# Exercise sheet #9

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**Exercise 1.** Let $p > 2$ be a prime number. Let $\alpha$ be a primitive 8th root of unity in $\overline{\mathbb{F}_p}$.
Let $\beta = \alpha + \alpha^{-1}$.

1. Show that $\beta^2 = 2$.
2. Show that $\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8}$.

**Exercise 2.** Let $p$ be a prime number such that $p = x^2 - 6y^2$, for some integers $x$ and $y$. Show that $p$ is congruent to 1, 5, 19 or 23 modulo 24.
*Hint:* $\left(\frac{6}{p}\right)$.

**Exercise 3.** We may extend the definition of the Legendre symbol in the following way: for any integers $a$ and $n \geqslant 1$ we define

$$\left(\frac{a}{n}\right) = \prod_{i=1}^{r} \left(\frac{a}{p_i}\right),$$

where $n = \prod_{i=1}^{r} p_i$ is the prime factorization of $n$ and where $\left(\frac{a}{p_i}\right)$ denotes the Legendre symbol. This new symbol is called *Jacobi symbol*.

1. Show that $\left(\frac{-1}{n}\right) = (-1)^{(n-1)/2}$.
2. Show that $\left(\frac{2}{n}\right) = (-1)^{(n^2-1)/8}$.
3. Show that for all coprime odd integers $a$ and $b$,

$$\left(\frac{a}{b}\right) = \left(\frac{b}{a}\right) (-1)^{(a-1)(b-1)/4}.$$

4. Compute $\left(\frac{1003}{10007}\right)$.