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Mathematical Tools for Engineering and Management
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Exercise sheet 2

The due date of the graded homework (Exercise 3) is November 9, 2011 **before** the exercise session starts. Solutions can also be sent by email to `stephan@math.tu-berlin.de`.

Exercise 1

For the German chemical industry, the import of Ethylen is a key issue: 5.7 Mio metric tons are imported each year. Consider the BASF Company with its production site in Ludwigshafen, Rheinland-Pfalz, Germany. Assume you can ship up to 10.000 t from the Port of Rotterdam to Ludwigshafen. You have the following options to do this: by inland shipping, by railways, or by a combination of these. After subtracting any costs, sending one ton by inland shipping gives you a net benefit of 5 monetary units per ton; using railways give you a net benefit of 4 monetary units per ton. Because of the higher speed of railways, at least 2.000 t must be shipped by railways, in order to start the production in time.

1. Formulate the above problem as a linear optimization model.
2. Solve this problem graphically.
3. Consider the following shipment policy: 5.000 t per ship and 5.000 t per rail. Is this a feasible solution? Does there exist any objective function (i.e. different values for the per ton benefits of the two modes of transport, possibly including "negative benefits") such that this particular shipment policy constitutes an optimal solution?

Exercise 2

(a) A brewery in the Netherlands has two plants located in Haarlem and Eindhoven and five customers in different cities. Truckloads of beer must be shipped from the plants to the customers dur- ing a particular period of time. Both the available supply at each plant and the required demand by each customer (measured in terms of truckloads) are known. The cost associated with moving one truck load from a plant to a customer is also provided. The objective is to make a least-cost plan for mov- ing the beer such that the demand is met and shipments do not exceed the available supply from each brewery.

The following table provides the data for the problem described in the previous paragraph.

Customers	Unit Transport Cost					
Plants	Amsterdam	Breda	Gouda	Amersfoort	Den Bosch	Supply
Haarlem	131	405	188	396	485	47
Eindhoven	554	351	479	366	155	63
Demand	28	16	22	31	12	

Table 1: Input data for beer transport problem

Formulate the above problem as a linear optimization model.

(b) Download the "Tutorial for Beginners" from the AIMMS website and work through it.

Graded Homework

Exercise 3

Campus Liquids Inc. (CLI) is engaged in the production and sale of two kinds of hard liquor. CLI purchases intermediate-stage products in bulk, purifies them by repeated distillation, mixes them, bottles the product under its own brand names and sells it. One product is a bourbon, the other a blended whiskey.

As the company's products are very popular on the market, CLI might sell more liquor than its production capacity allows. The bourbon requires 3 machine hours per bottle, while the blended whiskey requires 4 hours of machine time per bottle. There are 20000 machine hours available in the production period. The direct operating costs, which are mainly for labor and materials, are 3.00€ per bottle of bourbon and 2.00€ per bottle of blended whiskey. The working capital available to finance these costs is 4400€; however, 40% of the sales revenues will be collected during the production period and made available to finance ongoing operations.

The selling price is 5€ for a bottle of bourbon and 4.5€ for a bottle of blended whiskey.

1. (4 points) Set up a linear program in two variables x_1 and x_2 that maximizes CLI's profit in the production period to come, subject to limitations on machine capacity and working capital.

2. (4 points) Sketch the set of feasible solutions in the plane and give the coordinates of the vertices.

3. (2 points) What is the optimal production mix to schedule and how large is the company's profit with this schedule?

4. (4 points) Suppose CLI could spend some money to repair machinery and increase its available machine hours by 2000 hours (before production starts). Should the investment be made and if so, up to which price?
   Hint: How does this change affect the linear program and your sketch?