

## Exercise sheet 3

The due date of the graded homework (Exercise 3) is November 23, 2011 **before** the exercise session starts. Solutions can also be sent by email to [stephan@math.tu-berlin.de](mailto:stephan@math.tu-berlin.de).

### Exercise 1

Consider again the production planning problem for manufacturing Beetles and Cabrios.

$$\begin{array}{ll}
 \max 10000B + 20000C & \text{(Contribution to earnings)} \\
 \text{s.t. } 5B + 3C \leq 50 & \text{(Manufacturing capacity)} \\
 4B + 7C \leq 70 & \text{(Assembly capacity)} \\
 400B + 400C \leq 4500 & \text{(Available steel)}
 \end{array}$$

Assume this model is used to determine a daily plan for a planning horizon of a quarter. For the current quarter the company had contracted with a steel supplier for the delivery of 4500 kg of steel per day. Now suppose that the company is planning for the next quarter, and that they would like to determine how much steel to contract for with local suppliers for the next quarter. Suppose that steel contracts are typically arranged for daily deliveries over the entire quarter, and that the market price for such contracts is \$1100/1000kg of steel.

Defining  $S$  to be the amount of steel to contract for, for the next quarter, in kg/day, the company would like to determine the optimal value of  $S$ . However, for the next quarter you have to incorporate some uncertainties.

- The company has ordered new assembly machines to replace, as well as to augment, their existing assembly machines, but it is not known if these new machines will be delivered in time to be used next quarter. Let us assume that the assembly machine capacity will either be 70h/day (with probability 0.5) or 90h/day (with probability 0.5).
- The unit contribution to earnings of production of cabrios next quarter is uncertain, since a competitor has announced a new cabrio model so that your company is forced to decrease the prices. For simplicity assume that this unit earnings contribution will be either \$20000 (with probability 0.5) or \$14000 (with probability 0.5).

Task: Formulate this production planning problem with uncertainties as a linear program.

### Exercise 2

New Bedford Steel (NBS) is a steel producer located in Bedford, Pennsylvania. Coking coal is a necessary raw material in the production of steel, and NBS procures 1.0-1.5 million tons of coking coal per year. Its now time to plan for next year's production, and the coal supply chain manager for NBS has solicited and received bids from eight potential coal mining companies. Table 1 shows the relevant information on the bids from eight potential coal suppliers. Based on market

	Ashley	Bedford	Consol	Dunby	Earlam	Florence	Gaston	Hopt
Price (\$/ton)	49.50	50.00	61.00	63.50	66.50	71.00	72.50	80.00
Union/Non-Union	Union	Union	Non-Union	Union	Non-Union	Union	Non-Union	Non-Union
Truck/Rail	Rail	Truck	Rail	Truck	Truck	Truck	Rail	Rail
Volatility	15	16	18	20	21	22	23	25
Capacity(mtons/year)	300	600	510	655	575	680	450	490

Table 1: Bids received by NBS from potential coking coal suppliers.

forecasts and last year's production characteristics, NBS is planning to accept bids for 1,225 mtons (1,225,000 tons) of coking coal for the coming year. This coal must have an average volatility of at least 19%. Also, as a hedge against adverse labor relations, NBS has decided to procure at least 50% of its coking coal from union (United Mine Workers) mines. Finally, the coal supply chain manager needs to keep in mind that capacity for bringing in coal by rail is limited to 650 mton per year, and capacity for bringing in coal by truck is limited to 720 mton per year.

- (a) Construct a linear optimization model to determine how much coal NBS should contract from each supplier in order to minimize the cost of supply of coking coal.
- (b) Solve the model using AIMMS.
- (c) What is the cost of coking coal on the margin, i.e., how much does an extra of coking coal cost NBS?
- (d) Should NBS consider expanding their truck/rail capacity? If so, how much should they be willing to spend?
- (e) Should the coal supply chain manager be willing to negotiate a higher price in order to get more coal from Bedford and/or Gaston? If so, how high should he be willing to go?
- (f) NBS currently has a management policy of procuring at least 50% of their coking coal from union mines. How much is this policy costing NBS? Should they amending this policy?

## Graded Homework

### Exercise 3

Nature's Best Frozen Foods company produces four different mixes of frozen ready-to-eat vegetables. The mixes consist of five different vegetables: carrots, mushrooms, green peppers, broccoli, and corn. The company manufactures four different mixes each sold in 10 oz. bags. The mixes are: "Stir Fry", "Barbecue", "Hearty Mushrooms", and "Veggie Crunch", and their contributions to earnings (per bag) are \$0.22, \$0.20, \$0.18, and \$0.18, respectively. The monthly supplies of carrots, mushrooms, green peppers, broccoli, and corn are 150,000 oz., 80,000 oz., 135,000 oz., 140,000 oz., and 150,000 oz. per month, respectively. The compositions of the mixes are shown in Table 2. For example, one bag of "Stir Fry" mix contains 2.5 oz. of carrots, 3.0 oz. of mushrooms, 2.5 oz. of green peppers, 2.0 oz. of broccoli, and no corn. The company can sell all the mixes that they can produce.

- (a) (6 points) Construct a linear optimization model to determine the optimal product mix (i.e., how many bags of each mix to produce in order to maximize the contribution to earnings).
- (b) (4 points) Solve the linear optimization model using AIMMS. Attach a print-out of the optimal solution.
- (c) (2 points) Do you would accept a higher price for an additional charge of broccoli in order to increase your production of the vegetable mixes? If so, which cost increase (w.r.t. the current price) you would accept and how many additional ounces of broccoli you would order?
- (d) (2 points) Do you would accept a higher price for an additional charge of carrots in order to increase your production of the vegetable mixes? If so, which cost increase (w.r.t. the current price) you would accept and how many additional ounces of carrots you would order?

	"Stir Fry"	"Barbecue"	"Hearty Mushrooms"	"Veggie Crunch"
Carrots	2.5	2.0	0.0	2.5
Mushrooms	3.0	0.0	4.0	0.0
Green Peppers	2.5	2.0	3.0	2.5
Broccoli	2.0	3.0	3.0	2.5
Corn	0.0	3.0	0.0	2.5

Table 2: The composition of the products produced by Nature's Best Frozen Foods.