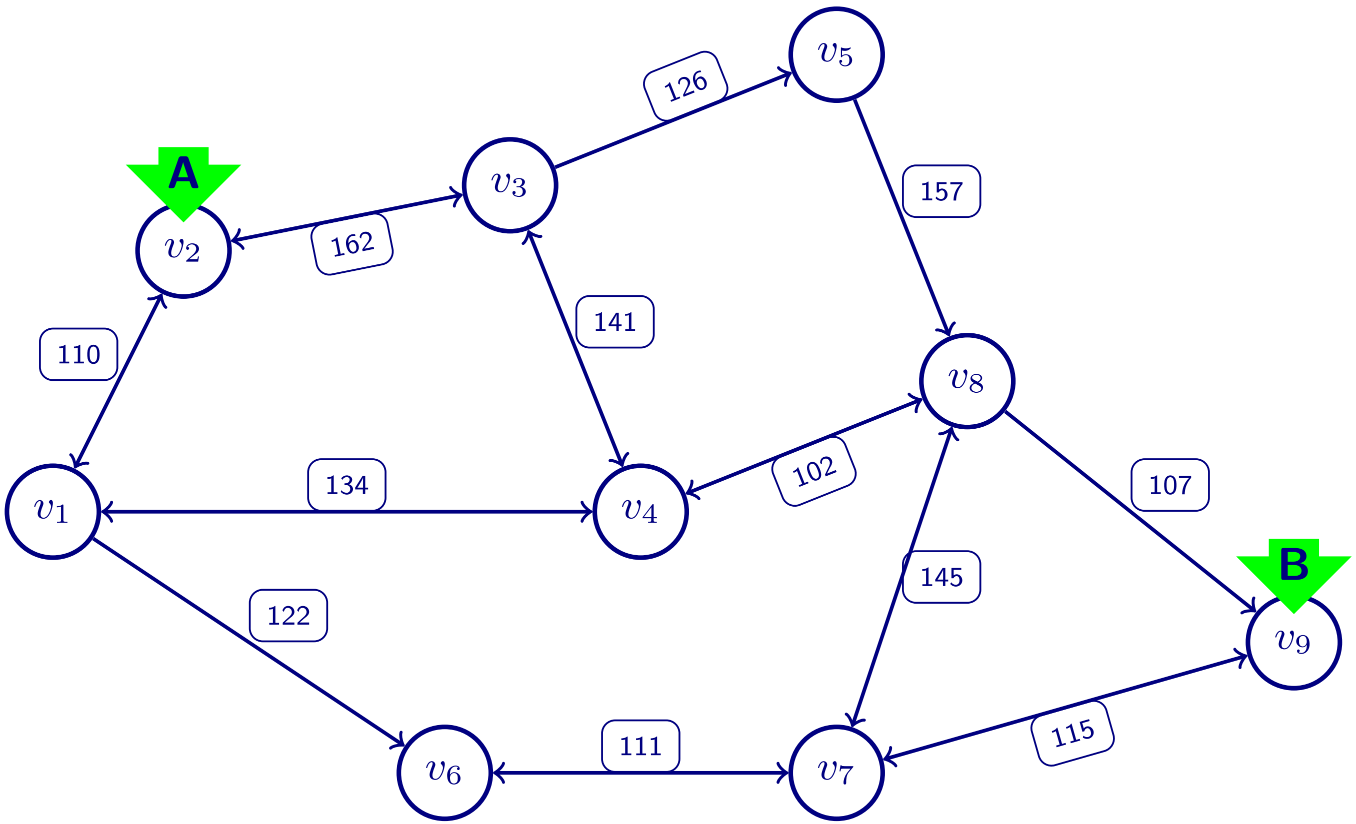

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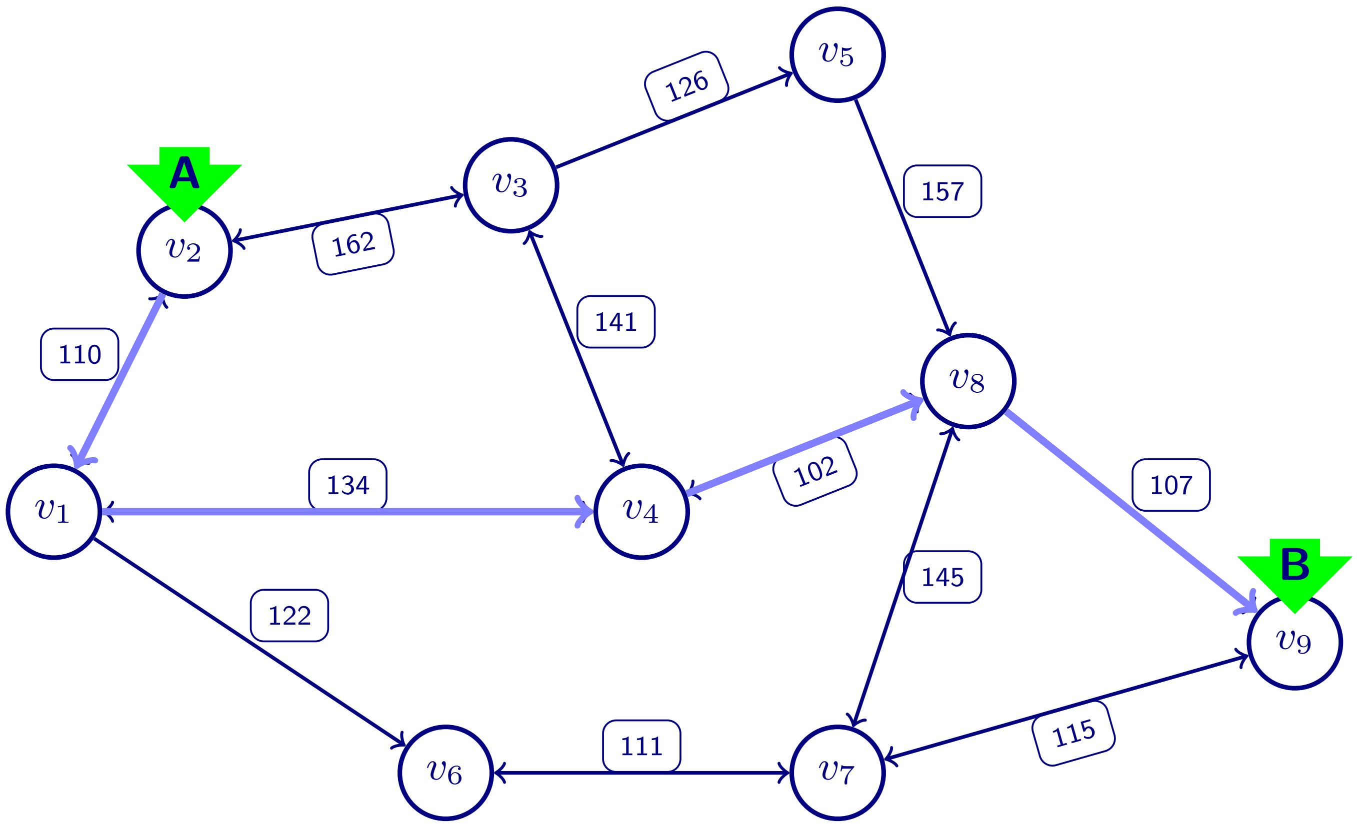

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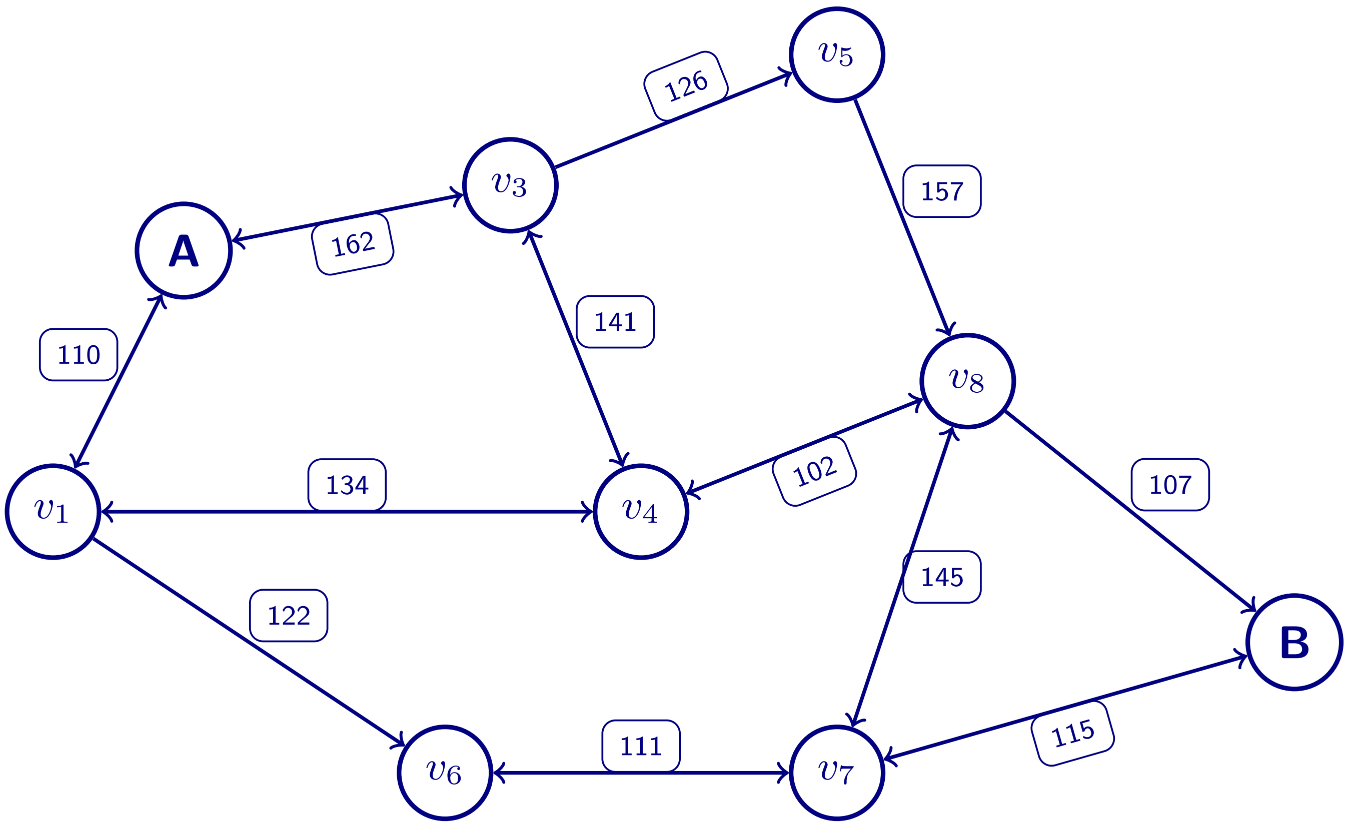

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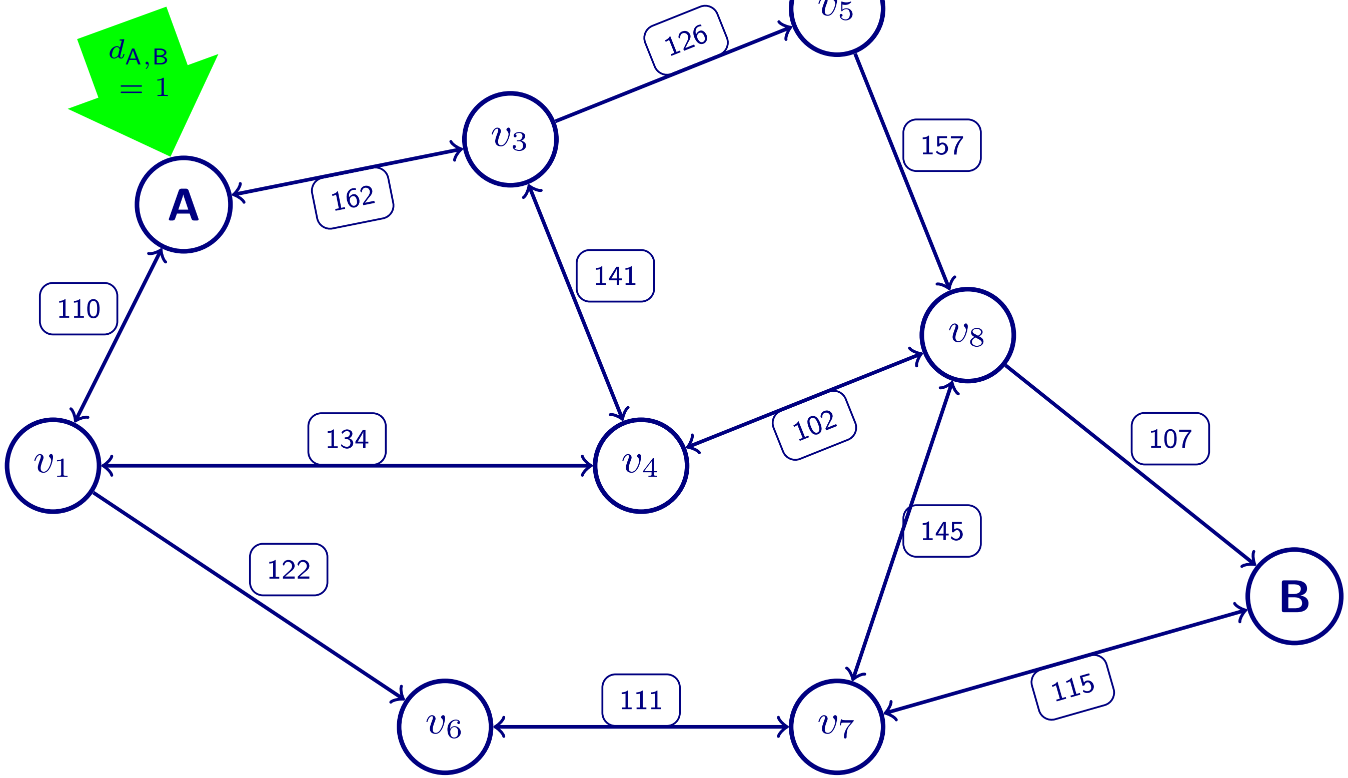

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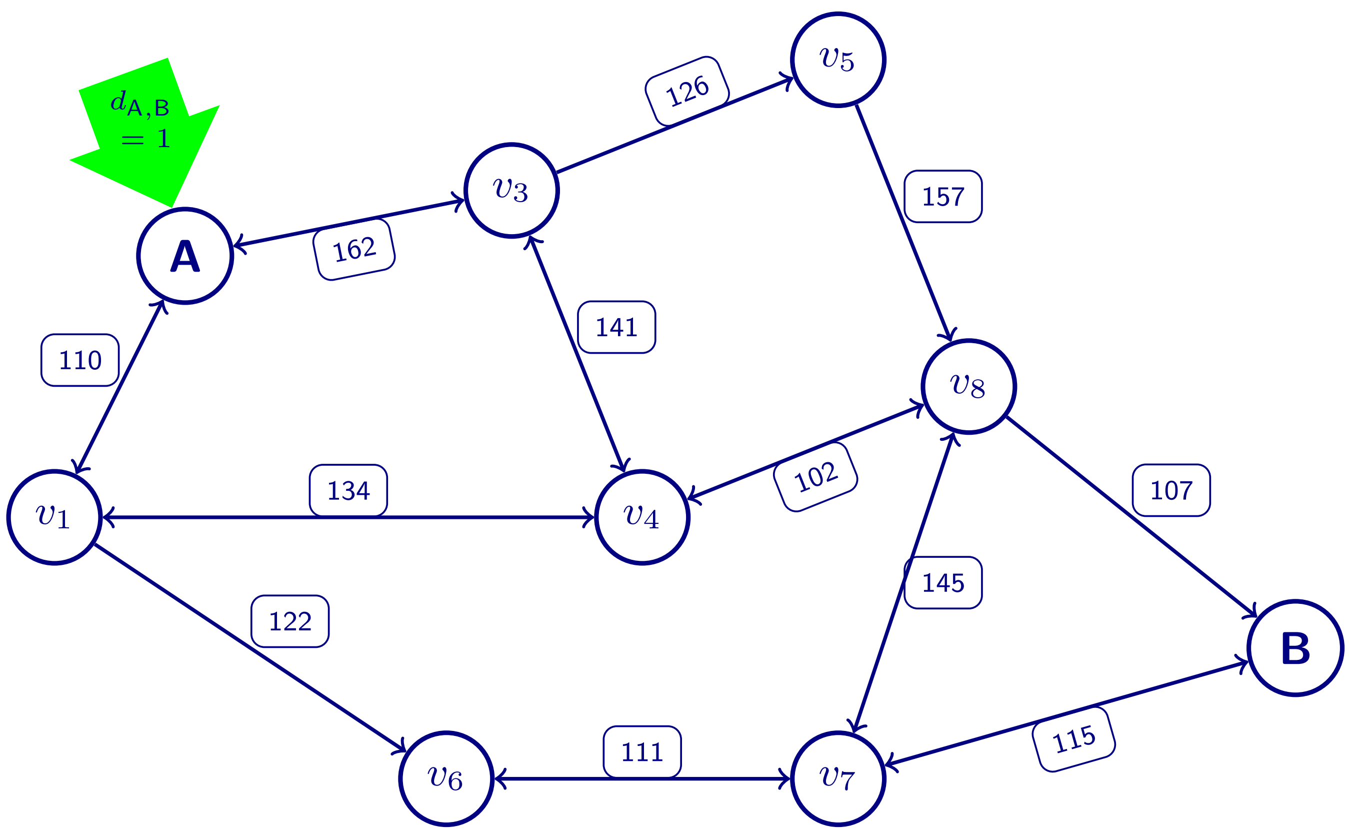

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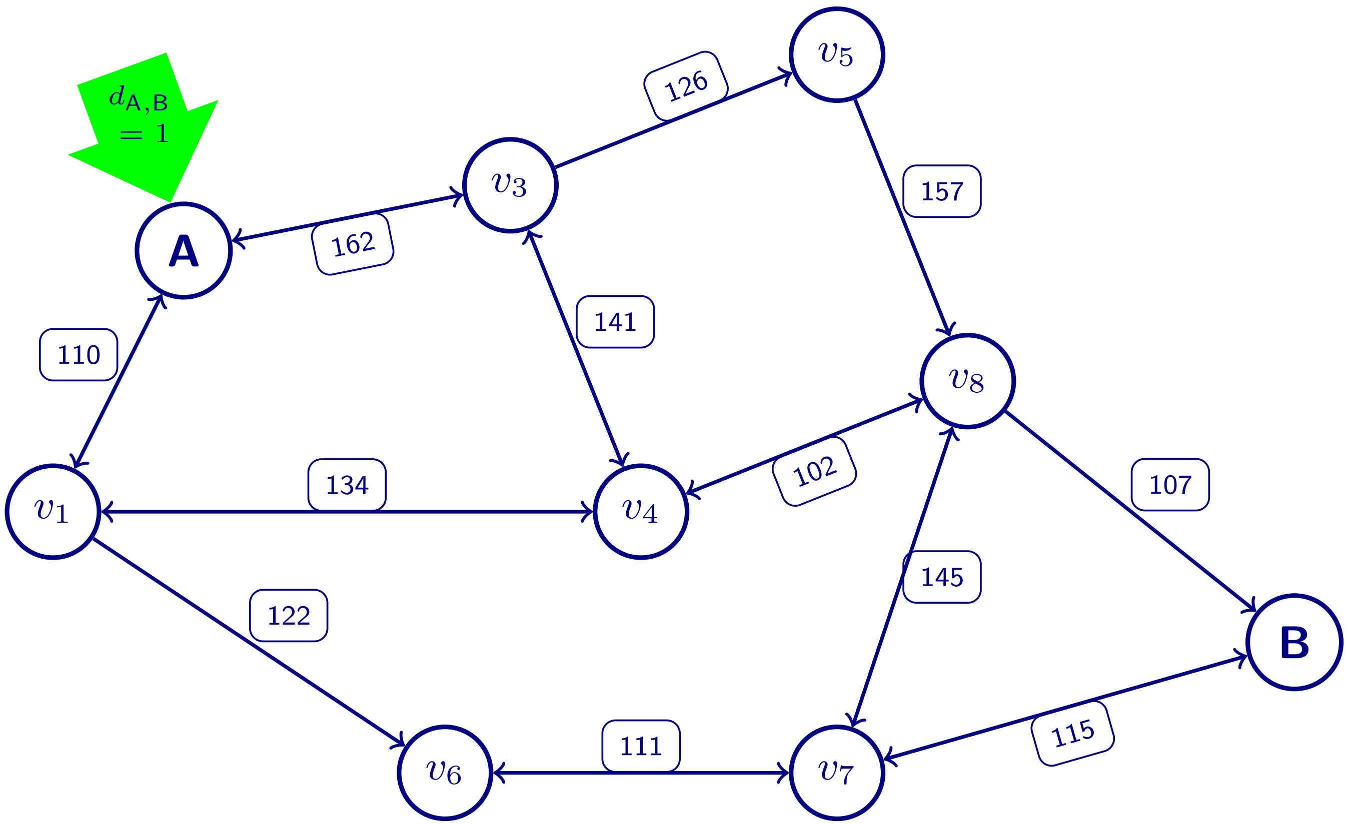

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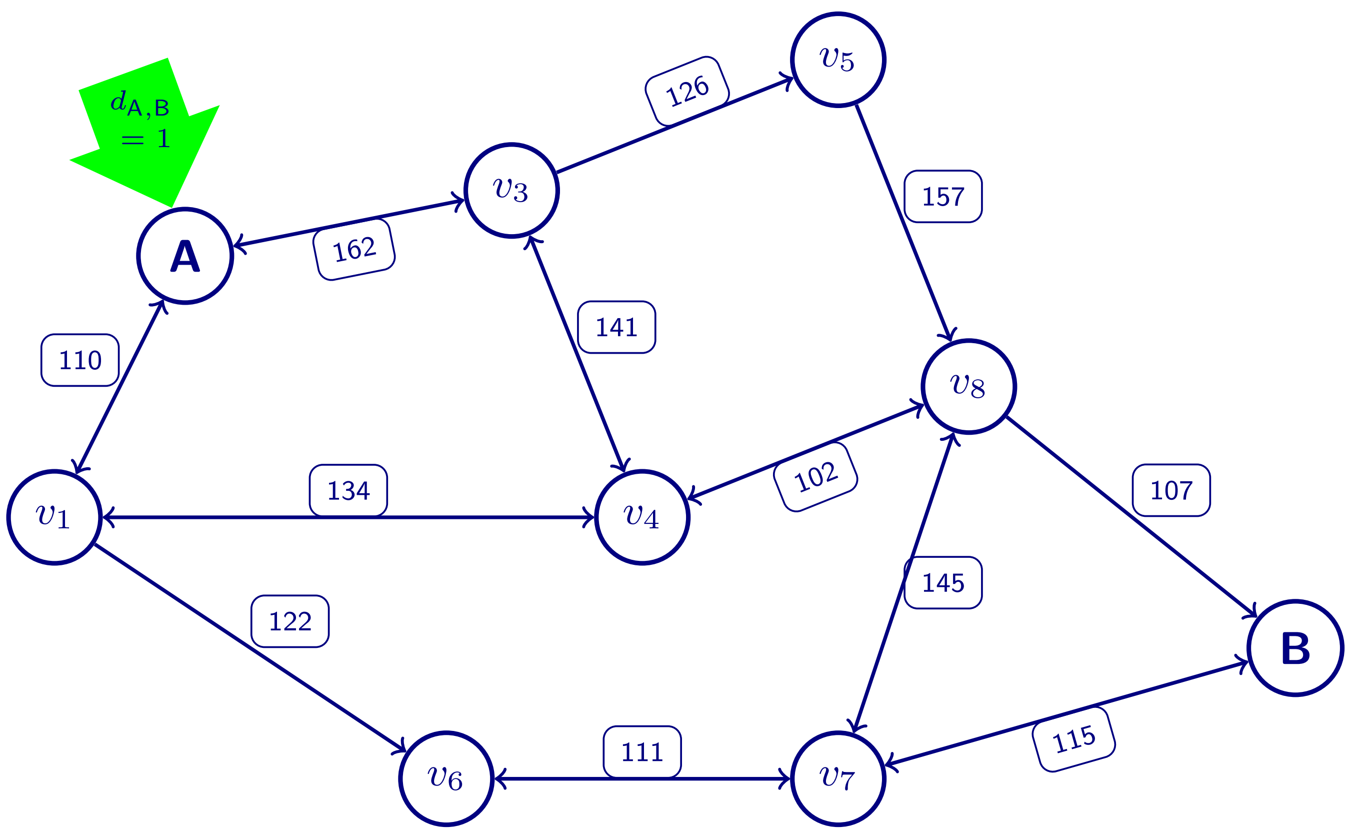

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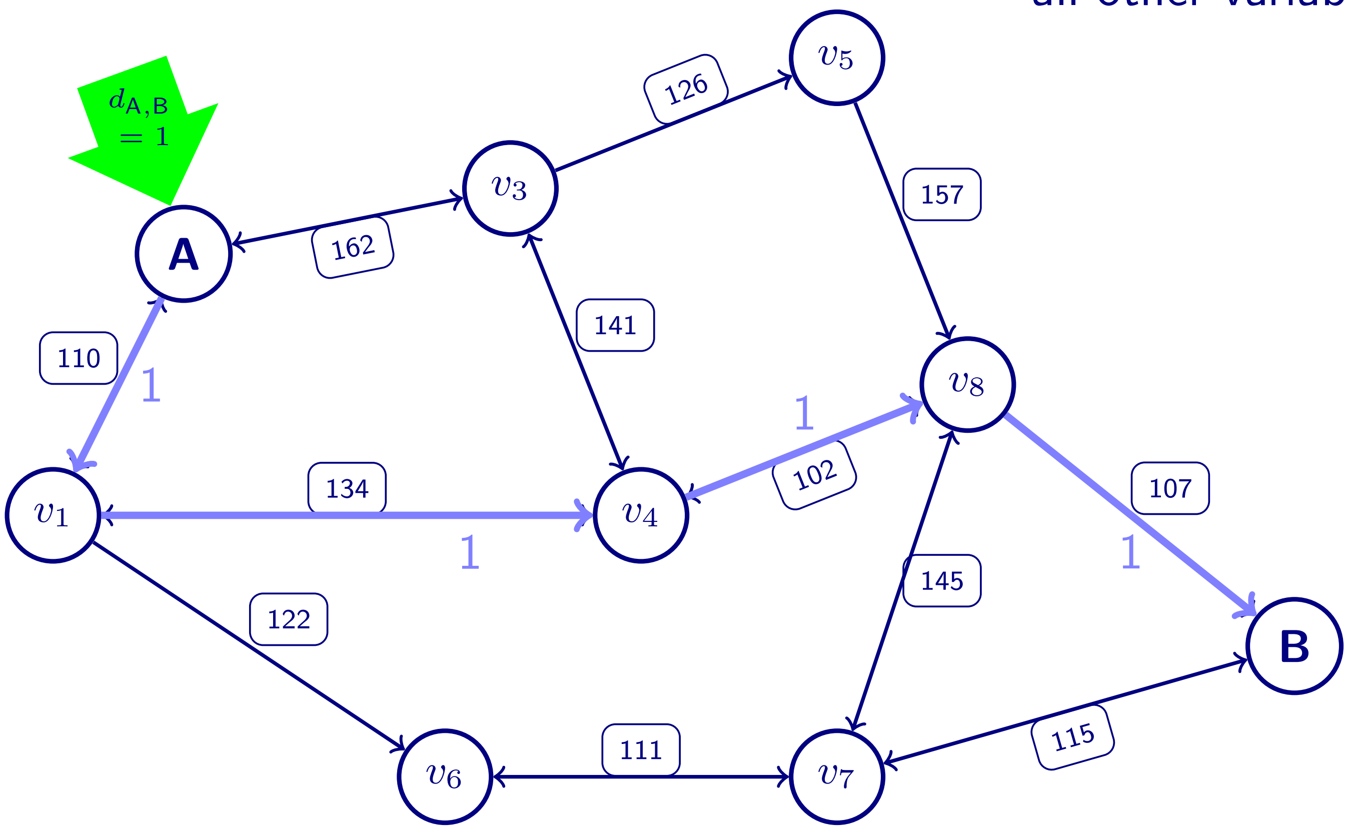

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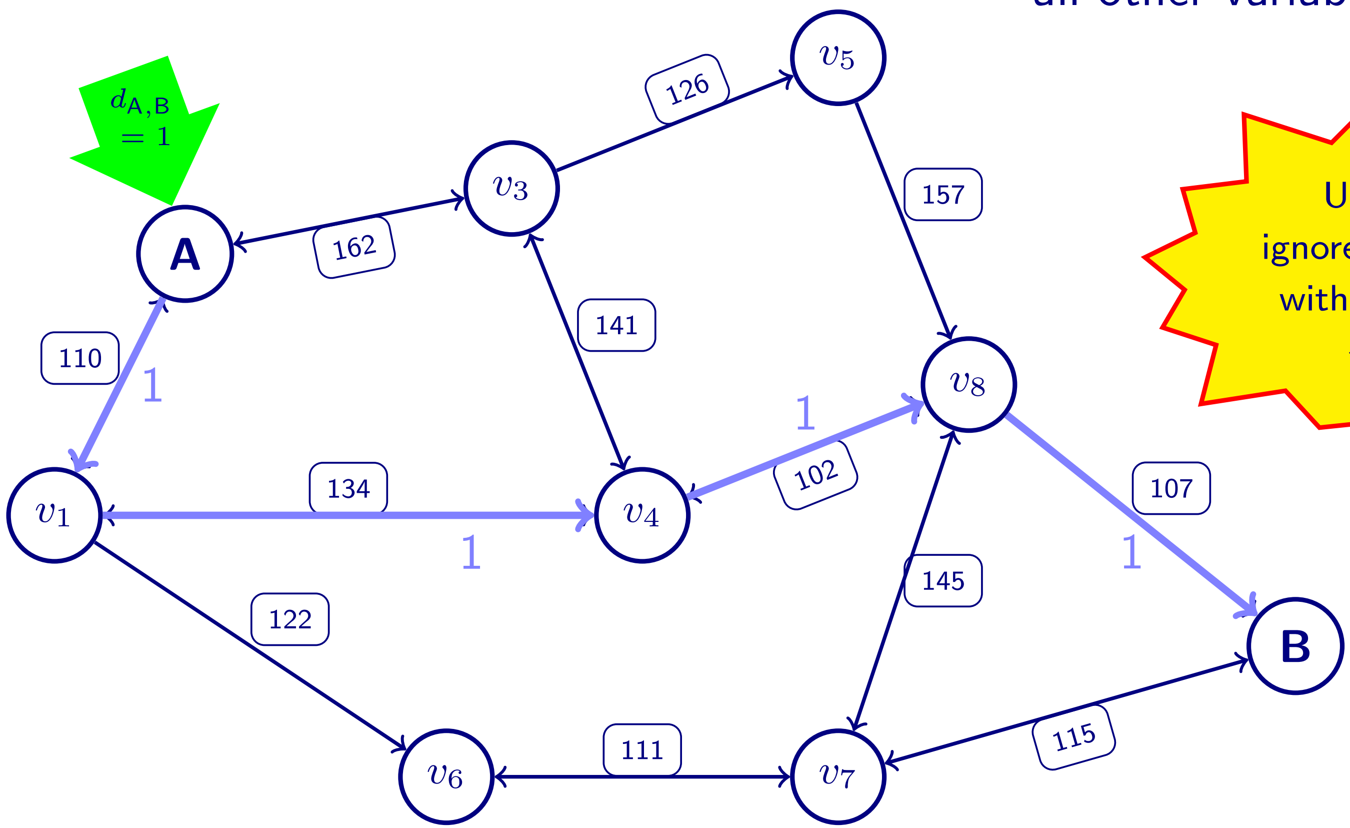

Unsplittability can be ignored if the model is solved with the simplex algorithm → linear program!

- Models, Data and Algorithms
- Linear Optimization
- Mathematical Background: Polyhedra, Simplex-Algorithm
- Sensitivity Analysis; (Mixed) Integer Programming
- MIP Modelling
- MIP Modelling: More Examples; Branch & Bound
- Cutting Planes; Combinatorial Optimization: Examples, Graphs, Algorithms
- TSP-Heuristics
- Network Flows
- Shortest Path Problem, Complexity Theory
- Nonlinear Optimization
- Scheduling, Lot Sizing
- Multicriteria Optimization
- Oral exam