Exercise sheet #7

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due on June 9 at the beginning of the lecture

Exercise A (10 points).

1. Show that every regular map $\mathbb{P}^1 \to \mathbb{A}^1$ is constant.
2. Let $n \geq 1$ be an integer. Show that every regular map $\mathbb{P}^n \to \mathbb{A}^1$ is constant. *(Hint: any two points in $\mathbb{P}^n$ are connected by a projective line.)*

Exercise B (10 points). Let $n, d \geq 1$ be integers. Let $f \in \mathbb{C}[X_1, \ldots, X_n]$ be a polynomial of degree d. The *homogenization* of f, denoted $\bar{f}$, is the polynomial

$$\bar{f}(X_0, X_1, \ldots, X_n) \overset{\text{def}}{=} X_0^d f\left(\frac{X_1}{X_0}, \ldots, \frac{X_n}{X_0}\right).$$

Let $Z = V(f) \subset \mathbb{A}^n$. Let i be the inclusion $(X_1, \ldots, X_n) \in \mathbb{A}^n \mapsto [1 : X_1 : \cdots : X_n] \in \mathbb{P}^n$. Show that $\overline{i(Z)} = V(\bar{f})$. *(Hint: consider first the case where f is irreducible.)*

Exercise C (10 points). Let $f \in \mathbb{C}[X_0, \ldots, X_n]$ be a homogeneous polynomial of positive degree. Show that $\mathbb{P}^n \setminus V(f)$ is isomorphic to an affine variety. *(Hint: use the Veronese embedding, Exercise C of Sheet #6.)*