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Exercises for Algebraic Geometry 1

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Exercise sheet 1

Try to really prove Exercises 1 and 2.
Exercises 3 and 4 are more of philosophical nature and "food for thought".

Exercise 1. Consider the following curve in $\mathbb{C}^3$:

$$C := \{(t^3, t^4, t^5) \mid t \in \mathbb{C}\}.$$

(1) Show that $C = \{(x, y, z) \in \mathbb{C}^3 \mid x^3 = yz,\ y^2 = xz,\ z^2 = x^2y\}$.

(2) Show that none of the 3 equation in (1) can be omitted!

Exercise 2. Consider the set X of all 2×3-matrices of rank at most 1, viewed as a subset of the $\mathbb{C}^6$ of all 2×3-matrices. Show that X has dimension 4, but that you need 3 equations to define X in the ambient 6-dimensional space $\mathbb{C}^6$.

Exercise 3. (1) Show that the space of lines in $\mathbb{C}^n$ has dimension $2n - 2$.

(2) Let $S \subset \mathbb{C}^3$ be a cubic surface, i.e., the zero locus of a polynomial of degree 3 in the three coordinates of $\mathbb{C}^3$. Why would you expect there to be finitely many lines in S (i.e., why would you expect the dimension of the space of lines in S to be 0-dimensional?)

Exercise 4. (1) What can you say about the *genus* of

$$C'_n := \{(x, y) \in \mathbb{C}^2 \mid y^2 = (x-1)(x-2)\cdots(x-(2n-1))\}?$$

(2) In the lecture, we argued that a polynomial of degree d in two complex variables gives rise to a surface of genus $\binom{d-1}{2}$. We have also seen that the polynomial $y^2 = (x-1)(x-2)\cdots(x-(2n))$ gives us a surface of genus $n-1$. Is this not a contradiction?