

Algebraic Geometry I

4h lecture at TU Berlin, WS 2017/18, Prof. P. Bürgisser

The course mainly followed the first half of the script “Algebraic Geometry” by Andreas Gathmann (2002/2003), but there were some deviations and some additional material was covered.

Chap. 0. What is algebraic geometry?

Chap. 1. Affine varieties

- 1.1 Algebraic sets and the Zariski topology
- 1.2 Hilbert’s Nullstellensatz
- 1.3 Irreducibility and dimension

Chap. 2. Functions, morphisms, and varieties

- 2.1 Functions on affine varieties
- 2.2 Sheaves
- 2.3 Morphisms between affine varieties
- 2.4 Prevarieties
- 2.5 Varieties

Chap. 3. Projective varieties

- 3.1 Projective spaces and projective varieties
- 3.2 Cones and the projective Nullstellensatz
- 3.3 Projective varieties as ringed spaces
- 3.4 The main theorem on projective varieties

Chap. 4. Dimension

- 4.1 The dimension of projective varieties

The rest of this chapter deviates from Gathmann’s script.

- 4.2 Algebraic intermezzo
 - (a) On Noether normalization
 - (b) Norm of field extensions
 - (c) Integral ring extension

- 4.2 The dimension of varieties
- 4.3 The structure of morphisms
- 4.4 Birational equivalence

Chap. 5. Tangent spaces and derivations

- 5.1 Tangent spaces
- 5.2 Regular points
- 5.3 Nonsingularity of fibres
- 5.4 Projective embeddings of smooth varieties

Chap. 6. Schemes

This chapter was only outlined; no detailed proofs were given.

- 6.1 Affine schemes
- 6.2 Morphisms and locally ringed spaces
- 6.3 Schemes and prevarieties
- 6.4 Fiber products (omitted)
- 6.5 Projective schemes

Chap. 7. First applications of scheme theory

- 7.1 Hilbert polynomials
- 7.2 Bézout's theorem