Technische Universität Berlin
Institut für Mathematik

Mathematical Tools for Engineering and Management
Winter Term 2011/2012

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January 11, 2012

Exercise sheet 10

Exercise 1

Look if you can find any relationships between the following problems. Which of them are $\mathcal{NP}$-complete?

1. The SUBSET-SUM-Problem: Given numbers $(a_1, \dots, a_n)$ and a number k, is there a subset S of $\{1, \dots, n\}$ such that $\sum_{s \in S} a_i = k$?
2. The PARTITION-Problem: Given numbers $(a_1, \dots, a_n)$, is there a subset S of $\{1, \dots, n\}$ such that $\sum_{s \in S} a_i = \sum_{s \notin S} a_i$?
3. The 3-PARTITION-Problem: Given numbers $(a_1, \dots, a_n)$ with n a multiple of 3, are there $\frac{n}{3}$ triples in $a_1, \dots, a_n$ which all have the same sum?
4. The KNAPSACK-Problem: Given items $I = (1, \dots, n)$ with weights $w(i)$ and values $v(i)$ and given an upper bound B on the weight, is there a subset S of I with $\sum_{s \in S} w(s) <= B$ and $\sum_{s \in S} v(i) >= K$?
5. The MAXIMUM-BIPARTITE-MATCHING-Problem: Given a bipartite graph $G = (X \cup Y, E)$ with edges only between X and Y and a number k, are there k edges such that each vertex is only adjacent to at most one of them?
6. The MAX-FLOW-Problem: Given a graph $G = (V, E)$ with edge capacities $c(e)$, a start vertex s and a target vertex t and a value k, is there a flow of k units from s to t?
7. The VERTEX-COVER-Problem: Given a graph $G = (V, E)$ and a number k, are there k vertices in G such that every edge has at least one endpoint in one of the chosen k vertices?
8. The STABLE-SET-Problem: Given a graph $G = (V, E)$ and a number k, are there k vertices in G such that no edge exists between any of the the chosen k vertices?
9. The CLIQUE-Problem: Given a graph $G = (V, E)$ and a number k, are there k vertices in G such that there is an edge between every two of the chosen k vertices?
10. The SHORTEST-PATH-Problem: Given a graph $G = (V, E)$ and lengths $l(e)$ for the edges, a start vertex s and a target vertex t and a number k, is there a path from s to t with total length at most k?
11. The SHORTEST-PATH-Problem with nonnegative edge weights: Given a graph $G = (V, E)$ and lengths $l(e) \geq 0$ for the edges, a start vertex s and a target vertex t and a number k, is there a path from s to t with total length at most k?
12. The HAMILTON-PATH-Problem: Given a graph $G = (V, E)$ and a node s, is there a path in G which starts at node s and visits every node exactly once?