Exercise sheet #12

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due on July 14 at the beginning of the lecture

Let K be an algebraically closed field.

Exercise A (20 points)**.**

1. Show that any finite set $S \subset \mathbb{A}^2$ is the zero set of two polynomials.
2. Show that any finite set $S \subset \mathbb{P}^2$ is the zero set of two homogeneous polynomials.

Exercise B (30 points)**.** Let $F \in K[X_0, \dots, X_N]$ be a homogeneous polynomial of degree $m > 0$ and let $Z = V(F)$. Let V be the closed subset of $\mathbb{P}^N \times \mathbb{P}^N$

$$V = \left\{ ([x],[y]) \in \mathbb{P}^N \times \mathbb{P}^N \;\middle|\; \forall \lambda, \mu \in K, F(\lambda x + \mu y) = 0 \right\}.$$

1. Show that $\dim V \geq 2N - m - 1$.

Let $\Delta = \left\{ (x,x) \;\middle|\; x \in \mathbb{P}^N \right\}$.

2. Show that $\dim(\Delta \cap V) = N - 1$.
3. Show that if $m < N$, then Z contains a projective line.
4. Show that if $m < N$, then Z contains infinitely many projective lines.